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Notes on the Comparative Study of the Reliability Estimation for Standby System with Rayleigh Lifetime Distribution

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Abstract

We shall propose maximum likelihood, Bayesian and generalized maximum likelihood estimation for the reliability of the two-unit hot standby system with Rayleigh lifetime distribution that switch is perfect. Each estimation will be compared numerically in terms of various mission times, parameter values and asymptotic relative efficiency through Monte Carlo simulation.

Keywords: Asymptotic relative efficiency, Bayesian estimation, Generalized maximum likelihood, Maximum likelihood, Standby system

1. Introduction

The two-unit standby redundant system configuration is a form of paralleling where only one component is in operation; if the operating component fails, the another component is brought into operation, and the redundant configuration continues to function. Depending failure characteristic, standby redundancy is classified into three types. Hot standby system, where each component has the same failure rate regardless of whether it is standby or in operation; Cold standby system, where components do not fail when they are in standby; Warm standby system, where a standby component can fail but it has a smaller failure rate than the principal component.

Reliability computations for a two-unit standby redundant systems with constant

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failure rate are found by Osaki and Nakagawa(1971). Fujii and Sandoh(1984) considered the Bayesian estimation for reliability of a two-unit hot standby redundant system. Kapur and Garg(1990) considered the technique of Markov renewal process to obtain various reliability measures for a two-unit standby system with perfect switch and Shen and Xie(1991) considered the effect of standby redundancy at the system and the component level. Oh and Berger(1992) suggested the adaptive sampling in Monte Carlo integration. Kim(2003) considered the comparative study of the reliability estimation for standby system with perfect switch under exponential lifetime component.

The classical statistical estimation procedure, for example maximum likelihood estimation, have been applied to many situations. But recently there are many cases in which the Bayesian methods and generalized maximum likelihood estimation are frequently used. The main contribution of this paper is to propose some Bayesian estimators and generalized maximum likelihood estimators and to compare them with maximum likelihood estimator in the sense of asymptotic relative efficiency(ARE) for the reliability of standby system.

In this paper, we shall find maximum likelihood estimator(MLE), generalized maximum likelihood estimator(GMLE) and Bayesian estimator for reliability of a two-unit hot standby system with Rayleigh lifetime distribution under perfect switch. Also we shall compare these estimators by ARE of GMLE and Bayesian estimator for MLE through generating random number of the proposed estimators and numerical integration.

2. Reliability for Standby System

We consider an Rayleigh distribution of lifetime governed by the probability density function

$$f(t \mid \beta) = \begin{cases} \beta t e^{-\frac{1}{2}\beta t^2}, & 0 \le t,\ 0 < \beta \\ 0, & \text{otherwise.} \end{cases} \tag{2.1}$$

The Rayleigh distribution was first presented by Rayleigh(1919). It has been frequently used as a model with increasing hazard function of the form βt, $\beta > 0$. Here we shall consider the reliability estimation for the two-unit hot standby system with perfect switch in a Rayleigh distribution. The assumptions are:

1. The system consists of two independent and identically distributed units and a switch.
2. One unit serves as a hot standby when the other is in use.
3. The switch is instantaneous when the one in use fails.
4. The times to failure of both units in use and standby are independent and Rayleigh distributed with Rayleigh slope β.

5. The unit and the switch are independent.
6. The switch is failure free.

Then the reliability for a two-unit hot standby system at specified mission time t_o is given by

$$R(t_0 \mid \beta) = e^{-\frac{1}{2}\beta t_0^2}\left(1 + \frac{1}{2} t_0\sqrt{\pi\beta} \cdot e^{\frac{1}{4}\beta t_0^2} \cdot erf(\frac{1}{2} t_0\sqrt{\beta})\right), \tag{2.2}$$

where

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

3. The Method of Reliability Estimation

3.1 The Method of Maximum Likelihood Estimation

Let $T_1, \cdots, T_n$ be a simple random sample from a Rayleigh distribution with Rayleigh slope β and $G = \sum_{i=1}^{n} T_i^2$. If the time G is accumulated on all items including those that failed and those that did not fail prior to test termination at the given mission time t_0. Then the G is as follows;

$$G = \sum_{i=1}^{F} T_i^2 + (n - R)t_0^2, \tag{3.1}$$

where F is the number of failures.

In this case MLE for the Rayleigh slope β is as follows;

$$\widehat{B} = \frac{2n}{G}. \tag{3.2}$$

By the invariance property of MLE, the MLE of standby system is as follows;

$$\widehat{R_M}(t_0) = e^{-\frac{1}{2}\hat{B}t_0^2}\left(1 + \frac{1}{2}t_0\sqrt{\pi\hat{B}} \cdot e^{\frac{1}{4}\hat{B}t_0^2} \cdot erf(\frac{1}{2}t_0\sqrt{\hat{B}})\right) \tag{3.3}$$

3.2 The Method of Bayesian Estimation

Now we shall consider Bayesian estimation of reliability (2.2) under the squared error loss. Let the random variable of Rayleigh slope β be B with prior probability density function(p.d.f.) $\pi(\beta)$. Then the Bayesian estimator $\widetilde{R}(t_0)$ of

$R(t_0)$ is posterior mean because the loss function is squared error loss.

First we assume that B has an uniform distribution U(0, α) with p.d.f.

$$\pi_U(\beta \mid \alpha) = \begin{cases} \frac{1}{\alpha}, & 0 < \beta < \alpha \\ 0, & \text{otherwise}. \end{cases} \tag{3.4}$$

Then the posterior p.d.f. of B given the time G is

$$g_U(\beta \mid t, \alpha) = \left(\frac{G}{2}\right)^{n+1} \cdot \frac{\beta^n e^{-\frac{1}{2}\beta G}}{\Gamma(n+1, \alpha G/2)}, \; 0 < \beta < \alpha, \tag{3.5}$$

where $\Gamma(a, z)$ represents the standard incomplete gamma function.

Hence the Bayesian estimator $\widetilde{R_U}(t_0)$ for the system reliability $R(t_0)$ is

$$\widetilde{R_U}(t_0) = \frac{1}{\Gamma(n+1, \alpha G/2)} \left(\frac{G}{2}\right)^{n+1} (I_{U1} + I_{U2}), \tag{3.6}$$

where

$$I_{U1} = \Gamma(n+1, \alpha(t_0 + G)/2)) \left(\frac{2}{t_0 + G}\right)^{n+1}$$

and

$$I_{U2} = \frac{1}{2} t_0 \sqrt{\pi} \int_0^{\alpha} \beta^n e^{-\frac{1}{4}\beta(2G + 2t_0 - t_0^2)} erf(\frac{1}{2} t_0 \sqrt{\beta}) d\beta.$$

Second we assume that B has a gamma distribution GAM(b, a) with p.d.f.

$$\pi_G(\beta \mid b, a) = \frac{1}{\Gamma(a) b^a} \beta^{a-1} e^{-\beta/b}, \tag{3.7}$$

where $0 < \beta < \infty$ and a is a positive integer.

Then the posterior p.d.f. of B given the time G is

$$g_G(\beta \mid t) = \frac{1}{\Gamma(n+a)\left(\frac{2b}{bG+2}\right)^{n+a}} \beta^{n+a-1} e^{-\beta\left(\frac{bG+2}{2b}\right)}. \tag{3.8}$$

Hence the Bayesian estimator $\widetilde{R_G}(t_0)$ for the system reliability $R(t_0)$ is

$$\widetilde{R_G}(t_0) = \frac{((bG+2)/2b)^{n+a}}{\Gamma(n+a)} (I_{G1} + I_{G2}) \tag{3.9}$$

where

$$I_{G1} = \Gamma(n+a) \cdot \left(\frac{2b}{bt_0 + bG + 2}\right)^{(n+a)}$$

and

$$I_{G2} = \frac{1}{2} t_0 \sqrt{\pi} \int_0^{\infty} \beta^{n+a-1/2} e^{-\frac{1}{2}\beta(G + t_0/2 + 2/b)} erf\left(\frac{1}{2} t_0 \sqrt{\beta}\right) d\beta.$$

Third we assume that B has an inverted gamma distribution IGAM(a, b) with p.d.f.

$$\pi_{IG}(\beta \mid a, b) = \frac{b^a}{\Gamma(a)}\left(\frac{1}{\beta}\right)^{a+1} e^{-b/\beta}, \quad a, b, \beta > 0. \tag{3.10}$$

Then the posterior distribution of B given the time G is

$$g_{IG}(\beta \mid t) = \frac{2^{-(n-a+2)/2}\beta^{n-a-1}e^{-(bG/2+b/\beta)}}{(bG^{-1})^{(n-a)/2}K(n-a,\sqrt{2bG})}, \tag{3.11}$$

where $K(n, x)$ is a modified Bessel function of the second kind of order n.

Hence the Bayesian estimator $\widetilde{R_{IG}}(t_0)$ for the system reliability $R(t_0)$ is

$$\widetilde{R_{IG}}(t_0) = \left(\frac{bG}{t_0+G}\right)^{(n+a)/2} \cdot \frac{K(n-a, \sqrt{2b(t_0+G)})}{K(n-a, \sqrt{2bG})} + \delta_n \cdot I_{IG} \tag{3.12}$$

where

$$\delta_n = \frac{2^{-(n-a+2)/2}(G/b)^{(n-a)/2}}{K(n-a,\sqrt{2bG})}$$

and

$$I_{IG} = \frac{1}{2}\, t_0\sqrt{\pi}\int_0^{\infty} \beta^{n-a-1/2} \cdot e^{-\frac{1}{4}(t_0+2G)\beta} \cdot erf(\frac{1}{2}\, t_0\sqrt{\beta})d\beta.$$

Fourth we assume that B has a truncated gamma distribution TGAM(b, a, β_T) with p.d.f.

$$\pi_{TG}(\beta \mid b, a, \beta_T) = \frac{1}{\Gamma(a, \beta_T/b)b^a}\,\beta^{a-1}e^{-\beta/b}. \tag{3.13}$$

Then the posterior distribution of B given the time G is

$$g_{TG}(\beta \mid t) = \frac{\beta^{n+a-1}e^{-\beta/(\frac{2b}{bG+2})}}{\Gamma\left(n+a, \beta_T/(\frac{2b}{bG+2})\right) \cdot \left(\frac{2b}{bG+2}\right)^{n+a}}. \tag{3.14}$$

Hence the Bayesian estimator $\widetilde{R_{TG}}(t_0)$ for the system reliability is

$$\widetilde{R_{TG}}(t_0) = \frac{1}{\Gamma\left(n+a, \beta_T/(\frac{2b}{bG+2})\right)\left(\frac{2b}{bG+2}\right)^{n+a}} \cdot (I_{TG1} + I_{TG2}) \tag{3.15}$$

where

$$I_{TG1} = \Gamma\left(n+a, \beta_T/(\frac{2b}{bG+bt_0+2})\right) \cdot \left(\frac{2b}{bG+bt_0+2}\right)^{(n+a)}$$

and

$$I_{TG2} = \frac{1}{2}\, t_0^2\sqrt{\pi}\int_0^{\beta_T} \beta^{n+a-1/2} \cdot e^{-\beta((bG+bt_0/2+2)/2b)} erf(\frac{1}{2}\, t_0\sqrt{\beta})d\beta.$$

3.3 The Method of Generalized Maximum Likelihood Estimation

Now we shall consider GMLE of reliability (2.2). Let the random variable of Rayleigh slope failure rate β be B with prior p.d.f. $\pi(\beta)$. Then GMLE $\widehat{R_{GMLE}}(t_0)$ of $R(t_0)$ is MLE that is replaced β by $\widehat{B}$, which maximizes the posterior distribution $g(\beta)$ in reliability (2.2).

First we assume that B has an uniform distribution U(0, α). Then MLE of B is the same as the $\hat{B}$ in (3.2). Hence GMLE $\widehat{R_U}(t_0)$ of the system reliability $R(t_0)$ is the same as the MLE $\widehat{R_M}(t_0)$.

Second we assume that B has a gamma distribution GAM(b, a).

Then MLE of B is

$$\widehat{B_G} = \frac{2b(n+a-1)}{bG+2}. \tag{3.16}$$

Hence GMLE $\widehat{R_G}(t_0)$ for the system reliability $R(t_0)$ is

$$\widehat{R_G}(t_0) = e^{-\frac{1}{2}\hat{B}_G t_0^2}\left(1 + \frac{1}{2}t_0\sqrt{\pi\widehat{B_G}} \cdot e^{\frac{1}{4}\hat{B}_G t_0^2} \cdot erf\left(\frac{1}{2}t_0\sqrt{\widehat{B_G}}\right)\right) \tag{3.17}$$

Third we assume that B has an inverted gamma distribution IGAM(a, b).

Then MLE of B is

$$\widehat{B_{IG}} = ((n-a-1) + \sqrt{(n-a-1)^2 - 2bG})/G. \tag{3.18}$$

Hence GMLE$\widehat{R_{IG}}(t_0)$ for the system reliability $R(t_0)$ is

$$\widehat{R_{IG}}(t_0) = e^{-\frac{1}{2}\hat{B}_{IG} t_0^2}\left(1 + \frac{1}{2}t_0\sqrt{\pi\widehat{B_{IG}}} \cdot e^{\frac{1}{4}\hat{B}_{IG} t_0^2} \cdot erf\left(\frac{1}{2}t_0\sqrt{\widehat{B_{IG}}}\right)\right) \tag{3.19}$$

Fourth we assume that B has a truncated gamma distribution TGAM(b, a, β_T).

Then MLE of B is the same as the $\widehat{B_G}$ in (3.16).

Hence GMLE $\widehat{R_{TG}}(t_0)$ of the system reliability $R(t_0)$ is the same as the GMLE .$\widehat{R_G}(t_0)$

4. Numerical Examples and Conclusion

Tables 1.1 through 4.3 show the simulated values for the asymptotic relative efficiency(ARE) of the proposed reliability estimators for MLE under the two-unit hot standby system with perfect switch when $\beta=(0.2, 0.4, 0.6, 0.8, 1.0, 1.2)$, $t_0=(0.5, 0.7, 1.0)$, various parameter values of the prior distribution, the sample size n=30, and simulations were replicated 500 times. We can know from the Table 1.1 through 1.3, GMLE with respect to a uniform prior distribution on Rayleigh slope β is more efficient than the Bayesian estimator about the given mission time t_0 and parameter value β.

We can know from the Table 2.1 through 2.3, GMLE with respect to a gamma prior distribution is more efficient than the Bayesian estimator as the mission time t_0 increase and the parameter β of the gamma prior distribution decrease except large t_0 and small β together. We can know from the Table 3.1 through 3.3, GMLE with respect to a incomplete gamma prior distribution is more efficient than the Bayesian estimator about the given mission time t_0 and the parameter value β. We can know from the Table 4.1 through 4.3, GMLE with respect to the truncated gamma prior distribution is more efficient than the Bayesian estimator about the given mission time t_0 and parameter value β. Also the another Bayesian method such that the noninformative or nonparametric approach are remained for future works.

[Table 1.1] The simulated ARE's of GMLE $\widehat{R_U}(t_0)$ and Bayesian estimator $\widetilde{R_u}(t_0)$ for MLE $\widehat{R_M}(t_0)$ on system reliability under the U(0,α) prior on β when α=3

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_U}$, $\widehat{R_M}$)	ARE ($\widetilde{R_U}$, $\widehat{R_M}$)	ARE ($\widehat{R_U}$, $\widehat{R_M}$)	ARE ($\widetilde{R_U}$, $\widehat{R_M}$)	ARE ($\widehat{R_U}$, $\widehat{R_M}$)	ARE ($\widetilde{R_U}$, $\widehat{R_M}$)
0.2	1.0000	0.0584	1.0000	0.0436	1.0000	0.0544
0.4	1.0000	0.0644	1.0000	0.0510	1.0000	0.0685
0.6	1.0000	0.0695	1.0000	0.0574	1.0000	0.0824
0.8	1.0000	0.0744	1.0000	0.0635	1.0000	0.0938
1.0	1.0000	0.0791	1.0000	0.0696	1.0000	0.1037
1.2	1.0000	0.0852	1.0000	0.0763	1.0000	0.1163

[Table 1.2] The simulated ARE's of GMLE $\widehat{R_U}(t_0)$ and Bayesian estimator $\widetilde{R_U}(t_0)$ for MLE $\widehat{R_M}(t_0)$ on system reliability under the U(0,α) prior on β when α=5.

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_U}, \widehat{R_M}$)	ARE ($\widetilde{R_U}, \widehat{R_M}$)	ARE ($\widehat{R_U}, \widehat{R_M}$)	ARE ($\widetilde{R_U}, \widehat{R_M}$)	ARE ($\widehat{R_U}, \widehat{R_M}$)	ARE ($\widetilde{R_U}, \widehat{R_M}$)
0.2	1.0000	0.0301	1.0000	0.0275	1.0000	0.0532
0.4	1.0000	0.0321	1.0000	0.0306	1.0000	0.0665
0.6	1.0000	0.0341	1.0000	0.0330	1.0000	0.0763
0.8	1.0000	0.0357	1.0000	0.0347	1.0000	0.0831
1.0	1.0000	0.0377	1.0000	0.0369	1.0000	0.0884
1.2	1.0000	0.0392	1.0000	0.0391	1.0000	0.0920

[Table 1.3] The simulated ARE's of GMLE $\widehat{R_U}(t_0)$ and Bayesian estimator $\widetilde{R_U}(t_0)$ for MLE $\widehat{R_M}(t_0)$ on system reliability under the (0,α) prior on β when α=10.

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_U}, \widehat{R_M}$)	ARE ($\widetilde{R_U}, \widehat{R_M}$)	ARE ($\widehat{R_U}, \widehat{R_M}$)	ARE ($\widetilde{R_U}, \widehat{R_M}$)	ARE ($\widehat{R_U}, \widehat{R_M}$)	ARE ($\widetilde{R_U}, \widehat{R_M}$)
0.2	1.0000	0.0187	1.0000	0.0259	1.0000	0.0532
0.4	1.0000	0.0196	1.0000	0.0283	1.0000	0.0664
0.6	1.0000	0.0203	1.0000	0.0299	1.0000	0.0762
0.8	1.0000	0.0209	1.0000	0.0317	1.0000	0.0834
1.0	1.0000	0.0217	1.0000	0.0327	1.0000	0.0877
1.2	1.0000	0.0224	1.0000	0.0337	1.0000	0.0902

[Table 2.1] The simulated ARE's of GMLE $\widehat{R_G}(t_0)$ and Bayesian estimator $\widetilde{R_G}(t_0)$ for MLE $\widehat{R_M}(t_0)$ on system reliability under the GAM(b, 10) prior on β when b=0.5.

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_G}, \widehat{R_M}$)	ARE ($\widetilde{R_G}, \widehat{R_M}$)	ARE ($\widehat{R_G}, \widehat{R_M}$)	ARE ($\widetilde{R_G}, \widehat{R_M}$)	ARE ($\widehat{R_G}, \widehat{R_M}$)	ARE ($\widetilde{R_G}, \widehat{R_M}$)
0.2	1.7548	0.0232	0.9512	0.0309	0.6586	0.0557
0.4	1.7782	0.0254	0.9669	0.0365	0.6661	0.0778
0.6	1.8109	0.0276	0.9861	0.0420	0.6729	0.1008
0.8	1.8384	0.0293	1.0031	0.0467	0.6805	0.1250
1.0	1.8819	0.0315	1.0256	0.0517	0.6835	0.1475
1.2	1.9212	0.0332	1.0455	0.0558	0.6920	0.1724

[Table 2.2] The simulated ARE's of GMLE $\widehat{R_G}(t_0)$ and Bayesian estimator $\widetilde{R_G}(t_0)$ for MLE $\widehat{R_M}(t_0)$ on system reliability under the GAM(b, 10) prior on β when b=1.0.

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_G}$, $\widehat{R_M}$)	ARE ($\widetilde{R_G}$, $\widehat{R_M}$)	ARE ($\widehat{R_G}$, $\widehat{R_M}$)	ARE ($\widetilde{R_G}$, $\widehat{R_M}$)	ARE ($\widehat{R_G}$, $\widehat{R_M}$)	ARE ($\widetilde{R_G}$, $\widehat{R_M}$)
0.2	0.9283	0.0200	0.6570	0.0273	0.5401	0.0509
0.4	0.9374	0.0218	0.6638	0.0323	0.5408	0.0703
0.6	0.9460	0.0234	0.6677	0.0363	0.5380	0.0892
0.8	0.9538	0.0248	0.6736	0.0406	0.5314	0.1068
1.0	0.9618	0.0261	0.6789	0.0446	0.5224	0.1231
1.2	0.9744	0.0277	0.6800	0.0474	0.5145	0.1395

[Table 2.3] The simulated ARE's of GMLE $\widehat{R_G}(t_0)$ and Bayesian estimator $\widetilde{R_G}(t_0)$ for MLE $\widehat{R_M}(t_0)$ on system reliability under the GAM(b, 10) prior on β when b=2.0.

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_G}$, $\widehat{R_M}$)	ARE ($\widetilde{R_G}$, $\widehat{R_M}$)	ARE ($\widehat{R_G}$, $\widehat{R_M}$)	ARE ($\widetilde{R_G}$, $\widehat{R_M}$)	ARE ($\widehat{R_G}$, $\widehat{R_M}$)	ARE ($\widetilde{R_G}$, $\widehat{R_M}$)
0.2	0.6486	0.0185	0.5395	0.0258	0.4876	0.0488
0.4	0.6520	0.0201	0.5422	0.0302	0.4847	0.0663
0.6	0.6551	0.0214	0.5435	0.0341	0.4775	0.0827
0.8	0.6592	0.0229	0.5428	0.0373	0.4685	0.0988
1.0	0.6595	0.0237	0.5422	0.0406	0.4578	0.1140
1.2	0.6638	0.0251	0.5401	0.0433	0.4439	0.1265

[Table 3.1] The simulated ARE's of GMLE$\widehat{R_{IG}}(t_0)$ and Bayesian estimator $\widetilde{R_{IG}}(t_0)$ for MLE$\widehat{R_M}(t_0)$ on system reliability under the IGAM(10 , b) prior on β when b=0.2.

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_{IG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{IG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{IG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{IG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{IG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{IG}}$, $\widehat{R_M}$)
0.2	4.7305	0.0134	4.7767	0.0138	4.9190	0.0149
0.4	4.7464	0.0138	4.8422	0.0149	5.2737	0.0163
0.6	4.7704	0.0144	4.9866	0.0153	5.9036	0.0176
0.8	4.8151	0.0148	5.1623	0.0165	6.8513	0.0189
1.0	4.8731	0.0153	5.4161	0.0171	8.5398	0.0192
1.2	4.9534	0.0156	5.7568	0.0176	11.1109	0.0197

[Table 3.2] The simulated ARE's of GMLE$\widehat{R_{IG}}(t_0)$ and Bayesian estimator $\widetilde{R_{IG}}(t_0)$ for MLE$\widehat{R_M}(t_0)$ on system reliability under the IGAM(10 , b) prior on β when b=0.5.

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_{IG}}, \widehat{R_M}$)	ARE ($\widetilde{R_{IG}}, \widehat{R_M}$)	ARE ($\widehat{R_{IG}}, \widehat{R_M}$)	ARE ($\widetilde{R_{IG}}, \widehat{R_M}$)	ARE ($\widehat{R_{IG}}, \widehat{R_M}$)	ARE ($\widetilde{R_{IG}}, \widehat{R_M}$)
0.2	4.7875	0.0133	4.8789	0.0139	5.1436	0.0148
0.4	4.7964	0.0138	4.9500	0.0147	5.5016	0.0165
0.6	4.8248	0.0143	5.0759	0.0158	6.1871	0.0175
0.8	4.8709	0.0147	5.2860	0.0163	7.2109	0.0190
1.0	4.9327	0.0151	5.5395	0.0171	9.1877	0.0187
1.2	5.0054	0.0156	5.9009	0.0174	11.7325	0.0199

[Table 3.3] The simulated ARE's of GMLE$\widehat{R_{IG}}(t_0)$ and Bayesian estimator $\widetilde{R_{IG}}(t_0)$ for MLE$\widehat{R_M}(t_0)$ on system reliability under the IGAM(10 , b) prior on β when b=1.0.

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_{IG}}, \widehat{R_M}$)	ARE ($\widetilde{R_{IG}}, \widehat{R_M}$)	ARE ($\widehat{R_{IG}}, \widehat{R_M}$)	ARE ($\widetilde{R_{IG}}, \widehat{R_M}$)	ARE ($\widehat{R_{IG}}, \widehat{R_M}$)	ARE ($\widetilde{R_{IG}}, \widehat{R_M}$)
0.2	4.8747	0.0180	5.0590	0.0237	5.5554	0.0349
0.4	4.8843	0.0185	5.1357	0.0246	5.9592	0.0367
0.6	4.9140	0.0190	5.2693	0.0257	6.6880	0.0382
0.8	4.9565	0.0195	5.4769	0.0264	7.9744	0.0382
1.0	5.0163	0.0200	5.7544	0.0271	9.8479	0.0388
1.2	5.1003	0.0203	6.1085	0.0276	13.6588	0.0374

[Table 4.1] The simulated ARE's of GMLE$\widehat{R_{TG}}(t_0)$ and Bayesian estimator $\widetilde{R_{TG}}(t_0)$ for MLE $\widehat{R_M}$ on system reliability under the TGAM(1,10 , β_T) prior on β when β_T=5.

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_{TG}}, \widehat{R_M}$)	ARE ($\widetilde{R_{TG}}, \widehat{R_M}$)	ARE ($\widehat{R_{TG}}, \widehat{R_M}$)	ARE ($\widetilde{R_{TG}}, \widehat{R_M}$)	ARE ($\widehat{R_{TG}}, \widehat{R_M}$)	ARE ($\widetilde{R_{TG}}, \widehat{R_M}$)
0.2	0.9281	0.0134	0.6568	0.0138	0.5397	0.0148
0.4	0.9369	0.0139	0.6638	0.0148	0.5400	0.0163
0.6	0.9450	0.0143	0.6680	0.0155	0.5375	0.0177
0.8	0.9542	0.0148	0.6723	0.0163	0.5305	0.0184
1.0	0.9647	0.0153	0.6780	0.0172	0.5221	0.0188
1.2	0.9724	0.0156	0.6801	0.0176	0.5107	0.0189

[Table 4.2] The simulated ARE's of GMLE $\widehat{R_{TG}}(t_0)$ and Bayesian estimator $\widetilde{R_{TG}}(t_0)$ for MLE $\widehat{R_M}$ on system reliability under the TGAM(1,10 , β_T) prior on β when β_T=10.

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)
0.2	0.9288	0.0134	0.6573	0.0139	0.5395	0.0147
0.4	0.9360	0.0138	0.6620	0.0146	0.5401	0.0163
0.6	0.9452	0.0143	0.6686	0.0156	0.5378	0.0177
0.8	0.9554	0.0149	0.6716	0.0162	0.5340	0.0191
1.0	0.9642	0.0153	0.6775	0.0171	0.5244	0.0193
1.2	0.9731	0.0157	0.6813	0.0178	0.5175	0.0200

[Table 4.3] The simulated ARE's of GMLE $\widehat{R_{TG}}(t_0)$ and Bayesian estimator $\widetilde{R_{TG}}(t_0)$ for MLE $\widehat{R_M}$ on system reliability under the TGAM(1,10 , β_T) prior on β when β_T=20.

β	$t_0=0.5$		$t_0=0.7$		$t_0=1.0$	
	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widehat{R_{TG}}$, $\widehat{R_M}$)	ARE ($\widetilde{R_{TG}}$, $\widehat{R_M}$)
0.2	0.9274	0.0133	0.6570	0.0138	0.5404	0.0150
0.4	0.9359	0.0138	0.6630	0.0148	0.5416	0.0166
0.6	0.9449	0.0143	0.6691	0.0157	0.5383	0.0179
0.8	0.9567	0.0150	0.6731	0.0164	0.5322	0.0187
1.0	0.9640	0.0153	0.6767	0.0171	0.5246	0.0194
1.2	0.9731	0.0157	0.6806	0.0177	0.5162	0.0198

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