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A Study on the Comparison of Storage Sharing Schemes in Queueing System with Finite Capacity Buffer

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유한 용량의 버퍼를 가지는 대기행렬에서의 저장공간 공유방안 비교에 관한 연구

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본 논문의 목적은 유한 저장공간을 가지는 대기행렬 시스템에서 완전공유(Complete Sharing), 완전분할(Complete Partitioning), 최소할당공유(Sharing with Minimum Allocation)와 같은 다양한 저장공간 공유방안들을 비교·분석하는 것으로, 이를 위하여 먼저 각각의 공유방안에서의 대기행렬 안정상태확률을 효율적으로 구할 수 있는 방법이 제시되었다. 다음으로 각각의 저장공간 공유방안을 특징짓는데 필요한 몇 가지 성질들이 규명되었으며, 이를 토대로 각각의 저장공간 공유방안에 대하여 시스템 성능척도인 생산률들을 도출하는 한편, 이들의 대소관계를 파악하고, 수치실험을 통하여 이를 입증하였다.

Keywords : storage sharing schemes, throughput, steady-state probability, reversibility

1. Introduction

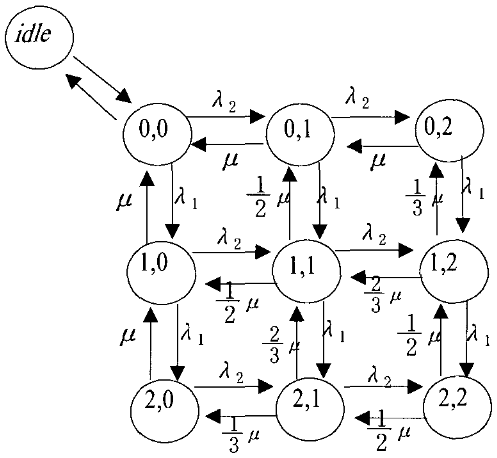
A queueing network with blocking, a set of arbitrarily linked finite queues, is widely used for modelling telecommunications, computer systems and manufacturing systems. An important feature of these systems is that a server can become blocked when the capacity limitation of another queue (i.e., destination) is reached. Because of complexity and dependency among the queues, most analyses are based on approximation method which is usually decomposing queueing networks into individual queues and investigating each individual queue with revised arrival and service processes in isolation [10, 11, 12, 13, 14].

In view of increasing server utilization and reducing work-in-process inventory, it is required to have some storage of proper size at each server, and it may be desirable to provide storage allocation schemes. Often, storage allocation schemes are used to achieve distinct levels of performance for different classes of customers, e.g. voice/data and command and control systems.

The purpose of this paper is to analyze and compare a few existing and/or intuitive storage sharing schemes. The first is the complete partitioning(CP) scheme where actually no sharing is provided, but where the entire finite storage is permanently partitioned for each customer class. At the other extreme is the second scheme, complete sharing(CS), which is such that an arriving customer is accepted if any storage space is available. The third is the sharing with minimum allocation(SMA) scheme where a minimum number of storage is always reserved for each customer class and a common pool of storages is to be shared among all customer class.

These schemes were studied for R M/M/1 queueing systems with first-come firest-served (FCFS) by Kamoun and Kleinrock [7]. Bondi[3] has compared the admitted arrival rates, queue lengths, and performance of finite capacity queues whose storage is segregated by priority class or completely shared.

In this paper, an efficient method for computing the steady state probabilities of a finite capacity queue with exponential service is exploited by using the theory of reversibility. Some interesting properties are derived and the performance measure (throughput) is then compared for all storage sharing schemes. And, some numerical results are presented.



<Figure 1> CP with n=2, b_i=2 and B=4

2. Model Description and Analysis

2.1 Model Description

The system is composed of a single server and finite sharing buffer(storage) with capacity of size B. Each class of customers arrives according to a Poisson process with rate λ_i $(i=1,\cdots,n)$, and the service time for classes of customers is exponentially distributed with rate μ. Accepted customers are served by server on a first-come first-served basis.

Let the states of the system be denoted by $(k_1,\cdots,k_n)$ and (*idle*), where k_i $(i=1,\cdots,n)$ denotes the number of class *i* customers and the state (*idle*) represents the situation that there is no customer within the system. Since the occurrence times are exponentially distributed, the system dynamics can then be described by a Markov process with state space $A=\{(idle), (k_1,\cdots,k_n) \mid 0\le k_i \le b_i\}$, where b_i denotes the maximum number of buffer allowed for class *i* customers.

2.2 Complete Partitioning Scheme

Complete partitioning scheme is that the entire finite buffer is permanently partitioned for each customer class. In case of n=2 (two classes of customers), b_i=2 (buffer capacity with size 2 for each customer) and B=4 (capacity of finite sharing storage), the steady state transition diagrams for the CP scheme is shown in <Figure 1.>

The steady state probabilities can be obtained by using the associated balance equations. However, as the number of classes and storage capacity increase, such a balance equation approach may get difficult to analyze the system due to the multi-dimensional inter-dependency complexity incurred on the system state space. In spite of the complexity nature, it is fortunate enough to show that the theory of reversibility can be applied to determine the steady state probabilities easily without trying to solve any balance equations directly.

The solution procedure based on the theory of reversibility is the same as those (Lemma 1, Theorem 1, Lemma 2, Theorem 2) in Sung and Kwon [13], and the steady state probabilities are derived as follows.

Lemma 1. (refer to Sung and Kwon [13])

In case of complete partitioning scheme, the steady state probability of the system is derived as

$$\Pi_{cp}(k_1,\cdots,k_n) = P(k_1,\cdots,k_n)/G_{cp}$$
$$\Pi_{cp}(idle) = P(idle)/G_{cp} \quad \cdots\cdots (2.1)$$

for states $(idle)$ and $(k_1,\cdots,k_n)\in\Phi$,

where

$$\Phi=\{(k_1,\cdots,k_n) \mid 0\le k_i\le b_i, 0\le i\le n\},$$
$$G_{cp}=\sum_{(k_1,\cdots,k_n)\in\Phi} P(k_1,\cdots,k_n)+P(idle),$$
$$P(k_1,\cdots,k_n)=(1-\rho)\cdot\rho^{(k_1+\cdots+k_n)} \cdot \frac{(k_1+\cdots+k_n)!}{k_1!\cdots k_n!}\cdot q_1^{k_1}\cdots q_n^{k_n}$$

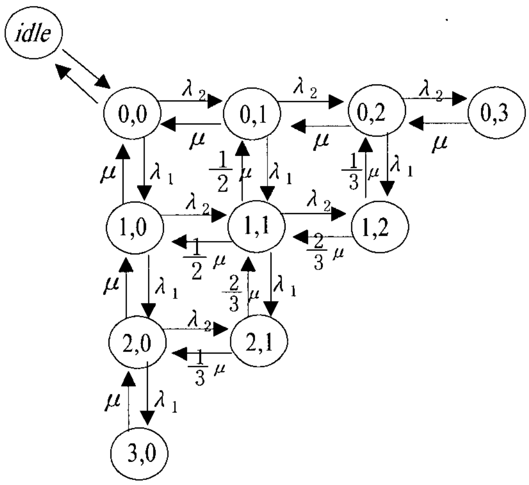
$$P(idle) = 1 - \rho,$$

$$\rho = \sum_{i=1}^{n} \lambda_i / \mu,$$

$$q_i = \lambda_i \ / \ \sum_{i=1}^{n} \lambda_i.$$

2.3 Complete Sharing Scheme

Complete sharing scheme is that all buffers are allocated on a FCFS basis. In case of n=2 (two classes of customers) and B=3(capacity of finite sharing storage), the steady state transition diagrams for the CS scheme is shown in Figure 2.



<Figure 2> CS with n=2 and B=3

The solution procedure is similar to Lemma 1 and the steady state probabilities are derived as follows.

Lemma 2.
In case of complete sharing scheme, the steady state probability of the system is derived as

$$\Pi_{cs}(k_1, \cdots, k_n) = P(k_1, \cdots, k_n) / G_{cs}$$
$$\Pi_{cs}(idle) = P(idle) / G_{cs} \quad \cdots\cdots (2.2)$$

for states $(idle)$ and $(k_1, \cdots, k_n) \in \Psi$,
where

$$\Psi = \left\{ (k_1, \cdots, k_n) \mid \sum_{i=1}^{n} k_i \le B \right\},$$

$$G_{cs} = \sum_{(k_1, \cdots, k_n) \in \Psi} P(k_1, \cdots, k_n) + P(idle),$$

$$P(k_1, \cdots, k_n) = (1-\rho) \cdot \rho^{(k_1 + \cdots + k_n)} \cdot \frac{(k_1 + \cdots + k_n)!}{k_1! \cdots k_n!} \cdot q_1^{k_1} \cdots q_n^{k_n}$$

$$P(idle) = 1 - \rho,$$

$$\rho = \sum_{i=1}^{n} \lambda_i / \mu,$$

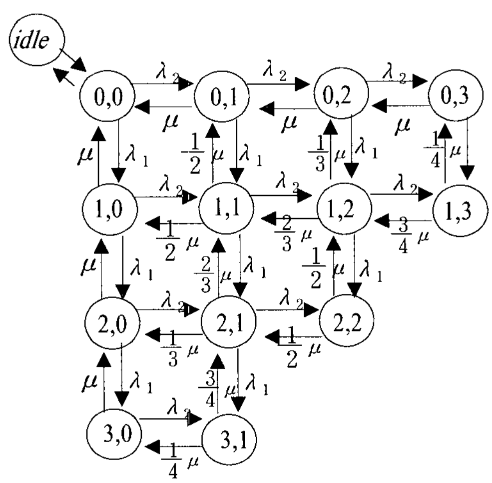
$$q_i = \lambda_i \ / \ \sum_{i=1}^{n} \lambda_i.$$

Proof.
The system of CS scheme has state space $\Psi = \left\{ (k_1, \cdots, k_n) \mid \sum_{i=1}^{n} k_i \le B \right\}$ which is truncated from the state space of infinite capacity buffer and the associated Markov process is reversible. From the result of Corollary 1.10(refer to Kelly [9]), the steady state probability of the system is derived in Eq. (2.2) for states $(idle)$ and $(k_1, \cdots, k_n) \in \Psi$.
This completes the proof.

2.4 SMA Scheme

The sharing with minimum allocation scheme is that a minimum number of buffers are reserved for each customer class and the remaining buffers are pooled and allocated on a FCFS basis. In case of n=2 (two classes of customers), m_i=1 (reserved buffer capacity with size 1 for each customer) and B=4 (capacity of finite sharing storage), the steady state transition diagrams for the SMA scheme is shown in Figure 3.

The solution procedure is similar to Lemma 1 and the steady state probabilities are derived as follows.



<Figure 3> SMA with n=2, m_i=1 and B=4

Lemma 3.

In case of SMA scheme, the steady state probability of the system is derived as

$$\Pi_{sma}(k_1, \cdots, k_n) = P(k_1, \cdots, k_n)/ G_{sma}$$
$$\Pi_{sma}(idle) = P(idle)/ G_{sma} \quad \cdots\cdots (2.3)$$

for states $(idle)$ and $(k_1, \cdots, k_n) \in \Omega$,
where

m_i = minimum buffer size for class i,

$$\Omega = \left\{ (k_1, \cdots, k_n) \mid \sum_{i=1}^{n} k_i \le B, \text{ and } 0 \le k_i \le m_i + B - \sum_{i=1}^{n} m_i \right\}$$

$$G_{sma} = \sum_{(k_1, \cdots, k_n) \in \Omega} P(k_1, \cdots, k_n) + P(idle),$$

$$P(k_1, \cdots, k_n) = (1-\rho) \cdot \rho^{(k_1 + \cdots + k_n)} \cdot \frac{(k_1 + \cdots + k_n)!}{k_1! \cdots k_n!} \cdot q_1^{k_1} \cdots q_n^{k_n}$$

$$P(idle) = 1 - \rho,$$

$$\rho = \sum_{i=1}^{n} \lambda_i / \mu,$$

$$q_i = \lambda_i \,/ \sum_{i=1}^{n} \lambda_i.$$

Proof.

The system of SMA scheme has state space $\Omega = \left\{ (k_1, \cdots, k_n) \mid \sum_{i=1}^{n} k_i \le B \text{ and } 0 \le k_i \le m_i + B - \sum_{i=1}^{n} m_i \right\}$ which is truncated from the state space of infinite capacity buffer and the associated Markov process is reversible. From the result of Corollary 1.10(refer to Kelly [9]), the steady state probability of the system is derived in Eq. (2.3) for states $(idle)$ and $(k_1, \cdots, k_n) \in \Omega$.

This completes the proof.

3. Comparison of Sharing Schemes

In this section, the throughput measure is obtained and compared for the finite storage sharing schemes. And, some interesting properties are derived that are useful for characterizing the finite storage sharing schemes.

Let TH_{cp}, TH_{cs} and TH_{sma} denote the throughput of CP, CS and SMA, respectively. By using the definition of throughput and steady state probabilities for each schemes, the throughput TH_i can be derived as follows :

$$TH_i = \mu \cdot \left(1 - \frac{1-\rho}{G_i}\right) \quad \cdots\cdots (3.1)$$

for all i (= cp, cs and sma)

And, the throughput TH_i is characterized as follows:

Property 1.

In the case of complete partitioning scheme, The throughput TH_{cp} is a monotonically increasing concave function of its buffer size.

Proof.

Let $TH_{cp}(b_1, \cdots, b_n)$ be the throughput of CP with buffer size b_i for each customer class. Then,

$$TH_{cp}(b_1, \cdots, b_n) = \mu \cdot \left(1 - \frac{1-\rho}{G_{cp}}\right) = \mu \cdot \left[1 - \frac{1}{Q(b_1, \cdots, b_n)}\right].$$

where,

$$G_{cp} = (1-\rho)\left[1 + \sum_{k_1=0}^{b_1} \cdots \sum_{k_n=0}^{b_n} \rho^{k+1} \cdot \frac{(k_1 + \cdots + k_n)!}{k_1! \cdots k_n!} q_1^{k_1} \cdots q_n^{k_n}\right],$$

$$Q(b_1, \cdots, b_n) = 1 + \sum_{k_1=0}^{b_1} \cdots \sum_{k_n=0}^{b_n} \rho^{k+1} \cdot \frac{(k_1 + \cdots + k_n)!}{k_1! \cdots k_n!} q_1^{k_1} \cdots q_n^{k_n},$$

$$k = k_1 + \cdots + k_n.$$

Let

$$Q_1 = \sum_{k_1=0}^{b_1} \cdots \sum_{k_i=b_i+1}^{b_i+1} \cdots \sum_{k_n=0}^{b_n} \rho^{k+1} \cdot \frac{(k_1 + \cdots + k_n)!}{k_1! \cdots k_n!} q_1^{k_1} \cdots q_n^{k_n},$$

$$Q_2 = \sum_{k_1=0}^{b_1} \cdots \sum_{k_i=b_i+2}^{b_i+2} \cdots \sum_{k_n=0}^{b_n} \rho^{k+1} \cdot \frac{(k_1 + \cdots + k_n)!}{k_1! \cdots k_n!} q_1^{k_1} \cdots q_n^{k_n}.$$

These lead to the relation $Q_1 > Q_2$, and it holds that

$$Q(b_1, \cdots, b_i+1, \cdots, b_n) = Q(b_1, \cdots, b_i, \cdots, b_n) + Q_1,$$

$$Q(b_1, \cdots, b_i+2, \cdots, b_n) = Q(b_1, \cdots, b_i, \cdots, b_n) + Q_1 + Q_2.$$

By the definition of $TH_{cp}(b_1, \cdots, b_n)$,

$$\begin{aligned} & TH_{cp}(b_1, \cdots, b_i+1, \cdots b_n) - TH_{cp}(b_1, \cdots, b_n) \\ &= \mu \cdot \frac{1}{Q(b_1, \cdots, b_n)} - \mu \cdot \frac{1}{Q(b_1, \cdots, b_i+1, \cdots, b_n)} \\ &= \mu \cdot \frac{1}{Q(b_1, \cdots, b_n)} - \mu \cdot \frac{1}{Q(b_1, \cdots, b_n) + Q_1} \\ &> 0 \quad \text{for all } x_i. \end{aligned}$$

And,

$$\begin{aligned} & 2 \cdot TH_{cp}(b_1, \cdots, b_i+1, \cdots, b_n) - TH_{cp}(b_1, \cdots, b_n) - TH_{cp}(b_1, \cdots, b_i+2, \cdots, b_n) \\ &= \mu\left[\frac{-2}{Q(b_1, \cdots, b_i+1, \cdots, b_n)} + \frac{1}{Q(b_1, \cdots, b_n)} + \frac{1}{Q(b_1, \cdots, b_i+2, \cdots, b_n)}\right] \\ &= \mu\left[\frac{-2}{Q(b_1, \cdots, b_n) + Q_1} + \frac{1}{Q(b_1, \cdots, b_n)} + \frac{1}{Q(b_1, \cdots, b_n) + Q_1 + Q_2}\right] \\ &= \mu \cdot [Q(b_1, \cdots, b_n)(Q_1 - Q_2) + Q_1^2 + Q_1 Q_2] \\ &\quad \cdot \frac{1}{[Q(b_1, \cdots, b_n) + Q_1][Q(b_1, \cdots, b_n]} \cdot \frac{1}{[Q(b_1, \cdots, b_n) + Q_1 + Q_2]} \\ &> 0 \quad \text{for all } b_i. \end{aligned}$$

Thus, the throughput is a monotonically increasing concave function of buffer size.
This completes the proof.

Property 2.
In the case of complete sharing scheme, the throughput TH_{cs} is a monotonically increasing concave function of its buffer size.

Proof.
Let $TH_{cs}(B)$ be the throughput of CS with buffer size K. Then,

$$\begin{aligned} TH_{cs}(B) &= \mu\left(1 - \frac{1-\rho}{G_{cs}(B)}\right) \\ &= \mu\left(1 - \frac{1-\rho}{1-\rho^{B+1}}\right) \end{aligned}$$

And,

$$\begin{aligned} & TH_{cs}(B+1) - TH_{cs}(B) \\ &= \mu\left(\frac{1}{1+\rho+\cdots+\rho^{B}} - \frac{1}{1+\rho+\cdots+\rho^{B+1}}\right) \\ &> 0. \end{aligned}$$

Also,

$$\begin{aligned} & 2\, TH_{cs}(B+1) - TH_{cs}(B) - TH_{cs}(B+2) \\ &= 2\mu\left(1 - \frac{1-\rho}{1-\rho^{B+2}}\right) - \mu\left(1 - \frac{1-\rho}{1-\rho^{B+1}}\right) - \mu\left(1 - \frac{1-\rho}{1-\rho^{B+3}}\right) \\ &= \mu(1-\rho)\left[\frac{1}{1-\rho^{B+1}} + \frac{1}{1-\rho^{B+3}} - \frac{2}{1-\rho^{B+2}}\right] \\ &> 0. \end{aligned}$$

Thus, the throughput is a monotonically increasing concave function of buffer size.
This completes the proof.

Property 3.
In the case of SMA scheme, the throughput TH_{sma} is a monotonically increasing concave function of its buffer size.

Proof.
It can easily be shown according to the procedure of previous properties.

This completes the proof.

The performance of classes is influenced not only by the buffer size but also by the choice of allocation scheme. Also, when the total arrival rate λ is given, the throughput is influenced by the ratio of class input rates. For the case of CS, the throughput is independent of the input rates ratio. However, in the case of SMA and CP, the throughput is dependent on the ratio of input rates and is maximized at equal ρ_i's.

Property 4.

In the case of SMA scheme, if the total arrival rate λ is given, the throughput is maximized at the same ρ_i $(i=1, \cdots, n)$ for each class customers.

Proof.

For simplification, the proof will be completed only for the case of n=2, B=3 and $m_1 = m_2 = 1$.

Let $TH_{sma}{}^{b}$ and $TH_{sma}{}^{nb}$ be the throughput of the balanced input rate case (that is, the same ρ_i), and the other one, respectively.

$$TH_{sma}{}^{b} - TH_{sma}{}^{nb}$$
$$= \mu(1-\frac{1-\rho}{G_{sma}{}^{b}}) - \mu(1-\frac{1-\rho}{G_{sma}{}^{nb}})$$
$$= \mu(1-\rho)\frac{(G_{sma}{}^{b} - G_{sma}{}^{nb})}{G_{sma}{}^{b} \cdot G_{sma}{}^{nb}}.$$

And, by the definition of G

$$G_{sma}{}^{b} - G_{sma}{}^{nb}$$
$$= (1-\rho)[(1+\rho+\rho^2+\rho^3+3\rho^4 q^b{}_1 q^b{}_2)$$
$$-(1+\rho+\rho^2+\rho^3+3\rho^4 q^{nb}{}_1 q^{nb}{}_2)]$$
$$= 3(1-\rho)\rho^4(q^b{}_1 q^b{}_2 - q^{nb}{}_1 q^{nb}{}_2).$$

Since $q^b{}_1 = q^b{}_2 = 1/2$, $q^{nb}{}_1 + q^{nb}{}_2 = 1$, and $q^{nb}{}_1 q^{nb}{}_2$ is maximized at $q^{nb}{}_1 = q^{nb}{}_2 = 0.5$, $q^b{}_1 q^b{}_2 - q^{nb}{}_1 q^{nb}{}_2 \geq 0$.

Therefore, $TH_{sma}{}^{b} - TH_{sma}{}^{nb} \geq 0$.

This complete the proof.

Property 5.

In the case of CP scheme, if the total arrival rate λ is given, the throughput is maximized at the same ρ_i $(i=1, \cdots, n)$ for each class customers.

Proof

For simplification, the proof will be completed only for the case of n=2, B=2 and $b_1 = b_2 = 1$.

Let $TH_{cp}{}^{b}$ and $TH_{cp}{}^{nb}$ be the throughput of the balanced input rate case (that is, the same ρ_i) and the other one, respectively.

$$TH_{cp}{}^{b} - TH_{cp}{}^{nb}$$
$$= \mu(1-\frac{1-\rho}{G_{cp}{}^{b}}) - \mu(1-\frac{1-\rho}{G_3{}^{nb}})$$
$$= \mu(1-\rho)\frac{(G_{cp}{}^{b} - G_{cp}{}^{nb})}{G_{cp}{}^{b}\, G_{cp}{}^{nb}}.$$

And, by the definition of G,

$$G_{cp}{}^{b} - G_{cp}{}^{nb}$$
$$= (1-\rho)[(1+\rho+\rho^2+2\rho^3 q^b{}_1 q^b{}_2)$$
$$-(1+\rho+\rho^2+2\rho^3 q^{nb}{}_1 q^{nb}{}_2)]$$
$$= 2(1-\rho)\rho^3(q^b{}_1 q^b{}_2 - q^{nb}{}_1 q^{nb}{}_2).$$

Since $q^b{}_1 = q^b{}_2 = 1/2$, $q^{nb}{}_1 + q^{nb}{}_2 = 1$, and $q^{nb}{}_1 q^{nb}{}_2$ is maximized at $q^{nb}{}_1 = q^{nb}{}_2 = 0.5$, $q^b{}_1 q^b{}_2 - q^{nb}{}_1 q^{nb}{}_2 \geq 0$.

Therefore, $TH_{cp}{}^{b} - TH_{cp}{}^{nb} \geq 0$.

This complete the proof.

Finally, the following result can be obtained by the comparison of the throughputs for finite storage allocation schemes.

Theorem 1.

If the capacity of finite sharing storage is the same as B for all buffer allocation schemes, the relationship of throughputs for each scheme is derived as follows:

$$TH_{cp} < TH_{sma} < TH_{cs} \quad \cdots\cdots (3.2)$$

Proof.

By using the Eq. (3.1),

$$TH_{cs} - TH_{sma}$$
$$= \mu(1-\frac{1-\rho}{G_{cs}}) - \mu(1-\frac{1-\rho}{G_{sma}})$$
$$= \mu(1-\rho)\frac{(G_{cs}-G_{sma})}{G_{cs}G_{sma}}$$

And,

$$G_{cs} - G_{sma}$$
$$= \sum_{(k_1,\cdots,k_n)\in\Psi} P(k_1,\cdots,k_n) + P(idle)$$
$$- \sum_{(k_1,\cdots,k_n)\in\Omega} P(k_1,\cdots,k_n) + P(idle)$$

Since $\Omega \subset \Psi$, $G_{cs} - G_{sma} > 0$.

Therefore, $TH_{cs} - TH_{sma} > 0$.

Moreover,

$$TH_{sma} - TH_{cp}$$
$$= \mu(1-\frac{1-\rho}{G_{sma}}) - \mu(1-\frac{1-\rho}{G_{cp}})$$
$$= \mu(1-\rho)\frac{(G_{sma}-G_{cp})}{G_{sma}G_{cp}}$$

And,

$$G_{sma} - G_{cp}$$
$$= \sum_{(k_1,\cdots,k_n)\in\Omega} P(k_1,\cdots,k_n) + P(idle)$$
$$- \sum_{(k_1,\cdots,k_n)\in\Phi} P(k_1,\cdots,k_n) + P(idle)$$

Since $\Phi \subset \Omega$, $G_{sma} - G_{cp} > 0$.

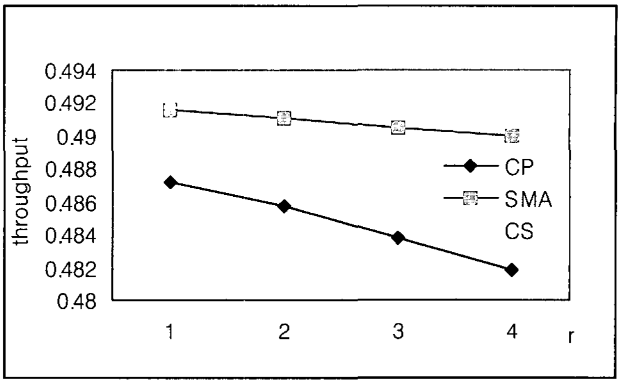
Therefore, $TH_{sma} - TH_{cp} > 0$

Thus, $TH_{cp} < TH_{sma} < TH_{cs}$.

This completes the proof.

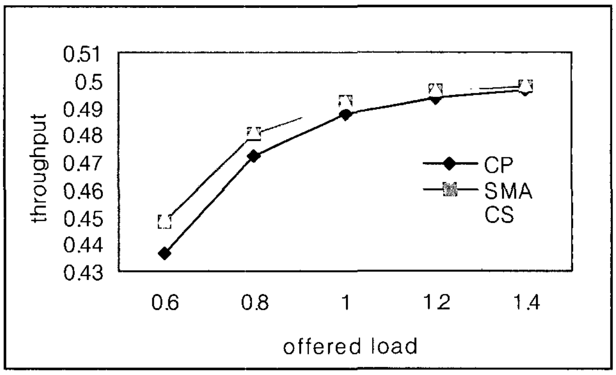
In order to illustrate the above results, the throughput of the finite storage sharing schemes are compared for the case of n=2, B=4, $m_1 = m_2 = 1$ and $b_1 = b_2 = 1$.

At first, the comparison of throughput according to the ratio of class input rates is considered with parameter set 1 (λ=1, μ=0.5). The computational results are shown in Figure 4, where r denotes the ratio of class input rates (= λ_2/λ_1). The results imply that the throughput is maximized at the same ρ_i (r=1) for each scheme, and $TH_{cp} < TH_{sma} < TH_{cs}$. That is, when the total arrival rate λ is given, the throughput is influenced by the ratio of class input rates and by the buffer allocation schemes.



<Figure 4> Throughputs according to r

At second, the comparison of throughput according to the offered load ($\rho = \lambda/\mu$) is considered with parameter set 2 (r=1, μ=0.5). The computational results present that $TH_{cp} < TH_{sma} < TH_{cs}$, and the throughput for each allocation scheme is a monotonically increasing concave function of the offered load as depicted in Figure 5.



<Figure 5> Throughputs according to the offered load

4. Conclusions

In this paper, the finite storage sharing schemes which are the complete sharing (CS), complete partitioning(CP) and sharing with minimum allocation (SMA) are analyzed and compared. And, an efficient method is exploited to compute the steady state probabilities for each allocation schemes.

Moreover, some interesting properties are derived that are useful for characterizing the finite storage sharing schemes. By using the results, the system performance measure (throughput) are obtained for all schemes.

Further research is to extend these results to the design and control of related system such as automated container terminals and automated storage/retrieval systems(AS/RS).

5. References

[1] Akyildiz, I.F. ; "Exact Product Form Solution for Queueing Networks with Blocking," IEEE Transaction on Computers, 36(1) : 122-125, 1987.

[2] Baskett, F., Chandy, M., Muntz, R.R. and Palacios, F.G.; "Open, Closed and Mixed Networks of Queues with Different Classes of Customers," Journal of Association for Computing Machinery, 22 : 233-248, 1975.

[3] Bondi, A.B. ; "An Analysis of Finite Capacity Queues with Priority Scheduling and Common or Reserved Waiting Areas," Computers & Operations Research, 16(3) : 217-233, 1989.

[4] Dowdy, L.W., Eager, D.L., Gordon, K.D. and Saxton, L.V. ; "Throughput Concavity and Response Time Convexity," Information Processing Letters, 19 : 209-212, 1984.

[5] Gross, D. and Harris, C.; Fundamentals of Queueing Theory, John Wiely & Sons, New York, 1985.

[6] Haskose, A., Kingsman B.G., and Worthington, D. ; "Modelling Flow and Jobbing Shops as a Queueing Network for Workload Control," International Journal of Production Economics, 78 : 271-285, 2002

[7] Kamoun, F., and Kleinrock, L. ; "Analysis of Finite Storage in a Computer Network Node Environment under General Traffic Condition," IEEE Transactions Communication, 28(7) : 992-1003, 1980.

[8] Kapadia, A.S., Kazmi, M.H., and Mitchell, A.C. ; "Analysis of Finite Capacity Non-Preemptive Priority Queue," Computers & Operations Research, 11(3) : 337-343, 1984.

[9] Kelly, F.P.; Reversibility and Stochastic Networks, John Wiley & Sons, New York, 1979.

[10] Lee, H.S., and Pollock, S.M. ; "Approximation Analysis of Open Acyclic Exponential Queueing Networks with Blocking," Operations Research, 38 : 1123-1134, 1990.

[11] Ma, J. and Matsui, M. ; "Perfomance Evaluation of a Flexible Machining /Assembly System and Routing Comparisons" International Journal of Production Research, 40(7) : 1713-1724, 2002.

[12] Rupe, J. and Kuo, W.; "Performability of FMS based on Stochastic Process Models," International Journal of Production Research, 39(1) : 139-155, 2001.

[13] Sung, C.S. and Kwon, S.T. ; "Performance Modelling of an FMS with Finite Input and Output Buffers," International Journal of Production Economics, 37 : 161-175, 1994.

[14] Xiaobo, Z., Gong, Q. and Wang, J. ; "On Control Strategies in a Multi-Stage Production System," International Journal of Production Research, 40(5) : 1155-1171, 2002