

# Robust Current Control for Permanent Magnet Synchronous Motors by the Inverse LQ Method

## – An Evaluation of Control Performance Using Servo-Locks at Low Speed –

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### ABSTRACT

This paper describes the optimal current-control of a permanent magnet synchronous motor by the use of robust and simple current controllers, based upon the analytical procedure known as the inverse LQ (ILQ) design method. The ILQ design method is a strategy for finding the optimal gains based on pole assignment without solving the Riccati equation. It is very important to keep the motor in robust servo-lock. By experiments and simulations, we will show that the ILQ optimal servo-system with servo-lock is more insensitive at low speeds to variations in armature inductance than the standard PI servo-system. Variations in armature inductance have the greatest influence on the responses of a servo-system.

**Keywords:** permanent magnet synchronous motor, current control, optimal servo system, LQ problem, ILQ design method

### 1. Introduction

Due to the development of strong permanent magnets and advanced control technologies, PWM inverter fed permanent magnet synchronous motors (PMSMs) are compact and have high efficiency<sup>[1]</sup>. As a result, their applications, including industrial machines, electrification products, electric vehicles, health and environmental monitoring equipment and information devices, have recently been expanding remarkably<sup>[2]</sup>.

A PMSM is a non-linear system in which the  $d$ - and  $q$ -axis systems interact with one another. A current control method which allows the systems to be

simultaneously decoupled and linearised has been proposed<sup>[3][4]</sup>. For applications with relatively large control delay times such as railway trains, it is necessary to add a lagging compensator to the basic control method in order to prevent performance loss and instability from control delays<sup>[5]-[7]</sup>.

PI controllers have a very simple structure and have many practical uses. However, the current control systems of PMSMs regulated by the PI controller are vulnerable to variations in the motor's parameters which lead to poor robust-performance and little robust-stability. This is because the PI controller introduces objective responses by canceling out the poles of the plant with the largest time-constant by the zeros of the controller<sup>[8]-[10]</sup>.

Generally, an optimal regulator derived from the linear quadratic (LQ) problem satisfies the return difference condition and it has the following advantages:

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- 1) the gain margin is infinity,
- 2) the reduction in gain is at least 50%,
- 3) the phase margin is not less than  $\pm 60^\circ$ ,
- 4) it is not sensitive to variations of parameters.

But the Riccati equation must be solved to find the optimal gains. The procedure for solving the Riccati equation is complex and the relations between the solution and the specification of responses in the servo system are not clear<sup>[11][12]</sup>.

The inverse LQ (ILQ) design method<sup>[11]-[14]</sup> is a strategy to find the optimal gains on the basis of pole assignment without having to solve the Riccati equation. It has the following advantages:

- 1) the transfer function of the servo-system can be asymptotically designated for the required specifications,
- 2) the optimal solutions are analytically obtained and their optimality is guaranteed,
- 3) the optimal gains can easily be adjusted to obtain desired responses at the point of use.

For these reasons the ILQ design method can be used to construct an excellent controller. Thus, many applications of the ILQ design method have been reported<sup>[15]-[19]</sup>.

In this paper we will present a design for a simple and responsive current controller for use with PMSMs which does not use a lagging compensator. This will be accomplished by using an extended ILQ design method which will enable us to obtain an analytical solution. The optimal gains to minimize the settling time for step response without overshoot have already been calculated<sup>[9]</sup>. In spite of its simple structure, the proposed ILQ designed servo-controller is more insensitive to the motor's parameter changes than standard PI-controllers. This has already been confirmed by numerical simulations<sup>[10]</sup>. Moreover, a unique cost function has been introduced which uses  $\mu$ -analysis to provide a quantitative evaluation of the stability and performance of an ILQ optimal current controller<sup>[20]</sup>. We can demonstrate analytically that the ILQ controller performs much better and more stably than standard PI-controllers despite variations of the motor's parameters.

This paper verifies experimentally that the ILQ optimal current-controller for PMSMs is more robust at low speeds than the standard PI servo-controller. This even

applies to variations of armature inductance which have the largest influence on the responses of the servo system. Experimental results correspond well with the numerical simulations. It is confirmed that the ILQ optimal current controller is highly insensitive to changes in the motor's parameters.

## 2. ILQ optimal current-controller<sup>[21][22]</sup> in a PMSM

In the current control of a PMSM, decoupled control leads to a linear state equation in a rotor flux ( $dq$ ) reference frame rotating with the rotor angular velocity<sup>[23]</sup>. Then state variables of the state equation are independent on each other and the state equation is given as follows<sup>[8]</sup><sup>[9]</sup>:

$$\frac{d}{dt}\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \quad \mathbf{y} = \mathbf{C}\mathbf{x} \quad (1)$$

and

$$\mathbf{x} \in \mathcal{R}^2 = [i_{1d} \ i_{1q}]^T, \quad \mathbf{u} \in \mathcal{R}^2 = [v_{1d}^c \ v_{1q}^c]^T, \quad \mathbf{y} \in \mathcal{R}^2:$$

$$\text{output, } \mathbf{A} \in \mathcal{R}^{2 \times 2} = \text{diag}(a_1, a_2) = -r_1 \mathbf{B}, \quad \mathbf{B} \in \mathcal{R}^{2 \times 2} \\ = \text{diag}(1/L_{1d}, 1/L_{1q}), \quad \mathbf{C} = \mathbf{I}_2$$

where,  $\mathcal{R}^2$  and  $\mathcal{R}^{2 \times 2}$  denote a 2-vector and a  $2 \times 2$ -matrix with real numbers, respectively.  $\mathbf{I}_2$  is a  $2 \times 2$  identity matrix, and  $[\ ]^T$  is the transpose of the matrix.  $i_{1d}$  and  $i_{1q}$  represent the  $d$ - and  $q$ -axis components of the armature current in the  $dq$  reference frame, respectively.  $v_{1d}^c$  and  $v_{1q}^c$  denote the decoupled and linearised  $d$ - and  $q$ -axis components of the armature voltage in the  $dq$  reference frame, respectively.  $r_1$  is armature resistance.  $L_{1d}$  and  $L_{1q}$  are the  $d$ - and  $q$ -axis self-inductances of the armature winding in the  $dq$  reference frame.

The system: 1) is controllable and observable, 2) has a minimal-phase, 3) has no zeros at the origin. Thus, we can apply the ILQ design method to the current control for the PMSM<sup>[8]</sup>.

**Proof<sup>[22]</sup>:**

- 1) Controllable and observable system

The Following controllable and observable

conditions can be derived:

$$\text{rank}[B \ AB]=2 \text{ and } \text{rank} \begin{bmatrix} C \\ CA \end{bmatrix} = 2 .$$

Because, from (1) both matrices  $B$  and  $C$  are nonsingular.

2) Minimal-phase system

In the transfer function  $G(s)=C(sI_2 - A)^{-1}B$  of (1), a solution  $z$  satisfying  $\text{rank}[G(s)]<2$  represents the zeros of the function  $G(s)$ , that is, the lines of  $G(s)$  form a linear dependence. And the following condition can be formed:

$$\text{rank} \begin{bmatrix} zI_2 - A & -B \\ C & 0 \end{bmatrix} = 4 .$$

Thus this system does not have unstable zeros and is a minimal-phase system.

3) No zero at origin

In (1), the matrix  $A$  is also nonsingular, that is

$$\text{rank} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = 4 .$$

Therefore, this system has no zeros at the origin.

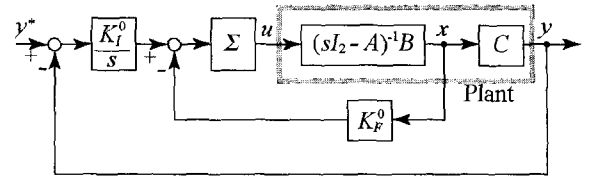


Fig. 1 ILQ optimal servo-system

When  $\sigma_i$  is sufficiently larger than  $|a_i|$ , from the second term of (4) the variations in  $a_i$  are cancelled by  $\sigma_i$ . Therefore, the ILQ servo-system itself is essentially robust.

Figs. 2 (a) and (b) show a typical-ILQ and a standard-PI servo-controller with decoupled control laws to decouple the currents  $i_{1d}$  and  $i_{1q}$ .  $i_{1d}^*$  and  $i_{1q}^*$  represent commands of the currents  $i_{1d}$  and  $i_{1q}$  in the  $dq$  reference frame.  $v_{1d}^*$  and  $v_{1q}^*$  denote commands of the motor voltages  $v_{1d}$  and  $v_{1q}$  in the  $dq$  reference frame.  $\omega_r$  is the rotor speed represented in electrical angular velocity, and  $\lambda_f$  is the PM flux interlinkage.

Both ILQ and PI controllers have one-degree-of-

Fig. 1 shows a typical ILQ optimal servo-system. The constants  $K_F^0 \in \mathcal{R}^{2 \times 2}$  and  $K_I^0 \in \mathcal{R}^{2 \times 2}$  are basic feedback and integral gains, respectively. A positive-definite diagonal-matrix  $\Sigma \in \mathcal{R}^{2 \times 2} = \text{diag}(\sigma_1, \sigma_2)$ ,  $\sigma_1, \sigma_2 > 0$  is the gain adjusting parameter.

Basic optimal gains are obtained as follows<sup>[9] [10]</sup>:

$$\begin{bmatrix} K_F^0 & K_I^0 \end{bmatrix} = \begin{bmatrix} L_{1d} & 0 & -L_{1d}s_1^* & 0 \\ 0 & L_{1q} & 0 & -L_{1q}s_2^* \end{bmatrix} \quad (2)$$

where  $s_1^*$  and  $s_2^*$  denote assigned poles.

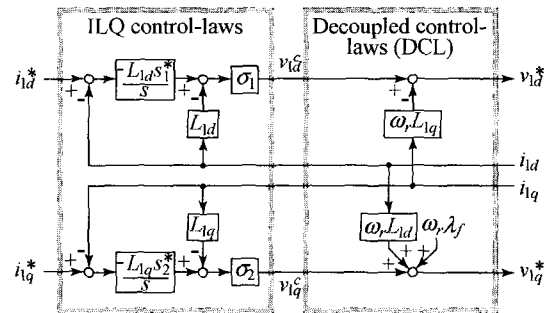
Moreover, the optimal condition is obtained as follows:

$$\sigma_i > \underline{\sigma}_i = \begin{cases} 2(a_i - s_i^*) & \text{if } s_i^* < a_i \\ 0 & \text{if } a_i \leq s_i^* < 0 \end{cases} \quad (3)$$

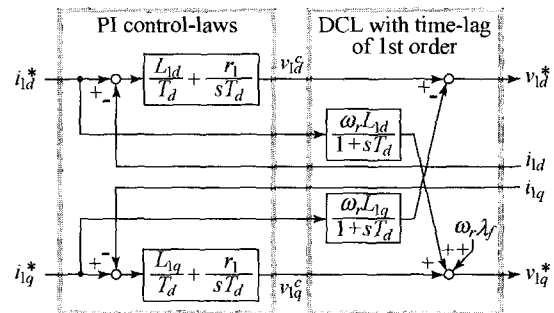
where  $i=1, 2$  and  $\underline{\sigma}_i$  is the lowest limit of  $\sigma_i$ .

The characteristic equation is:

$$s^2 + (\sigma_i - a_i)s - \sigma_i s_i^* = 0 . \quad (4)$$



(a) ILQ servo-controller



(b) PI servo-controller

Fig. 2 Servo-controllers for current control of the PMSM

freedom because there is no feed-forward loop in Figs.2 (a) and (b).

In Fig. 2 (b), the standard-PI servo-controller acts as a filter on the two current components with first-order time-lag causing the system to operate stably, where  $T_d$  is a time constant of the first-order time-lag. Comparing Fig. 2 (a) with Fig. 2 (b), the ILQ servo-controller does not use the motor's  $r_1$  parameter which leads to a simplified structure.

Table 1 shows the rated parameters, while Table 2 shows the rated constants of a tested PMSM. In Table 2 the superscript "0" denotes nominal values. The motor is a cylindrical rotor type, so that we can set a condition  $L_1 = L_{1d} = L_{1q}$ , where  $L_1$  is self inductance of the armature winding.

Table 1 Rated parameters of the tested motor.

Output power	1.3 kW
Speed	1500 rpm
Torque	8.34 N-m
No. of poles	8
Phase current	10.7 A

Table 2 Constants of the tested motor.

$r_1^0$ (at 11.5degree)	0.3162 $\Omega$
$L_1^0 = L_{1d}^0 = L_{1q}^0$	0.00178783 H
$\lambda_f^0$	0.11896 Wb
EMF (at 1000 rpm)	49.83 V
$J$	0.00123 kg-m <sup>2</sup>
$D$	0.036607 N-m/rad/s

### 3. Simulations and experiments for variation of the motor's parameters

Fig. 3 is a block diagram showing the set up for speed control of the PMSM. The control blocks are composed of a speed regulator, an ILQ or PI servo-controller with decoupled control and  $dq$  transformations ( $d$ - $q$  axes to three-phase and three-phase to  $d$ - $q$  axes). The simple PI controller is adopted as a speed regulator and the current

command  $i_{1d}^*$  is fixed at zero. Controlling the motor at  $i_{1d} = 0$  has many advantages, including: 1) the demagnetizing effect of  $i_{1d}$  on the PM will be suppressed; 2) the torque control law is simplified, that is, the controlled torque is proportional to the current  $i_{1q}$ .

In the experiments, it is difficult to change the motor's parameters in such a way that the controller's parameters are changed. This make it hard to evaluate robust performance.

### 3.1 Numerical simulations

Fig. 4 and Fig. 5 show the numerical simulations of the step responses to step speed command for the ILQ and PI servo-systems, respectively. The solid lines on the plots show responses, while the dotted lines show commands. The voltage and kinetic equations of the PMSM are solved at a sampling time of  $1\mu s$  and calculations of the current controllers are done at a sampling time of 0.1ms using Microsoft Visual C/C++.

In Fig. 4 and Fig. 5, Fig. (a) shows the results at the rated value of the motor's parameters. Figs. (b) and (c) are the results at one-half the rated value and at two times the rated value of  $r_1$ , and Figs. (d) and (e) are the results at one-half the rated value and at two times the rated value of  $L_1$ , respectively. Figs. (f) and (g) show the results at 0.8 times the rated value and at 1.2 times the rated value of  $\lambda_f$ .

Comparing Figs. 4 (b) and (c) with Figs. 5 (b) and (c), the variations of state variables due to the error of armature resistance  $r_1$  are very small in both the ILQ and the PI servo systems.

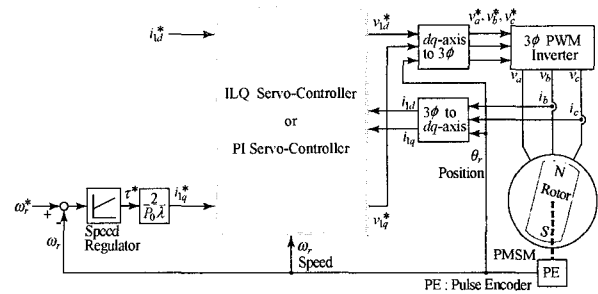


Fig. 3 Block-diagram of the system for speed control of the PMSM

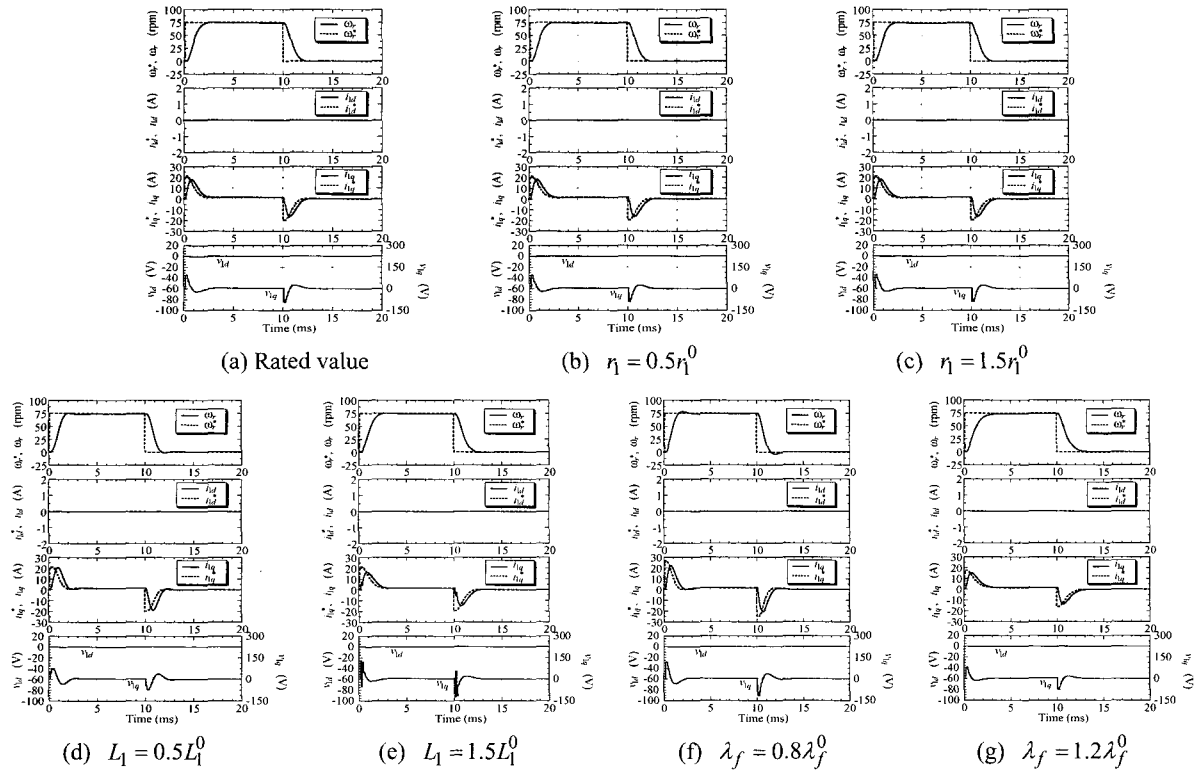


Fig.4 Simulated results (ILQ servo-system)

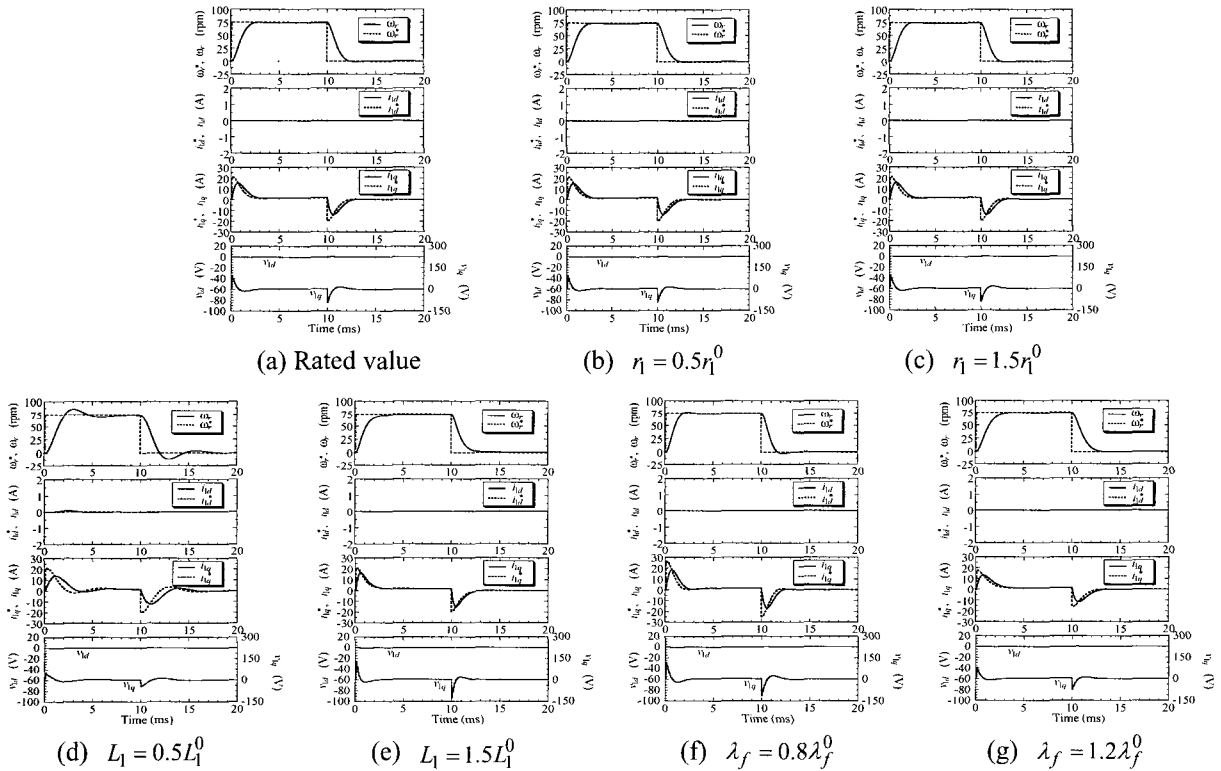


Fig.5 Simulated results (PI servo-system)

In Fig.4, the variations of current  $i_{1d}$  are very small and responses of the motor speed are very robust. While in Fig.5, the variations of current  $i_{1d}$  are large and the initial values converge to zero slowly. The motor speed in particular has a large error in Fig.5 (d). Thus, the PI servo-system is more sensitive to variations of armature inductance  $L_1$ .

### 3.2 Experimental verifications

Fig. 6 shows a block diagram for the experiment. The control system is composed of:

- 1) A floating point digital signal processor (DSP), TMS320C32, 50MHz,
- 2) A PWM modulator,
- 3) A personal computer for development and administration of the DSP,
- 4) An interface (I/F) to connect the PC to the DSP system,
- 5) A 12bit A/D converter,
- 6) Counters to count pulse signals from the pulse encoder,
- 7) A three-phase IGBT inverter,
- 8) A pulse encoder for measuring motor speed and position.

All the control computations are carried out at intervals of 0.1ms by using a DSP (TI, TMS320C32, 50MHz). The carrier frequency of PWM in the IGBT inverter is set at 10kHz.

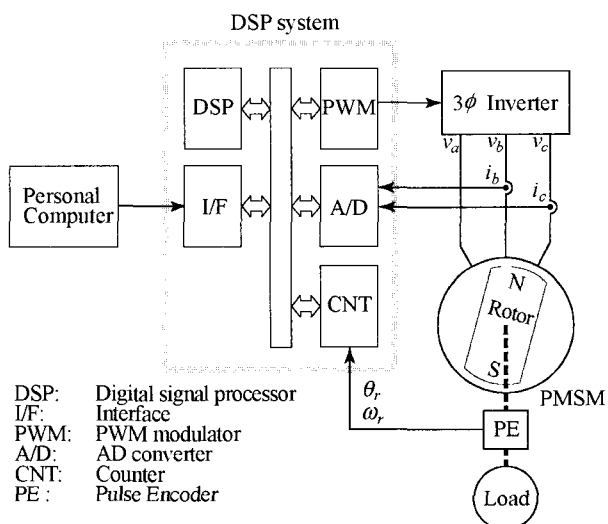


Fig.6 Experimental system

Fig. 7 and Fig. 8 show the experimental results of the step responses to step speed command for the ILQ and PI servo-systems, respectively. The solid lines on the plots show responses, while the dotted lines show commands.

In Fig. 7 and Fig. 8, Fig. (a) shows the result at the rated value of the motor's parameters. Figs. (b) and (c) are the results at one-half the rated value and at two times the rated value of  $\eta_1$ , and Figs. (d) and (e) are the results at one-half the rated value and at two times the rated value of  $L_1$ . Figs. (f) and (g) show results at 0.8 times the rated value and at 1.2 times the rated value of  $\lambda_f$ .

In the ILQ servo-system of Fig. 7, the variations of current  $i_{1d}$  are very small and the initial values converge to zero quickly, and all the responses have almost no difference. In the PI servo-system of Fig. 8, the responses have remarkable differences. The speed response in the error of  $0.5L_1^0$  of Fig. 8 (d) has larger overshoot than that of the nominal system in Fig. 8 (a).

Comparing Fig. 7 with Fig. 8, we see that the ILQ servo-system is more insensitive to changes in stability and performance caused by variations in armature inductance than the PI servo-system. Variations in armature inductance have a large influence on the servo-system and determine the performance of the controllers. The experimental results correspond well with the simulated results in Fig. 4 and Fig. 5. However, in Fig. 5 (d) and Fig. 8 (d), some of the waveforms are different from each other. The main reason for this is that the friction coefficient of the motor shaft at low speed, varies widely and nonlinearly. Therefore, the speed response is very delicate.

## 4. Conclusions

In spite of its simple structure, the proposed ILQ designed servo-controller is more insensitive to variations in armature inductance than standard PI-controllers. Variations in armature inductance have the greatest influence on the response of the servo-system. The experiments correspond well with the numerical simulations. It has been confirmed that the ILQ optimal current controller is more robust than the standard PI controller. In this paper experiments and simulations are

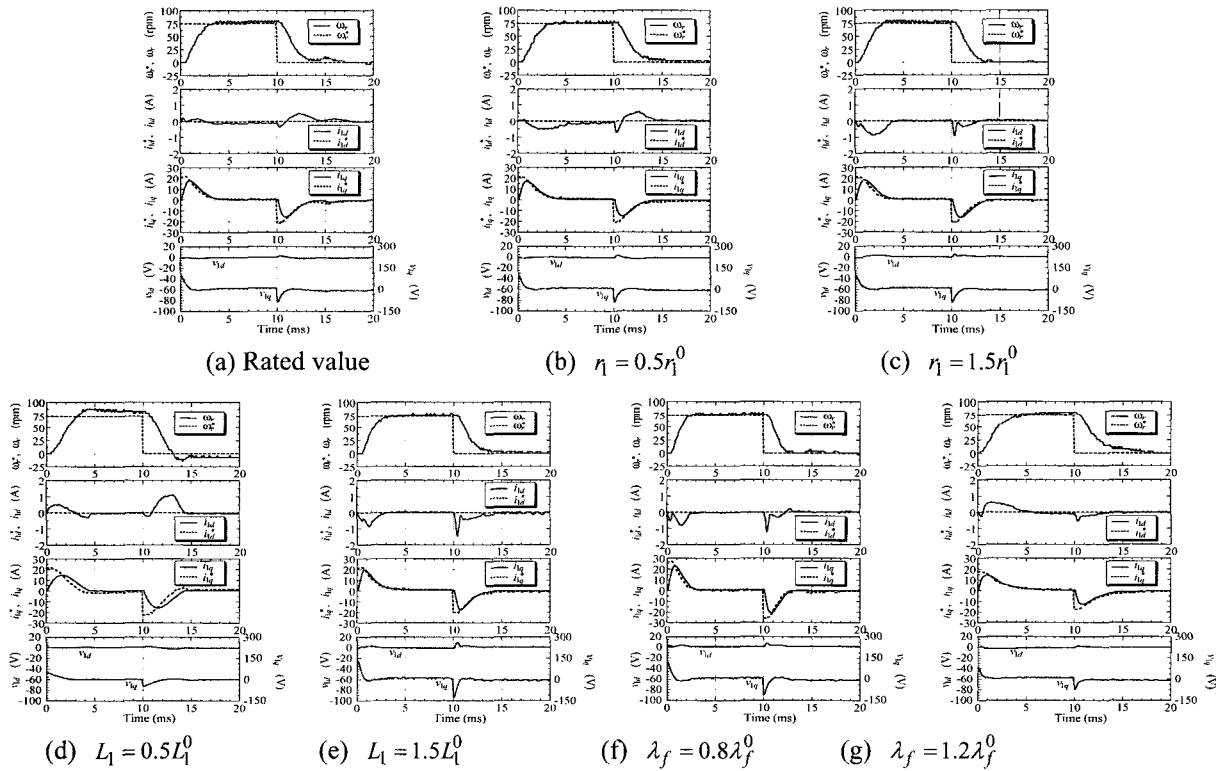
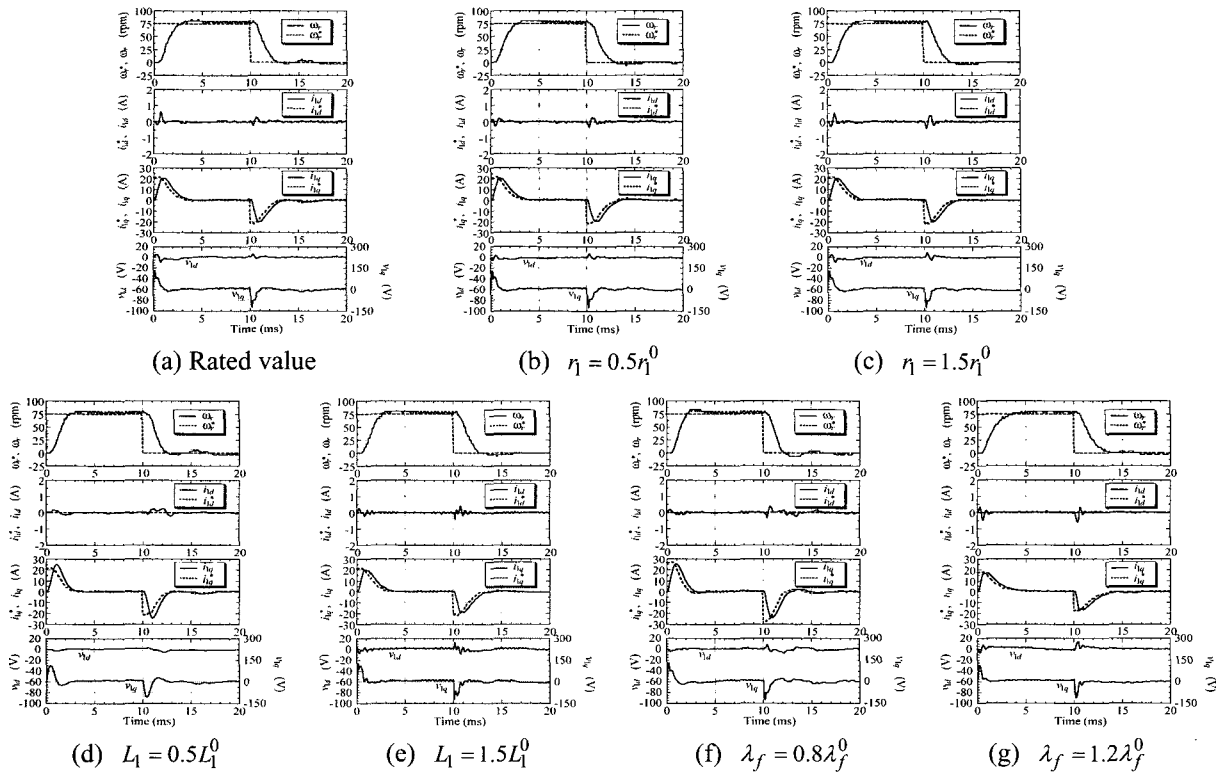


Fig. 8 Experimental results (PI servo-system)

carried out at low speed, but the proposed system can easily be operated at nominal speed and beyond by using the field-weakening control.

Robustness of the servo-system can not be guaranteed when decoupled control is not satisfied due to temperature variation and core saturation. We have been carrying out a quantitative evaluation of the robust-stability and robust-performance of nonlinear servo-systems such as decoupled controllers using  $\mu$ -analysis. Our evaluation shows that the ILQ servo-system is superior to the standard PI servo-system with regard to both performance and stability. These results will be shown in the next paper.

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