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지반재료 구성모델에 있어서의 데미지

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Damage in Constitutive Modeling for Soils

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요 약 본 연구에서는 점성토에 있어서 범용으로 적용 가능한 시간의존적 구성방정식을 유도하였다. 데미지 법칙을 유도된 구성방정식에 포함하였으며 이를 활용하여 비배수 크리프거동을 예측하였다. 구성방정식의 수학적 물리적 유도는 가능한 적은 모델정수를 포함시키는 원칙에서 수행되었다. 유도된 구성방정식을 활용하여 예측한 크리프 거동은 점성토의 중요한 시간의존적 거동인 비배수 크리프파괴를 포함하는 실험결과와 잘 부합하였다. 유도된 구성모델은 단순함에 비해 크리프 예측 능력이 뛰어난 것으로 평가된다.

Abstract In this study, a time-dependent constitutive model was developed for cohesive soils. A damage law was included in the model, using which the undrained creep behavior was predicted. The mathematical and physical derivation of the model was performed in the sense of adopting only few model parameters. The model prediction was well agreed with the experimental result of creep testing including creep rupture.

Key Words : damage, creep rupture, cohesive soil, constitutive model

1. Introduction

Creep is an important time-depenedent phenomenon in stability analysis for soil mass. The primary consolidation and the secondary compression have been respectively regarded as drained and undraind creeps. Specially the rapid large creep strain in undrained creep test may occur after the steady stage with relatively small creep strain. This is called the undrained creep rupture and it significantly affects the overall stability and should be considered in geotechnical analysis.

The proper simulation of the creep behavior has been studied through the mathematical and physical modeling. Most of them have adopted viscous theory and regarded soils in macroscopic view, in other words, soils were assumed very homogeneous throughout the entire soil mass; however, soils, in practice, can be damaged in patches during deformation. In that sense, a few efforts to properly describe the creep behavior have been made especially in attempting to incorporate the damage into the constitutive models[1-6].

In this research, a combined elastic-plastic-viscous constitutive relation has been developed. The generalized Hooke's law and Banerjee model were respectively used for the elastic and plastic parts[7]. The Banerjee model employs the solid critical state theory, and the model has given good predictions with few model parameters. The generalized viscous theory has the advantages that it has successfully simulated the time-dependent behavior of soils with few model parameters, and the mathematical form of the theory can be easily formulated with the classical elastic-plastic derivation[8]. The generalized viscous theory was simplified, in this research, for effective viscous modeling and obvious mathematical formulation in an elastic-plastic-viscous combination scheme. A damage law has been incorporated into the constitutive relation[3]. The physical and mathematical formulation of the combined model was performed from the point of view that fewer parameters better be employed. The model predic-

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tions have been compared with the experimental results of creep tests. The concepts of the constitutive models, the damage law, the theoretical and mathematical formulation, and the comparison and investigation of the results are described in following sections.

2. Simplification of Generalized Viscous Theory

In this section, the slight revision of the generalized viscous theory made in this research is presented.

The assumption that only the plastic response is rate sensitive makes it possible to assume that the total strain rate can be resolved into elastic and inelastic parts, i.e.,[8].

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{vp}_{ij} \qquad (1)$$

where $\dot{\varepsilon}^{e}_{ij}$ and $\dot{\varepsilon}^{vp}_{ij}$ are the elastic and the inelastic viscoplastic strain rate tensors, respectively. It is clear that eq. (1) is similar to that of inviscid plasticity, except for the fact that the inelastic strain tensor represents the combined viscous and plastic effects.

The elastic strain rate is obtained from the generalized Hooke's law. The viscoplastic strain rate tensor, on the other hand, is given by:

$$\dot{\varepsilon}^{vp}_{ij} = <\Phi(F)>\frac{\partial f}{\partial \sigma_{ij}} \qquad (2)$$

where $< >$ is Macauley bracket and the F represents the initial yield function, and is also called "the static yield function". The viscoplastic strain rate tensor is assumed a function of the "excess stress" or "overstress" above the initial yield condition. The f defines so called "dynamic loading surface". The viscoplastic strain rate is always directed along the normal to the dynamic loading surface as shown in Fig. 1. The magnitude of the viscoplastic strain rate is controlled by the value of the viscous or overstress flow function $<\Phi(F)>$.

The functional form of the overstress flow function is selected based on experimental results. Two forms that are commonly used are:

$$\Phi(F) = F^{n} \text{ or } \Phi(F) = \exp F - 1 \qquad (3)$$

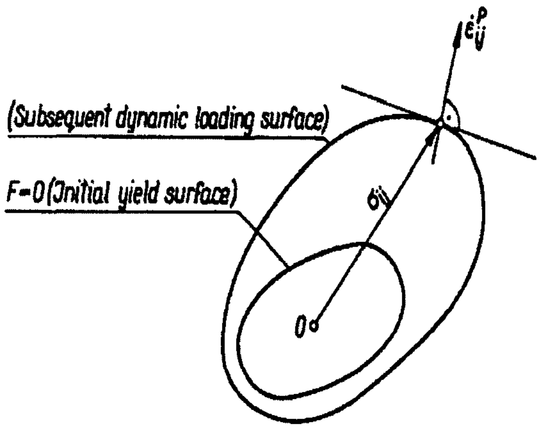


Fig. 1. Dynamic Loading Surface and Strain Rate Vector [8].

where n is a model parameter.

Both the inviscid plasticity and the generalized viscous theory assume decomposition of strain into elastic and inelastic deformations; however, two fundamental differences can be observed. First, unlike the inviscid plasticity theory, where the stress point must be on or within the yield surface, the stress point can be outside the initial yield surface, instead, the dynamic loading surface passes through the loading point. On this surface, the viscoplastic strain rate is not zero, and its magnitude depends on the overstress flow function $\Phi(F)$. Its direction is given by the gradient vector $\partial f/\partial \sigma_{ij}$, and that like in the associated flow rule of inviscid plasticity, is in the outward normal direction of the dynamic loading surface Fig. 1. The initial yield surface evolves exactly as in plasticity and serves only to separate the region of stress space where deformation is both elastic and viscoplastic from the region where only elastic deformation takes place and viscoplastic deformation is zero. The dynamic loading surface evolves similarly to the initial yield surface but it is also dependent on the rate of loading in addition to stress and strain. The second major difference lies in the way in which the inelastic strain is evaluated. In inviscid plasticity, plastic strain is obtained from a consistency condition applied to the yield function. In the generalized viscous theory, on the other hand, it is assumed that the inelastic strain (viscoplastic strain) is a function of an overstress on the initial yield function, i.e., the magnitude of the viscoplastic strain rate depends on the overstress flow function.

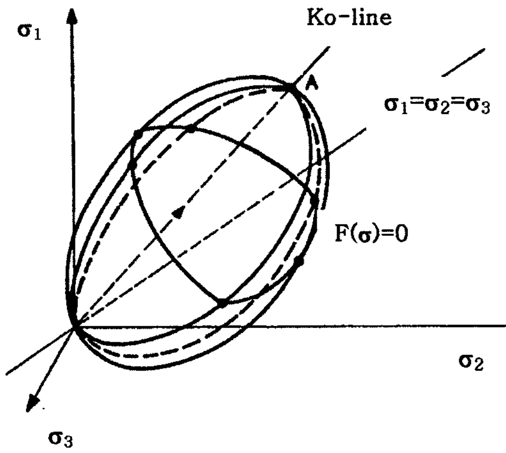
In this research, the generalized viscous theory stated above is revised as: the initial yield surface and the dynamic loading surface are not differentiated so only one loading surface exists to separate only elastic deformation at the stress state inside the surface from both elastic and viscoplastic deformation at the stress state on the surface. Which is same as in inviscid plasticity. The loading surface has the exactly same functional form and hardening rules with the plastic loading function. The right side of eq. (3), with no model parameter, was used as the overstress flow function since this research mainly focuses on the damage rather than time-dependent or viscous model itself.

3. Application of Banerjee Model

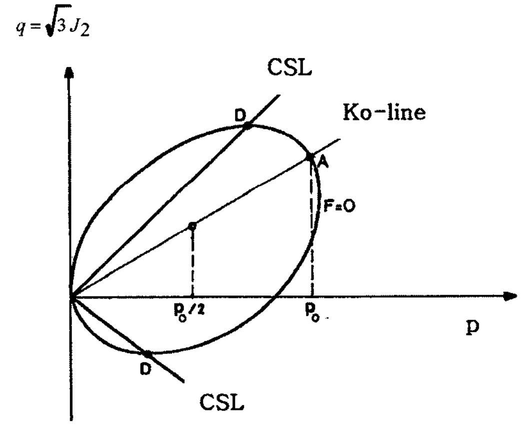
The Banerjee model is used to describe the plastic behavior in this study[7]. In the model, the yield surface is assumed to be a distorted ellipsoid and the hardening rule adopted is a combination of isotropic and kinematic hardening rules Fig. 2.

Due to the kinematic hardening, the material developes a new state of anisotropy at every stage of the loading process. The yield surface passes through the origin and intersects the failure surface as well as the hydrostatic stress axis. During a loading process, the yield surface rotates and the consistency condition is automatically satisfied at the instant of failure. All formulations including the consistency condition and the obtaining procedure of the loading function L obey the classical plasticity.

Geotechnical processes and the stresses relevant to the formation of naturally deposited soils are mainly responsible for the development of the inherent anisotropy. Subsequent application of a given stress history introduces further anisotropy, induced anisotropy, in the mechanical properties of real soils. The plastic model stated above has the advantage that it can account for both the inherent and induced anisotropy with relatively few model parameters, which are the traditional λ(slope of virgin compression), κ(slope of unloading-reloading line), ν(poisson's ratio), ϕ(internal friction angle).



(a) Yield surface in general stress space



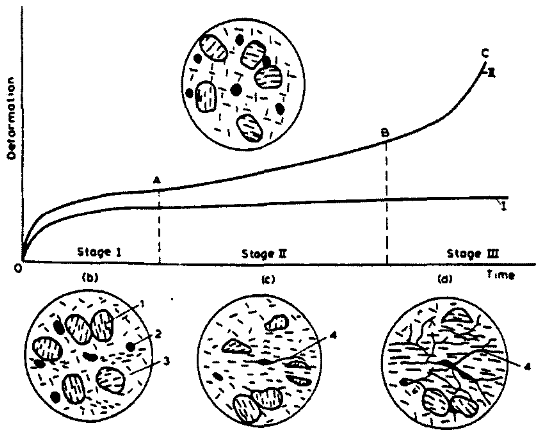
(b) Yield surface in p-q space

Fig. 2. Yield surface in general stress space and p-q space [7].

4. Damage for Soils

Fig. 3 illustrates a creep response curve which is typical of most clays, and the changes in the microstructure of soil at various stages of creep deformation. The creep curve has usually three stages: (1) primary creep where the strain rate gradually decreases with time. (2) steady state or secondary creep of creep at constant rate, leading to a period of linear strain versus time response. (3) tertiary creep where creep rate begins to rapidly increase.

Vyalov(1986) presented a general hypothesis that quantitatively describes the complete creep process of cohesive soils under a sustained load[3]. Accord-



1=micro-aggregates of particles,

2=cavities and voids

3=cementing clay

4=micro and macro-cracks

Fig. 3. Creep Mechanism [3].

ingly, soils are composed of randomly assembled solid particles and voids filled with water and air. Each particle is linked at the neighboring particles by at least two bonds. When there is only one bond, the soil structure is considered disturbed, so-called "discontinuity or disturbance", in the soil skeleton. Such disturbance takes place when the distance between particles is greater than the range of Van der Waals' forces. Any point of disturbance or discontinuity of the bonding in the soil skeleton is termed "defect". Under this definition, voids containing free water are not considered defects. In stiff clay soils, defects manifest themselves in the form of micro-cavities, voids, microcracks, or cleavage, etc. These defects in soil structure are considered the prime factors causing creep rupture. However, among these factors, microcracks have the most profound effect. Vyalov(1986)'s experimental investigation revealed that, during the primary creep stage, microcracks tend to close, and cavities and voids contract and expand in the direction of shear. This leads to compaction of the soil and the formation of new interparticle bonds. As a result, the deformation attenuates in time. During the secondary creep stage, more microcracks begin to propagate, and more particles are oriented with their basal surfaces in the direction of shear. Therefore, there is a continuous "healing" of the defects, as in primary creep stage, but also a fresh disturbance in the form of microcracks is formed. Finally, as deformation progresses, and following the onset of the tertiary creep stage, an intensive propagation of microcracks takes place. Eventually, these microcracks form larger cracks, causing failure of the specimen. Based on microscopic observation as stated above, it has been concluded that there are two phenomena responsible for soil creep, i.e., the hardening and the softening of soil. If hardening is dominant, the deformation attenuates in time, and the creep curve response possesses only the primary and secondary stages. If, on the other hand, softening prevails, the creep rate will accelerate and the tertiary stage of creep may terminate in rupture.

Whether the soil will harden or soften under a sustained load depends on structural changes which in turn depend on the level of applied load. When soil is loaded, the stress concentration produces overstrain and breaks some of the interparticle bonds. The breakage of the bond occurs at the weakest points of the structure, which then result in those particles with damaged bonds moving into more stable positions. Therefore, new bonds are formed, and the structural defects reduce in size and their number is decreased. This then leads to an overall reduction in soil volumes, consequently, the soil hardens. However, the "healing" of structural defects is now accompanied by new disturbances in the soil which will weaken the structural bonds. Thus the final and overall response will depend on the level of applied load. If the load is low, the number of newly formed bonds and "healed" defects will be larger than that of the broken bonds and fresh defects. Hence, the progress of hardening will prevail, and creep deformation will attenuate. For moderately high loads, the phenomenon of soil hardening occurs at the stage of primary creep only where the weakening of bonds is offset by a strengthening of the structure. As the creep progresses, the number of broken bonds continues to increase and soil resistance to the applied load rapidly deteriorates. Thus the soil softens. However, after a certain period of elapsed time, the softening is compensated for by a hardening, and the creep reaches a stage of steady state wherein the rate is approximately constant. Finally, when the applied load is high, the structural damage accumulated during the course of deformation will be large, which then leads to a prevalence of softening as opposed to hardening. Consequently, the deformation rate increases, marking the offset of the tertiary creep stage.

It is quite obvious that the acceleration of deformation during the tertiary stage is a macroscopic manifestation of microscopic degradation, or deterioration of the material. As a result, the material exhibits a softening effect in the tertiary creep stage when the strain rate continuously increases for constant stress. It was suggested that this increase in strain rate be described by the introduction of an additional variable into the constitutive equation[9]. This additional variable, termed ω, is considered to be a measure of "damage" incurred by the material under sustained loading. As time passes, damage accumulates, and the value of ω evolves according to a certain rate equation.

This concept of damage has been widely used in modeling the strength deterioration of metals, which is due in part to its simplicity. In this research, the same approach was adopted, where soil damage is represented by the single damage variable ω. During the tertiary creep stage, structural defects develop at a drastic rate and, once the structural damage ω, reaches a critical value ω, the soil is considered to have creep failed.

5. Development of Model

In this section, the combination and mathematical formulation of elastic, plastic, and viscous constitutive equations is described. The basic concept is as eq. (4) [10]. The revision and application of each model, made in this research, was performed so that each model may employ fewer parameters for the practically easy usage of the model and for the avoidance of the confusion with the complicated parameters. The superscripts e, p, vp denote elastic, time-independent plastic, and time-dependent viscoplastic parts, respectively. The time-dependent bahavior is assumed to be attributed to only the coupling of plastic and viscous parts, i.e., viscoplastic part.

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{p}_{ij} + \dot{\varepsilon}^{vp}_{ij} \tag{4}$$

The elastic strain rate is time-independent and obtained using the generalized Hooke's law. The time-independent plastic strain rate is represented by the associated flow rule, where the Banerjee loading function, is used as the loading function f.

$$\dot{\varepsilon}^{p}_{ij} = \langle L \rangle \frac{\partial f}{\partial \sigma_{ij}} \tag{5}$$

The time-dependent viscoplastic strain rate is estimated using eq. (3). In this research, only one loading surface is assumed to separate elastic deformation at the stress state inside the surface from both elastic and viscoplastic deformation at the stress state on the surace. Regarding the viscoplastic behavior, the loading surface is assumed to have the same functional form and hardening rules with the plastic loading function in eq. (5). The right side of eq. (3) is used as the overstress flow function.

In this research, soil damage is represented by the single damage variable ω, in the form of eq. (6)[3].

$$\omega = 1 - \frac{(1-\omega_o)}{(1+t)^{\Lambda\bar{\tau}}} \quad 1 - \omega = (1-\omega_o)(1+t)^{-\Lambda\bar{\tau}} \tag{6}$$

where ω is the degree of damage at any time t, ω_o is the initial structural damage, Λ is constant, and $\bar{\tau}$ is a dimensionless stress function representing the magnitude of an applied deviatoric stress. The t is a dimensionless quantity equal to the value of period of deformation t divided by t^m, where t^m is a parameter measured in units of time, and may be taken to be equal to one. The quantities $1-\omega_o$ and $1-\omega$ define the undamaged areas of soil at the initial condition and at any time t, respectively. If ω_f is used, it indicates the degree of damage at the moment of failure ($t=t_f$).

According to a proper assumption of $\bar{\tau}$, eq. (6) yields: [3]

$$\omega = 1 - \frac{(1-\omega_o)}{(1+t)^{\frac{\Lambda\tau}{\tau_o-\tau}}} \tag{7}$$

where τ_o is the hypothetical instantaneous strength of the soil and the value should be determined from the creep test; however, τ_o is assumed to be undrained shear strength so that the value of which can be practically determined from the time-independent triaxial test. The τ is the applied deviatoric stress. Eq. (7) defines the damage incurred by the soil at any time t and for any given applied load τ. It is obvious that the higher the hypothetical instantaneous strength of the soil, the smaller the changes in ω Also, as expected, the degree of structural damage increases as the applied stress increases. The significance of eq. (7) rises from the fact that all parameters except Λ have a quite definite physical meaning. The initial degree of damage ω_o and the constant Λ can be evaluated through microscopic investigation of a soil sample. Due to the difficulty in determining the microscopic data, it is often and successfully assumed that the values of ω_o and Λ are obtained from conventional creep test. This represents that the constants, τ_o, ω_o and Λ were included originally for predicting the change of the structural damage ω Here, the essence of these constants changed from those of microscopically-based parameters to macroscopic parameters. If it is further assumed that soil is initially not damaged, then ω_o=0, and eq. (7) becomes

$$\omega = 1 - \frac{1}{(1+t)^{\frac{\Lambda\tau}{\tau_o - \tau}}} \qquad (8)$$

Eq. (8) indicates the degree of structural damage ω under any applied load τ and at any elapsed time t. One approach for incorporating the damage law into the proposed model is based on the concept of net stress. The quantity 1-ω represents the intact area of a unit cross-section. The damage accumulates and the amount of material available for carrying the applied load is reduced thus net stress increases. Based on this proposition, the stress are given by eq. (9) [11].

$$\check{\sigma}_{ij} = \frac{\sigma_{ij}}{1-\omega} \qquad (9)$$

Eqs. (7) and (8) were adopted in this research as they have physical meanings and have been accepted for various kinds of material.

6. Creep Simulation and Discussion

Soils may fail under a sustained load, depending on the mechanical characteristics, the magnitude of the applied load, and the stress history but the physical mechanism that causes undrained creep rupture in cohesive soils is not yet fully understood. In this section, the validity of the developed model in simulating some available creep test results, including creep rupture, is examined. The experimental program carried out for San Francisco Bay mud have investigated the undrained creep behavior of normally consolidated cohesive soils[12,13].

The creep behavior of anisotropically consolidated soils has been rarely studied. A series of anisotropic triaxial undrained creep tests on undisturbed San Francisco Bay mud was conducted[12,13]. The soil specimens were first consolidated isotropically to a confining pressure of 0.3kg/cm^2. Then the axial load was increased in small increments until the stress ratio was equal to 2.0. Thereafter, the specimens were anisotropically consolidated by simultaneously increasing the axial load and the cell pressure so that the stress ratio was maintained at 2.0 throughout the consolidation. The load increments were applied at approximately 8 hour interval, and each increment was less than 10% of the undrained shear strength. Once the proper consolidation stress was reached, a prescribed deviatoric stresses (0.53, 0.55, 0.57, 0.60kg/cm^2) were applied and the creep tests initiated.

The observed and predicted creep curves for the four creep tests are shown in Fig. 4 and Fig. 5. The set of the input parameters listed in Table 1. The values of the input parameters were evaluated using the previous test results[12,13]. The hypothetical instantaneous strength parameter t was assumed to be equal to the undrained shear strength (1.4kg/cm^2) obtained from the test results. It was reported that the two creep tests with deviatoric stress equal to 0.57 and 0.60kg/cm^2 clearly resulted in creep rupture eventhough it is true that the other two tests are likely to fail in rupture. The damage parameter Λ was determined by matching the results of these two creep rupture tests.

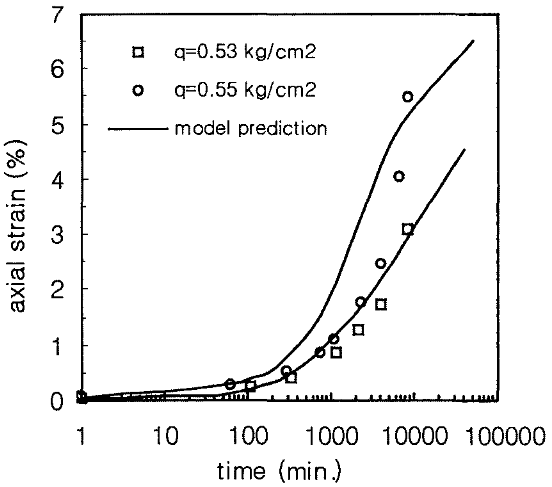


Fig. 4. Creep behavior (deviatoric stress 0.53, 0.55kg/cm^2).

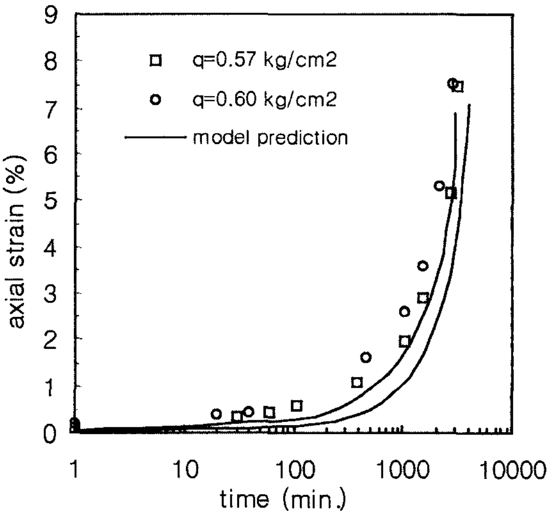


Fig. 5. Creep behavior (deviatoric stress 0.57, 0.60 kg/cm^2).

From Fig. 4 and Fig. 5, it can be noted that agreement between the experimental and predicted behavior is acceptable, considering the relatively simple constitutive model. For the other two tests in Fig. 5, where creep rupture clearly took place, the model predictions underestimate the experimental creep strain. It was possible, though, to use a larger value for the damage parameter Λ, which would have improved the prediction. However, that leads to an overestimation of the final values of the creep strain. For this reason, Λ was set equal to zero in predicting the results of the creep tests with deviatoric stresses

Table 1. Input parameters

parameter	identification	value
M	slope of CSL	1.40
λ	slope of NCL	0.37
κ	slope of swelling line	0.054
ν	poisson's ratio	0.3
e_o	initial void ratio	2.52
ϕ	internal friction angle	18°ý
τ_o	damage parameter	1.4
Λ		1.5x10^{-4}

equal to 0.53 and 0.55kg/cm^2. Otherwise, the model prediction of these tests would end in rapid creep rupture. Another more realistic alternative approach for the creep behavior with rupture is to assume that the initial damage ω_o is not zero. In this case, two different values of ω_o can be used for the creep tests with rupture, on the other hand, a fixed value for Λ is used for all of the tests with and without rupture. For the case of Bay mud, the first approach using a relatively small value of Λ was adopted instead of the second approach using the initial damage ω_o.

It is important to point out the used of the anisotropic Banerjee model instead of a isotropic model. Isotropic models would not have given as good as it appears in Fig. 4 and Fig. 5.

The indentification and the determination of the model parameter is practically most important in the usage of a constitutive model. In the sense, the relatively simple and basic form of the time-dependent constitutive relation was developed and used in this research since the complexity due to the model parmeters was not prefered in the mathematical viscous formulation and determining values. The constitutive model could give better predictions if more precise but complicated viscous model was used. Regarding the viscous constitutive modeling, the respective consideration of the initial yield surface and the dynamic loading surface might be another development. The contribution of the anisotropic Banerjee model could be appreciated in that the normally consolidated specimen, on which the model is principally based, was used for the test and the prediction.

It is undesirable that the value of parameter Λ, in this research, was determined from the best match of the test whose result was predicted using the parameter value, eventhough the parameter value was used to simulate the results of the other tests; however, this is proper for the case that the specimen from a construction site is tested then the parameter values determined from the test results are used to predict the soil behavior due to the actual construction.

7. Conclusions

A damage law was incoporated into the combined elastic-plastic-viscous constitutive model, that was developed based on a simple formulation scheme. The generalized Hooke's law was used as the elastic part. The anisotropic Banerjee model was used for the plastic part. The generalized viscous theory was simplified and used for the viscous constitutive part. The mathematical formulation and development of the model was performed from the point of view that fewer parameters better be employed. The creep behavior with or without creep rupture was predicted using the developed model for cohesive soils. Comparing the model prediction and the experimental result for the Bay mud, the following conclusions can be made.

1) The prediction conducted using the proposed constitutive model generally agreed well with the experimental creep strain of undrained creep test for normally consolidated cohesive soils.

2) The anisotropic Banerjee model improved the model accuracy since it is fundamentally based on the normally consolidated clay.

3) The simplification of the generalized viscous theory was successfull for the simulation of the creep behaviors adopted in this research.

4) The inclusion of a damage law made it possible for the model to satisfactorily simulate undrained creep rupture under different stress levels.

5) Despite the simplicity of the constitutive model, it performs well as long as the time to failure ratio of the creep rupture tests is within the same order of magnitude. Otherwise, it becomes necessary to use different damage parameter values for different tests.

Acknowledgments

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