◎ 논 문

Thermal Contact Resistance of Two Bodies in Contact

Hong Sup Kwak*[1], Jae Tack Jeong*[2]

접촉하는 두 물체 사이의 접촉 열저항

곽홍섭, 정재택*

전도 열전달 분야에서 두 물체가 접해 있는 경우, 접촉 열저항은 고려해야 할 중요한 요소이다. 특히 최근에는 전자부품의 과열방지를 위한 열 소산과 관련하여 접촉 열저항 문제는 중요하게 대두되고 있으며 이에 관련한 많은 이론적 연구와 응용연구가 수행되고 있다. 접촉 열저항은 주로 거친 두 물체표면의 불완전 접촉에 기인한다. 본 연구에서는, 접촉하는 두 물체사이의 접촉면을 이상화시킨 비교적 간단한 문제를 이론적으로 해석함으로써 접촉면의 틈새 형상 및 비접촉면적비(비접촉면적/외관접촉면적)의 크기에 따른 접촉 열저항의 크기를 구하였다.

Key Words: Thermal contact resistance, Contact surface, Conductivity, Noncontact area ratio, Constriction alleviation factor

Nomenclature

AR : noncontact area ratio (noncontact area/apparent contact area)
a : height of non-contact slit
b : radius of non-contact circle
c : width of non-contact annulus
A_n, B_n, C_n : coefficients
F : constriction alleviation factor
H : half periodicity of periodic gaps
k_A, k_B, k_C : thermal conductivity
q : heat flux
R : radius of cylinder
R_c : thermal contact resistance
T : temperature

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*1 학생회원 전남대학교 대학원 기계공학과
*2 정회원, 전남대학교 기계시스템공학부, 자동차연구소 (jtjeong@chonnam.ac.kr)

1. Introduction

When two nominally flat solid surfaces are brought into contact, the actual contact takes place only on parts of the interface because of the surface roughness. At the interfacial regions where the actual contact does not exist, air may be present in most instances. However, the heat conduction across the air gap is negligible since the thermal conductivity of air is very low. Consequently, the heat conduction from the warmer solid to the cooler one is restricted within the actual contact area only. The constriction of heat flow then results in the "thermal contact resistance" at the interface.

Thermal contact resistance problems are remained as important unresolved problems in many applications such as electronics, materials science, and so on. Due to the complexity of the problem originated from the dependence of contact resistance on the surface roughness, joint pressure of the two bodies, elastic or plastic deformations at the interface, accurate prediction of heat transfer across the contact surface has been only marginally successful.

The study of thermal contact resistance dates back to several decades (Carslaw[1]), and reviews of this subject are provided by Snaith et al.[2] Yovanovich[3] and Fletcher[4]. Hunter and Williams[5], Gibson[6], and Negus and Yovanovich[7] have modeled the problem with an isothermal contact area on the top surface of an infinitely long circular cylinder of adiabatic lateral surface. The case of a cylinder of finite length has also been treated by Faltin[8] and Gladwell and Lemczyk[9], whose analysis covers a broad spectrum of boundary conditions. Another popular model is the problem of multiple contact areas on the surface of a semi infinite solid. In this regard, Greenwood[10] has considered the case of a single cluster of circular contacts, while Beck[11] has treated the case of regularly arranged circular contacts heated by a uniform heat flux. In a more recent study, Tio and Sadhal[12] have also treated the problem of a periodic array of isothermal contacts, in addition to contact areas heated by a uniform heat flux.

While the works cited above all involve discrete contact areas, the problem of a multiply connected contact region has also been studied. Tio and Sadhal[13] have modeled this problem with discrete circular gaps, i.e., regions of no contact, arranged periodically on the otherwise isothermal surface of a semi infinite solid. A two dimensional problem in which two solids are in partial contact at their interface due to the presence of interstitial substance has also been considered by Das and Sadhal[14]. Zhang et al.[15] proposed a rough contact surface profile model using random numbers and carried out numerical simulation to predict the thermal contact resistance.

The purpose of this paper is providing theoretical solutions of some modeled problems to investigate the thermal resistance of two contacting bodies. Two dimensional and axisymmetrical models are considered and it is assumed that the gaps are very thin and the air in the gaps is non conductive. For analysis, the air gap clearance is set to be zero, while the adiabatic boundary condition is applied on the gap.

2. Method of Solution

2.1 Model 1 : Two Dimensional Periodic Gaps

For the analysis of thermal contact problem, the contacting rough surfaces of two solid materials are modeled as Fig. 1. The contact surface of two solids consists of actual (real) contact surfaces and many noncontact gaps. Every gaps are assumed to be very thin 2 D slits of same width and equally spaced.

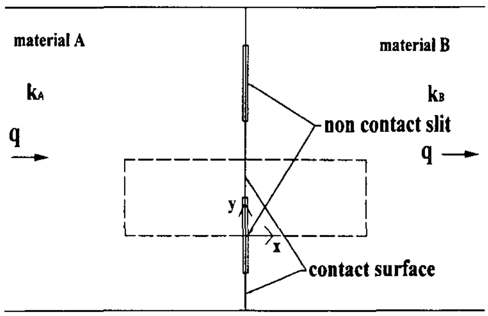

Fig. 1 Modeled contact surface of two bodies

To calculate the thermal resistance of this contact surface, we have to obtain temperature drop per unit heat flux in x direction, $\Delta T/q$, which is caused by existence of non contact gaps in the contact surface. From the periodicity and symmetry of the modeled geometry, it is enough to consider only a partial region of conduction as Fig. 2. Since there is no heat generation, the steady state temperature field $T(x,y)$ satisfies Laplace equation;

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0 \quad (0<x<\infty, 0<y<H) \tag{2.1}$$

where H is the distance between two adjacent symmetry lines and a is the half width of the gap.

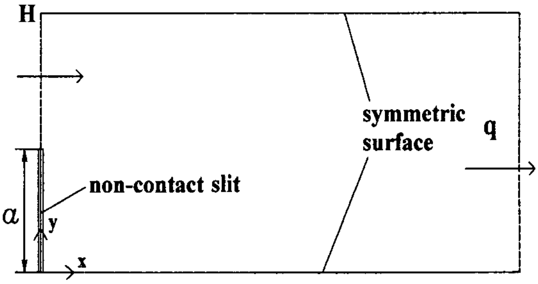

Fig. 2 Boundary condition of Model 1

Since the temperature field is symmetric about the symmetry lines of geometry (y=0,H), the adiabatic boundary condition is applied, i.e,

$$\frac{\partial T}{\partial y}=0, \quad (0<x<\infty, y=0,H) \tag{2.2}$$

There exists uniform heat flux q in x direction at $x\to\infty$,

$$\frac{\partial T}{\partial x}\to-\frac{q}{k} \quad (x\to\infty, 0<y<H) \tag{2.3}$$

Since the heat flux is in x direction at actual (real) contact surface, $\partial T/\partial y$ should be zero or T=const along the real contact surface. This constant temperature may be taken as 0, without loss of generality.

$$T=0 \quad (x=0, a<y<H) \tag{2.4}$$

The air in the non contact gap is assumed to be adiabatic, so boundary condition on this very thin gap is

$$\frac{\partial T}{\partial x}=0 \quad (x=0, 0<y<a) \tag{2.5}$$

We construct the solution of Eq.(2.1), which satisfies boundary conditions (2.2) and (2.3) by using separation of variables.

$$T(x,y)=\frac{qH}{k}\left\{-\frac{x}{H}+\sum_{n=0}^{\infty}A_n e^{-\frac{n\pi x}{H}}\cos\frac{n\pi y}{H}\right\} \tag{2.6}$$

Applying boundary condition (2.4) and (2.5) at appropriate N points on $0<y<H$, $x=0$, we obtain N linear equations for N unknown coefficients A_n (n=0,1···N 1) in Eq.(2.6). Solving these linear equation, we can obtain A_n.

2.2 Model 2 : Circular Gap in the Circular Contact Surface

As shown in Fig. 3, we consider a circular cylinder of radius R with a circular gap of radius b.

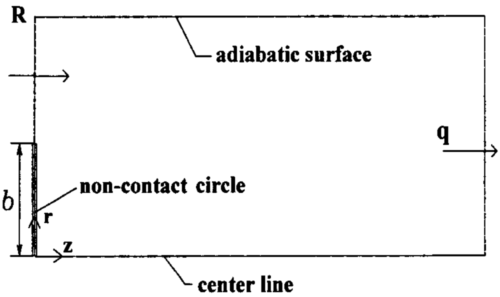

Fig. 3 Boundary condition of Model 2

In this case, temperature distribution is axisymmetric, and the conduction equation in r z coordinates is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}=0$$
$$(0<r<R,\ 0<z<\infty) \qquad (2.7)$$

with boundary conditions

$$\frac{\partial T}{\partial r}=0 \qquad (0<z<\infty, r=0,R), \qquad (2.8)$$

$$\frac{\partial T}{\partial z}=-\frac{q}{k} \qquad (0<r<R, z\to\infty), \qquad (2.9)$$

$$\frac{\partial T}{\partial z}=0 \qquad (b<r<R, z=0), \qquad (2.10)$$

$$T=0 \qquad (0<r<b, z=0). \qquad (2.11)$$

We construct a solution of (2.7), which satisfies boundary conditions (2.8) and (2.9) by using separation of variables.

$$T(r,z)=\frac{qR}{k}\left\{-\frac{z}{R}+\sum_{n=0}^{\infty}B_nJ_0\left(\frac{\lambda_n r}{R}\right)e^{-\frac{\lambda_n z}{R}}\right\}, \qquad (2.12)$$

where B_n's are roots of $J_1(\lambda_n)=0$ ($\lambda_0=0$, $\lambda_1=3.8317$, $\lambda_2=7.0156,\cdots\cdots$) and J_0 and J_1 are the first kind of Bessel functions of order 0 and 1, respectively.

Taking N points between of 0<r<R at z=0 and applying boundary conditions (2.10) and (2.11) to (2.12), we obtain N linear equations for N unknown coefficients B_n (n=0,1,2···N 1). Solving these linear equations, we can obtain B_n .

2.3 Model 3 : Annular Gap in the Circular Contact Surface

As shown in Fig. 4, we consider a circular cylinder of radius R with an annular gap of R c<r<R.

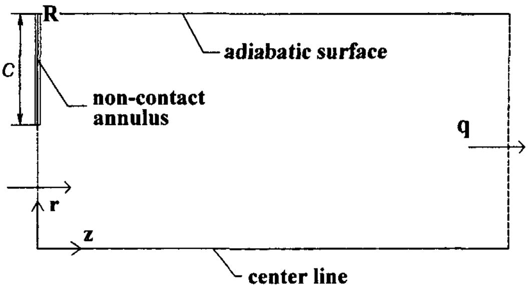

Fig. 4 Boundary condition of Model 3

In this case, temperature distribution is axisymmetric and analysis is very similar to Model 2 in section 2.2, except that the boundary conditions are

$$T=0 \qquad (0<r<c, z=0), \qquad (2.13)$$

$$\frac{\partial T}{\partial z}=0 \qquad (c<r<R, z=0), \qquad (2.14)$$

in stead of Eqs.(2.10) and (2.11). We can also obtain the solution form in this case as

$$T(r,z)=\frac{qR}{k}\left\{-\frac{z}{R}+\sum_{n=0}^{\infty}C_nJ_0\left(\frac{\lambda_n r}{R}\right)e^{-\frac{\lambda_n z}{R}}\right\}, \qquad (2.15)$$

which is similar to Eq.(2.12).

By using boundary conditions (2.13) and (2.14), we can determine truncated unknown constants C_n in Eq.(2.15) by the same way as in section 2.2.

3. Results and Discussion

Temperature distributions for 3 different model problems are obtained as in Eqs.(2.6), (2.12) and (2.15).

Isothermal lines (lines of T=const.) for Model 1, 2, 3 are shown in Fig. 5, 6, 7 respectively.

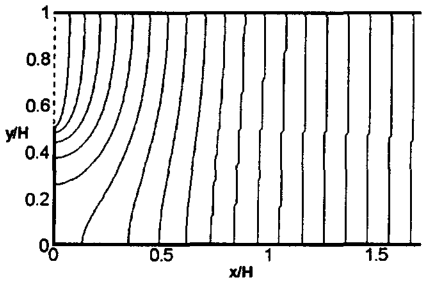

Fig. 5 Isothermal lines(b/H=0.5 of Model 1) with $k\Delta T/qH=0.1$

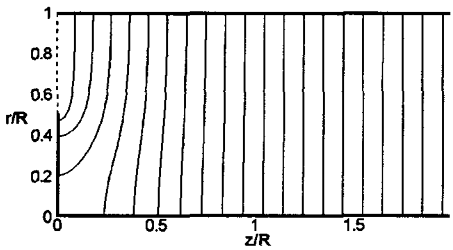

Fig. 6 Isothermal lines(b/R=0.5 of Model 2) with $k\Delta T/qR=0.1$

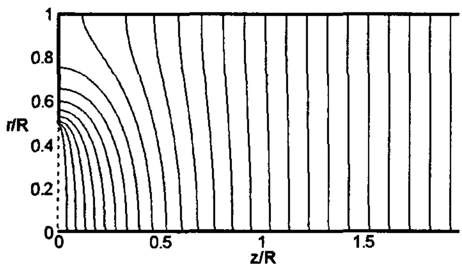

Fig. 7 Isothermal lines(c/R=0.5 of Model 3) with $k\Delta T/qR=0.1$

Typically, Temperature distribution on the center of cylinder for Model 3 is plotted in Fig. 8, when two bodies of same conductivity are contacted.

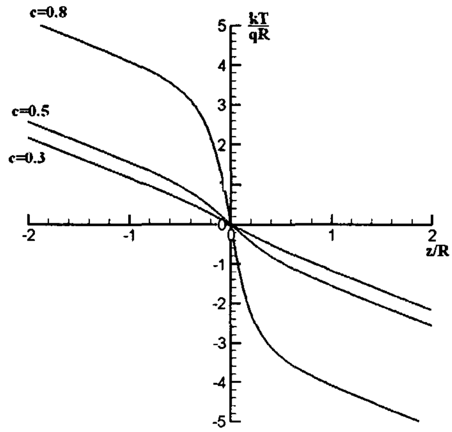

Fig. 8 Temperature distribution on the center line (Model 3 with c=0.3, 0.5, 0.8 and $k_A=k_B$)

In Fig. 8, we can see that the temperature drop except linear decrease in the temperature occurs due to the air gap induced by imperfect contact. From this temperature drop, we can estimate the thermal contact resistance $R_c(=\Delta T/q)$ of the contact surface.

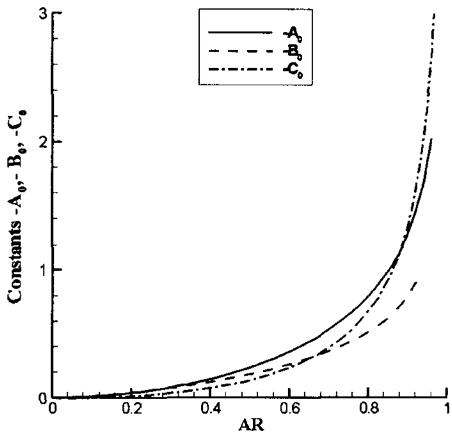

Fig. 9 Dimensionless constants A_0, B_0, C_0 in Eqs. (3.1)~(3.3)

From the analysis of 3 models, the thermal contact resistance in each model may be written as follows, for different k_A and k_B of two contacting solids.

Model 1

$$R_c = \frac{\Delta T}{q} = -A_0 H(\frac{1}{k_A} + \frac{1}{k_B}) = \frac{-2A_0 H}{k} \quad (3.1)$$

Model 2

$$R_c = \frac{\Delta T}{q} = -B_0 R(\frac{1}{k_A} + \frac{1}{k_B}) = \frac{-2B_0 R}{k} \quad (3.2)$$

Model 3

$$R_c = \frac{\Delta T}{q} = -C_0 R(\frac{1}{k_A} + \frac{1}{k_B}) = \frac{-2C_0 R}{k} \quad (3.3)$$

In Eqs.(3.1)~(3.3), the dimensionless negative constants A_0, B_0, C_0 are coefficients in Eqs.(2.6), (2.12), (2.15), respectively. Note that the temperature gradient may be discontinuous across z=0 for $k_A \neq k_B$ and that the right most terms in Eqs.(3.1)~(3.3) hold for $k_A=k_B=k$.

The values of constants A_0, B_0, C_0 are shown in Fig.9 as functions of the noncontact area ratio AR. From the result, it can be conjectured that thermal contact resistance is affected mainly by the noncontact area ratio AR rather than the geometry of the contacting surface.

In the Model 3, one of the major components of the contact resistance is the constriction alleviation factor F, defined in the study of other investigators as,

$$F \equiv \frac{4k}{\pi R}(1 - \frac{c}{R}) \times R_c \quad (3.4)$$

To verify the present work, we calculated the constriction alleviation factor F in Eq.(3.4) and compared it with the previous results as shown in Fig. 10.

As $c \rightarrow 0$ (perfect contact, AR=0), $R_c \rightarrow 0$ and F approaches well to zero in the present result as shown in Fig. 10. As $c \rightarrow R$ (no contact, $AR \rightarrow 1$), however, R_c tend to be infinity and present numerical calculation shows poor convergence. In this limit, other methods of approach should be considered.

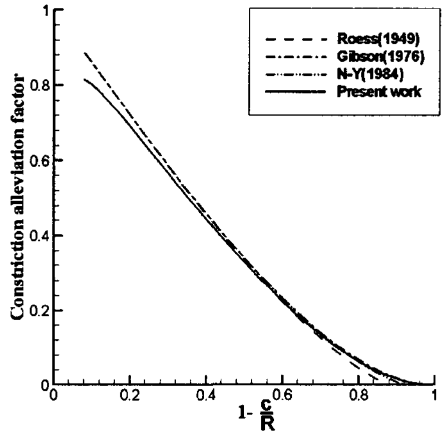

Fig. 10 Comparison of constriction alleviation factor F in Eq.(3.4) with previous results

4. Conclusions

We have carried out analytical study pertaining to the thermal contact resistance of two solid bodies in contact. For three different configuration models of contact surfaces, temperature fields are solved by using separation of variables and the effects of the noncontact area ratio AR on thermal contact resistance are studied. The results have indicated that the noncontact area ratio AR rather than geometrical configuration of contact surface has quite strong effects on the thermal contact resistance. Though it is very difficult to model the contact of rough surfaces, the results of our analysis for simple models may show some basic physical characteristics of thermal contact resistance.

Acknowledgments

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