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# Reliability for Series System in Bivariate Weibull Model under Bivariate Random Censorship[1)]

Jang Sik Cho[2)]

## Abstract

In this paper, we consider two-components system which the lifetimes have a bivariate Weibull distribution with bivariate random censored data. Here the bivariate censoring times are independent of the lifetimes of the components. We obtain estimators and approximated confidence intervals for the reliability of series system based on likelihood function and relative frequency, respectively. Also we present a numerical study.

**KeyWords** : Bivariate Weibull distribution, Maximum likelihood estimator, Relative frequency, Series system; Reliability.

## 1. Introduction

In many the aforementioned studies for the reliability of two-components system, the lifetimes of the components were assumed to be statistically independent for the sake of simplicity of mathematical treatment. But occasionally, independence assumption is not applicable in the practical situation. Naturally, it is more realistic to assume some forms of dependence among the components of the system. This dependence among the components arise from common environmental shocks and stress, or from components depending on common sources of power, and so on. (See Esary and Proschan (1970)).

As the forms of dependence among the components in two-components system, Lu and Bhattacharyya (1988, 1990) and Lu (1989) initially introduced some new construction of bivariate Weibull(BVW) distributions as extensions of the Freund and Marshall-Olkin's bivariate exponential distributions. Also they obtained many

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1) This Research was supported by Kyungsung University Research Grants in 2002.

2) Associate Professor, Department of Statistical Information Science, Kyungsung University, Busan, 608-736, Korea
E-mail : jscho@ks.ac.kr

important properties of the BVW distribution. Bivariate Weibull distribution(BVW) is a versatile family of life distributions in view of its physical interpretation and its flexibility for empirical fit, and has been extensively applied to analysis of life data concerning many types of manufactured items. The examples of a bivariate Weibull distribution can be visualized in many contexts, such as the times to first and second failures of a repairable device, the breakdown times of dual generators in a power plant, or the survival times of the organs in a two-organ system, such as lungs or kidneys, in the human body.

Cho, Cha and Lee(2003) obtained the system reliability from stress-strength relationship. Cho, Cho and Kang(2003) constructed large sample tests for independence and symmetry. Also Cho, Kim and Kang(2003) obtained estimator for the reliability under univariate random censored data. All the authors mentioned above considered complete sample or univariate censored sample cases.

In this paper, we derive maximum likelihood estimator and relative frequency estimator for the reliability of series system in the bivariate Weibull distribution with bivariate random censored data as extension of complete data and univariate censored data. And we construct approximated confidence intervals for the reliability of series system based on the asymptotic distribution of proposed estimators. Also we present a numerical example by giving a data set which is generated by computer.

## 2. Preliminaries

Let random vector $(X, Y)$ be lifetime of two components that follow a BVW distribution with parameter $(\delta_1, \delta_2, \delta_3, \psi)$. Then the joint probability density function of $(X, Y)$ is given as

$$f(x, y: \delta_1, \delta_2, \delta_3, \psi) = \begin{cases} \delta_1(\delta_2+\delta_3)\psi^2 x^{\psi-1} y^{\psi-1}\exp[-\delta_1 x^{\psi} - (\delta_2+\delta_3) y^{\psi}], & 0 < x < y < \infty, \\ \delta_2(\delta_1+\delta_3)\psi^2 x^{\psi-1} y^{\psi-1}\exp[-(\delta_1+\delta_3) x^{\psi} - \delta_2 y^{\psi}], & 0 < y < x < \infty, \\ \delta_3 \psi x^{\psi-1}\exp[-\delta x^{\psi}], & 0 < x = y < \infty, \end{cases} \tag{1}$$

where $\delta_1, \delta_2, \delta_3, \psi > 0$ and $\delta = \delta_1 + \delta_2 + \delta_3$.

And the joint survival function of $(X, Y)$ is given by

$$\begin{aligned} \overline{F}(x, y) &= P(X > x, Y > y) \\ &= \exp[-(\delta_1 x^{\psi} + \delta_2 y^{\psi} + \delta_3 \max(x, y)^{\psi})]. \end{aligned} \tag{2}$$

The above BVW distribution is not absolutely continuous with respect to Lebesgue measure on $R^2$. That is, there is provision for simultaneous failure of the both components, $P[X = Y] = \delta_3/\delta$. Also the marginal distribution of $X$ is given by

$$\overline{F}(x) = P(X > x) = \exp[-(\delta_1 + \delta_3)x^{\psi}], \tag{3}$$

which is the survival function of Weibull with parameters $(\delta_1+\delta_3, \psi)$. By similar method, the marginal distribution of $Y$ is given as $\overline{F}(y)=P(Y>y)$ $=\exp[-(\delta_2+\delta_3)y^{\psi}]$ which is the survival function of Weibull with parameters $(\delta_2+\delta_3, \psi)$.

From (1)-(3), random variables $X$ and $Y$ are independent if and only if $\delta_3=0$. And $X$ and $Y$ are identically distributed if and only if $\delta_1=\delta_2$. Also BVW leads to the Marshall-Olkin's bivariate exponential distribution if and only if $\psi=1$.

On the other hand, the reliability of series system for mission time $x_o$ is given by

$$R=P[\min(X, Y) > x_o] = \exp[-\delta x_o^{\psi}]. \tag{4}$$

Suppose that there are $n$ two-components units under study and $i$th pair of the components have lifetime $(x_i, y_i)$ and a bivariate random censoring time ( $t_{x_i}$, $t_{y_i}$).

For $j=1,2;\ k=1,2,3;\ i=1,2,\cdots,n$, we use following notations for convenience sake;

(I) ( $t_{x_i}$, $t_{y_i}$) : bivariate random censoring times for $i$th system.

(II) $C_{1i}=I(X_i>t_{x_i})$, $C_{2i}=I(Y_i>t_{y_i})$, $C^*_{ji}=1-G_{ji}$.

(III) $R_{1i}=I(X_i<Y_i)$, $R_{2i}=I(X_i>Y_i)$, $R_{3i}=I(X_i=Y_i)$, $R^*_{ki}=1-R_{ki}$.

(IV) $\Theta=(\delta_1, \delta_2, \delta_3, \theta_1, \theta_2)$.

Then $i$th observed lifetime $(x_i, y_i)$ is given by

$$(x_i, y_i) = \begin{cases} (x_i, y_i), & x_i<t_{x_i}\ ,\ y_i<t_{y_i} \\ (t_{x_i}, y_i), & x_i>t_{x_i}\ ,\ y_i<t_{y_i} \\ (x_i, t_{y_i}), & x_i<t_{x_i}\ ,\ y_i>t_{y_i} \\ (t_{x_i}, t_{y_i}), & x_i>t_{x_i}\ ,\ y_i>t_{y_i}\ , \end{cases} \tag{5}$$

where the distribution and reliability function of bivariate random censoring times $(t_{x_i}, t_{y_i})$ are $\overline{G_1}(t_{x_i}:\psi,\theta_1)\cdot\overline{G_2}(t_{y_i}:\psi,\theta_2)$ and $g_1(t_{x_i}:\psi,\theta_1)\cdot g_2(t_{y_i}:\psi,\theta_2)$, respectively. Where $g_1(t_{x_i}:\psi,\theta_1)$ and $g_2(t_{y_i}:\psi,\theta_2)$ have independent Weibull distributions with parameters $(\psi, \theta_1)$ and $(\psi, \theta_2)$, respectively.

If $t_{x_i}=t_{y_i}$ then the censoring scheme is univariate random censorship. And if $t_{x_i}$ and $t_{y_i}$ are fixed constants then the censoring scheme is bivariate type I censorship.

Hence the likelihood function is derived as follows;

$$
\begin{aligned}
L(\Theta) = \prod_{i=1}^{n} \{ & [f(x_i, y_i) \cdot \overline{G_1}(x_i;\psi,\theta_1) \cdot \overline{G_2}(y_i;\psi,\theta_2)]^{C^*_{1i}C^*_{2i}} \\
& \cdot [\overline{F}(x_i, y_i) \cdot g_1(x_i;\psi,\theta_1) \cdot g_2(y_i;\psi,\theta_{2i})]^{C_{1i}C_{2i}} \\
& \cdot [\overline{F}_{X\mid Y=y}(x_i) f_Y(y_i) \cdot g_1(x_i;\psi,\theta_1) \cdot \overline{G_2}(y_i;\psi,\theta_2)]^{C_{1i}C^*_{2i}} \\
& \cdot [\overline{F}_{Y\mid X=x}(y_i) f_X(x_i) \cdot \overline{G_1}(x_i;\psi,\theta_1) \cdot g_2(y_i;\psi,\theta_2)]^{C^*_{1i}C_{2i}} \}^{(R_{1i}+R_{2i}+R_{3i})} \\
= {} & \delta_1^{D_1}\delta_2^{D_2}\delta_3^{D_3}(\delta_1+\delta_3)^{D_4}(\delta_2+\delta_3)^{D_5}\psi^{D_6} \cdot \theta_1^{D_6} \cdot \theta_2^{D_7} \\
& \cdot \prod_{i=1}^{n} \{ x_i^{(\psi-1)(C_{1i}+C^*_{1i})} y_i^{(\psi-1)(C_{2i}+C^*_{2i})(1-R_{3i}C^*_{1i})} \} \\
& \cdot \exp[-(\delta_1+\theta_1)x_s - (\delta_2+\theta_2)y_s - \delta_3(x_s+y_s-t_s)], \qquad (6)
\end{aligned}
$$

where $f_X(x) = \psi(\delta_1+\delta_3)x_1^{\psi-1} \cdot \exp(-(\delta_1+\delta_3)x^{\psi})$,

$f_Y(y) = \psi(\delta_2+\delta_3)y^{\psi-1} \cdot \exp(-(\delta_2+\delta_3)y^{\psi})$,

$D_1 = \sum_{i=1}^{n}(R_{1i}C^*_{1i}C^*_{2i} + R^*_{2i}C^*_{1i}C_{2i})$, $D_2 = \sum_{i=1}^{n}(R_{2i}C^*_{1i}C^*_{2i} + R^*_{1i}C_{1i}C^*_{2i})$,

$D_3 = \sum_{i=1}^{n} R_{3i}C^*_{1i}C^*_{2i}$, $D_4 = \sum_{i=1}^{n} R_{2i}C^*_{1i}$, $D_5 = \sum_{i=1}^{n} R_{1i}C^*_{2i}$,, $D_6 = \sum_{i=1}^{n} C_{1i}$, $D_7 = \sum_{i=1}^{n} C_{2i}$

$x_s = \sum_{i=1}^{n} x_i^{\psi}$, $y_s = \sum_{i=1}^{n} y_i^{\psi}$, $t_s = \sum_{i=1}^{n} \min(x_i, y_i)^{\psi}$.

Also $D_1, D_2, \cdots, D_7$ are random variables. After some calculations, the expected value of each $D_i$, $i = 1, 2, \cdots, 5$ can be obtained as follows;

$$
\begin{aligned}
E(D_1) = \sum_{i=1}^{n} \{ & \delta_1/\delta - \delta_1 \exp(-\delta\, t^{\psi}_{x_i})/\delta + \exp(-\delta\, t^{\psi}_{y_i}) - \exp(-(\delta_2+\delta_3)\, t^{\psi}_{y_i}) \\
& + (1-\exp(-\delta_1\, t^{\psi}_{x_i})) \cdot \exp(-(\delta_2+\delta_3)\, t^{\psi}_{y_i}) \cdot I(t_{x_i} < t_{y_i}) \\
& + \delta_3(\exp(-\delta\, t^{\psi}_{y_i}) - \exp(-\delta\, t^{\psi}_{x_i}))/\delta \cdot I(t_{y_i} < t_{x_i}) \}.
\end{aligned}
$$

$$
\begin{aligned}
E(D_2) = \sum_{i=1}^{n} \{ & \delta_2/\delta - \delta_2 \exp(-\delta\, t^{\psi}_{y_i})/\delta + \exp(-(\delta_1+\delta_3)\, t^{\psi}_{x_i} - \delta_2\, t^{\psi}_{y_i}) \\
& - \exp(-(\delta_1+\delta_3)\, t^{\psi}_{x_i}) \\
& + (1-\exp(-\delta_2\, t^{\psi}_{y_i})) \cdot \exp(-(\delta_1+\delta_3)\, t^{\psi}_{x_i}) \cdot I(t_{y_i} < t_{x_i}) \\
& + \delta_3(\exp(-\delta\, t^{\psi}_{x_i}) - \exp(-\delta\, t^{\psi}_{y_i})))/\delta \cdot I(t_{x_i} < t_{y_i}) \},
\end{aligned}
$$

$$
E(D_3) = \sum_{i=1}^{n} \{ (\delta_3 - \delta_3 \exp(-\delta \min(t^{\psi}_{x_i}, t^{\psi}_{y_i})))/\delta \},
$$

$$
E(D_4) = \sum_{i=1}^{n} \{ \delta_2/\delta - \delta_2 \exp(-\delta\, t^{\psi}_{y_i})/\delta + \exp(-(\delta_1+\delta_3)\, t^{\psi}_{x_i} - \delta_2\, t^{\psi}_{y_i})
$$

$$- \exp(-(\delta_1+\delta_3)\, t^{\phi}{}_{x_i}) + [\exp(-\delta_2\, t^{\phi}{}_{y_i}) \cdot [1-\exp(-(\delta_1+\delta_3)\, t^{\phi}{}_{x_i})] + (\delta_1+\delta_3)\cdot(\exp(-\delta\, t^{\phi}{}_{x_i})-1)/\delta] \cdot I(t_{x_i} > t_{y_i})\},$$

$$E(D_5) = \sum_{i=1}^{n} \{\delta_1/\delta - \delta_1 \exp(-\delta\, t^{\phi}{}_{x_i})/\delta + \exp(-\delta\, t^{\phi}{}_{y_i}) - \exp(-(\delta_2+\delta_3)\, t^{\phi}{}_{y_i}) + \delta_1 \exp(-\delta\, t^{\phi}{}_{x_i})/\delta - \exp(-(\delta_2+\delta_3)\, t^{\phi}{}_{y_i} - \delta_1\, t^{\phi}{}_{x_i})\}.$$

In this paper, we focus only on BVW with fixed $\phi$. Now the log-likelihood function of the sample of size $n$ is given by

$$\begin{aligned}\log L(\Theta) &= D_1 \log\delta_1 + D_2\log\delta_2 + D_3\log\delta_3 + D_4\log(\delta_1+\delta_3) + D_5\log(\delta_2+\delta_3) \\ &\quad + D_6\log(\theta_1) + D_7\log(\theta_2) \\ &\quad + \sum_{i=1}^{n}\{(\phi-1)(C_{1i}+C_{1i}^{*})\log(x_i) + (\phi-1)(C_{2i}+C_{2i}^{*})(1-R_{3i}C_{1i}^{*})\log(y_i)\} \\ &\quad - (\delta_1+\theta_1)x_s - (\delta_2+\theta_2)y_s - \delta_3(x_s+y_s-t_s). \end{aligned} \tag{7}$$

Hence, the likelihood equations are given by

$$\frac{\partial}{\partial\delta_1}\log L(\Theta) = \frac{D_1}{\delta_1} + \frac{D_4}{\delta_1+\delta_3} - x_s = 0. \tag{8}$$

$$\frac{\partial}{\partial\delta_2}\log L(\Theta) = \frac{D_2}{\delta_2} + \frac{D_5}{\delta_2+\delta_3} - y_s = 0. \tag{9}$$

$$\frac{\partial}{\partial\delta_3}\log L(\Theta) = \frac{D_3}{\delta_3} + \frac{D_4}{\delta_1+\delta_3} + \frac{D_5}{\delta_2+\delta_3} - (x_s+y_s-t_s) = 0. \tag{10}$$

$$\frac{\partial}{\partial\theta_1}\log L(\Theta) = \frac{D_6}{\theta_1} - x_s = 0. \tag{11}$$

$$\frac{\partial}{\partial\theta_2}\log L(\Theta) = \frac{D_7}{\theta_2} - y_s = 0, \tag{12}$$

The likelihood equations (8)-(12) are not easy to solve. But we can obtain MLE's ( $\widehat{\delta}_1$, $\widehat{\delta}_2$, $\widehat{\delta}_3$) by either Newton-Raphson procedure or Fisher's method of scoring.

The Fisher information matrix is given by

$$I(\Theta) = ((I_{ij})),\ \text{where}\ I_{ij} = E\left[-\frac{\partial^2}{\partial\delta_i\partial\delta_j}\log L(\Theta)\right]\ ;\ i,j=1,2,3,$$

$$I_{11} = E(D_1)/\delta_1^2 + E(D_4)/(\delta_1+\delta_3)^2,\ \ I_{12}=0,\ \ I_{13} = E(D_4)/(\delta_1+\delta_3)^2,$$

$$I_{22} = E(D_2)/\delta_2^2 + E(D_5)/(\delta_2+\delta_3)^2,\ I_{23} = E(D_5)/(\delta_2+\delta_3)^2,$$

$$I_{33} = E(D_3)/\delta_3^2 + E(D_4)/(\delta_1+\delta_3)^2 + E(D_5)/(\delta_2+\delta_3)^2.$$

Thus $\sqrt{n}(\widehat{\underline{\delta}} - \underline{\delta})$ has asymptotic trivariate normal distribution with mean vector zero and covariance matrix $I^{-1}(\underset{\sim}{\delta}) = \frac{1}{n}((I^{ij}))$ ; $i, j = 1, 2, 3$. Here, $\widehat{\underset{\sim}{\delta}} = (\widehat{\delta}_1, \widehat{\delta}_2, \widehat{\delta}_3)$ and $\underset{\sim}{\delta} = (\delta_1, \delta_2, \delta_3)$.

## 3. Reliability Estimation for Series System

In this section, we obtain estimators for $R$ based on the likelihood function and the relative frequency, respectively. Also we obtain approximated confidence intervals for $R$ based on MLE and the relative frequency estimator, respectively.

For mission time $x_o$, we note that MLE for reliability with series system by invariant properties of MLE is given by

$$\widehat{R}_{MLE} = \exp[-\widehat{\delta}\cdot x_o^{\phi}], \quad \widehat{\delta} = \widehat{\delta}_1 + \widehat{\delta}_2 + \widehat{\delta}_3. \tag{13}$$

Hence, we can see that the distribution of $\widehat{R}_{MLE}$ is asymptotic normal distribution with mean $R$ and variance $\delta \cdot [I^{-1}(\delta_1, \delta_2, \delta_3)/n] \cdot \delta'$.

Here, $\delta = (-x_o^{\phi}\cdot \exp(-\delta x_o^{\phi}),\ -x_o^{\phi}\cdot \exp(-\delta x_o^{\phi}),\ -x_o^{\phi}\cdot \exp(-\delta x_o^{\phi}))$.

Therefore, $100(1-\alpha)\%$ approximated confidence interval for $R$ based on MLE is as follows;

$$\left(\widehat{R}_{MLE} - z_{\alpha/2}\cdot\sqrt{\widehat{\delta}\cdot I^{-1}(\widehat{\delta}_1, \widehat{\delta}_2, \widehat{\delta}_3)\cdot \widehat{\delta}'/n},\ \widehat{R}_{MLE} + z_{\alpha/2}\cdot\sqrt{\widehat{\delta}\cdot I^{-1}(\widehat{\delta}_1, \widehat{\delta}_2, \widehat{\delta}_3)\cdot \widehat{\delta}'/n}\right) \tag{14}$$

where $\widehat{\delta} = (-x_o^{\phi}\cdot \exp(-\widehat{\delta} x_o^{\phi}),\ -x_o^{\phi}\cdot \exp(-\widehat{\delta} x_o^{\phi}),\ -x_o^{\phi}\cdot \exp(-\widehat{\delta} x_o^{\phi}))$.

We next obtain the estimate and approximate confidence interval for $R$ based on relative frequency. Let $K$ be the number of observations with $\min(x_i, y_i) > x_o$ in the sample. Then we can see that the distribution of $K$ is binomial distribution with parameter $(n, R)$.

The relative frequency estimate of $R$ based on $K$ is given by

$$\widehat{R}_{RF} = K/n, \tag{15}$$

which is asymptotic normal distribution with mean $R$ and variance $R(1-R)/n$. Therefore, $100(1-\alpha)\%$ approximated confidence interval for $R$ based on the relative frequency estimate is as follows;

$$\left(\widehat{R}_{RF} - z_{\alpha/2}\cdot\sqrt{\widehat{R}_{RF}\cdot(1-\widehat{R}_{RF})/n},\ \widehat{R}_{RF} + z_{\alpha/2}\cdot\sqrt{\widehat{R}_{RF}\cdot(1-\widehat{R}_{RF})/n}\right). \tag{16}$$

## 4. Numerical Example

In this section, we present a numerical example by giving a data set which is generated by computer. We generate a random samples of size 30 from BVW with parameter ( $\delta_1 = 1.8,\ \delta_2 = 1.8,\ \delta_3 = 1.3,\ \phi = 2.0$). And we set the mission

time $x_o = 0.3$. Also we generate bivariate random censored data of sizes 30 corresponding the lifetimes from Weibull with parameters $\theta_1 = 1.0$, $\psi = 2.0$ and $\theta_2 = 1.0$, $\psi = 2.0$, respectively. Then the true reliability of series system is 0.6433.
The data is given as Table 1. In Table 1, * indicates censored data.

<Table 1> Generated samples from BVW

| $i$ | $x_i$ | $y_i$ | $i$ | $x_i$ | $y_i$ |
|---|---|---|---|---|---|
| 1 | 0.5615 | 0.5615 | 16 | 0.2170 | 0.2170 |
| 2 | 0.9988 | 0.3872 | 17 | 0.4562 | 0.9376 |
| 3 | 0.4279 | 0.4279 | 18 | 0.2385 | 0.6478* |
| 4 | 0.4925* | 0.6374 | 19 | 0.3649* | 0.4940* |
| 5 | 0.2284 | 0.7062* | 20 | 0.8273 | 0.8273 |
| 6 | 0.3118 | 0.9788 | 21 | 0.5209 | 0.4482 |
| 7 | 0.6650* | 0.4179* | 22 | 0.6323* | 0.5326 |
| 8 | 0.5040* | 0.4999* | 23 | 0.7964* | 0.2911 |
| 9 | 0.6340 | 0.6385* | 24 | 0.4251 | 0.1443 |
| 10 | 0.1756 | 0.0377 | 25 | 0.2136 | 0.2136 |
| 11 | 0.3626 | 0.0951 | 26 | 0.3475 | 0.2341* |
| 12 | 0.4775 | 0.5853 | 27 | 0.4342 | 0.2025 |
| 13 | 0.1026 | 0.1026 | 28 | 0.3254* | 0.7310 |
| 14 | 0.2326 | 0.4510 | 29 | 0.7197 | 0.0800* |
| 15 | 0.4406 | 0.4406 | 30 | 0.5992 | 0.5525* |

MLE's of the parameters in BVW model are $\widehat{\delta}_1 = 1.5788$, $\widehat{\delta}_2 = 1.4003$, $\widehat{\delta}_3 = 1.2721$. And $K = 17$. Hence, the MLE and relative frequency estimator of $R$ are $\widehat{R}_{MLE} = 0.6820$ and $\widehat{R}_{RF} = 0.5667$, respectively. Also 95% confidence intervals for $R$ based on $\widehat{R}_{MLE}$ and $\widehat{R}_{RF}$ are (0.6062, 0.7579) and (0.3893, 0.7439), respectively.

Hence, we note that $\widehat{R}_{MLE}$ based on MLE's perform better than $\widehat{R}_{RF}$ based on relative frequency estimate in viewpoint of bias and confidence interval, more or less.

In our discussions, we have concentrated on the bivariate Weibull model case. But we can apply our results, for a more general model.

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