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Polarization Phase-shifting Technique in Shearographic System with a Wollaston Prism

Soo-Gil Kim*

School of Electrical Eng., Hoseo Univ., San 29-1, Sechul-ri, Baebang-myun, Asan-city, Choongnam, 336-795, KOREA

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The method to obtain four speckle patterns with relative phase shift of $\pi/2$ by passive devices such as two waveplates and a linear polarizer, and to calculate the phase at each point of the speckle pattern in shearography with a Wollaston prism is presented, and the feasibility of the proposed method is theoretically demonstrated by Jones vector.

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I. INTRODUCTION

Shearography measures directly the deformation derivatives using a laser and hence obtains directly strain information. The most important laser speckle measuring methods are electronic speckle pattern interferometry (ESPI) and speckle pattern shearing interferometry (or shearography). The phase distribution in a speckle pattern is random, thus no fringes are visible in a single speckle pattern except for the bright and dark speckles. However, a visible fringe pattern depicting deformation in ESPI and deformation derivative in shearography does appear if two speckle patterns corresponding to two object states (before and after deformation) are recorded and a comparison between them is made. This procedure is the well-known double-exposure technique. This is the basic concept of ESPI and shearography for measurement of deformation and deformation derivatives.

Shearography can be used to reveal flaws or irregularities in mechanical components, electrical devices, and civil engineering structures, as well as in human and biological organs. Shearography detects both surface and internal flaws. This is because the internal defects, unless very remote from the surface, also affect the surface deformation. Leendertz and Butters reported on the direct measurement of the first derivative of deformation by using shearography in 1973 [1]. In 1982, Y. Y. Hung further developed this technique, and he published his results as obtained by shearography for NDT (Non-Destructive Test) and the numerical determination of the out-of-plane deformation gradient [2]. This methods required wet-chemical processing of the film and optical reconstruction of the shearogram. Contrary to the technique of film registration, digital shearography registers speckle patterns by CCD camera, and data are processed online by an image processor unit. By using this technique, wet-chemical processing of the film and optical reconstruction of the shearogram are not required, which makes it possible to observe the shearogram in real time(at video rate). Digital shearography can determine the out-of-plane components numerically, as in photographic shearography, and can be applicable to various fields [3-6].

The output of shearography is in the form of a fringe pattern which requires human interpretation. The displacement derivative at a point of interest is generally determined by multiplying the fringe order at the point with a constant of the system. One major difficulty in the fringe interpretation has been the identification of fringe orders. The difficulty stems from the ambiguities in determining fringe orders and their signs. At present, one common practice to identify fringe orders relies upon prior knowledge of the problem, such as boundary conditions and the nature of the problem. For years, this problem has been a major stumbling block to the automation of fringe phase determination in shearography.

Recently, a phase-shift technique has been developed which allows the determination of fringe phase distribution in digital shearography to be automated. The technique is based on superimposing a uniform phase on the original fringe pattern, thus producing a phase shift in the fringe pattern. The phase shift is achieved by tilting the direction of illumination, and the amount of phase shift can be controlled at will [7].

Phase-shifting technique is a method that determines the phase distribution of speckle pattern from the measured intensities by shifting the additional phase three times to five times [8,9].

Shearography is classified into two categories. The first type of shearography produces and interferes a pair of laterally sheared images, producing a speckle image in the image plane using shearing elements such as a glass wedge, Fresnel's biprism, glass plate with parallel faces, angular adjustable glass plates, and divided lens. The second type of shearography uses a Michelson interferometer as a shearing element. The reflected laser beam passes the Michelson interferometer and is focused on the CCD array of the image plane implemented in the CCD camera. By a small tilting of one mirror in the Michelson interferometer, two slightly shifted images of the object are generated. The interferometric superimposition of both speckle patterns yields a new specklegram in the CCD array.

To generate the required additional phase, the other mirror of the Michelson interferometer is driven by a piezoelectric crystal (PZT). A linear movement of the mirror controlled by the piezoelectric crystal results in the additional phase.

Shearography with a Wollaston prism has simple structure and is robust to large disturbance from environment [10,11]. But the shearography has a drawback that the application of phase-shifting technique is difficult. Recently, to solve the problem, the phase-shifting technique applied to the shearographic system with a Wollason prism has been reported. The technique uses a highly birefringent optical fiber to illuminate a test object with equal amounts of orthogonally polarized light, and the interferometer is readily phase stepped, without mechanical movement of components, by strain modulation of the relative phase of the polarization states in the fiber [12].

In this paper, the method to obtain four speckle patterns with relative phase shift of $\pi/2$ by passive devices such as two waveplates and a linear polarizer, and to calculate the phase at each point of the speckle pattern in shearography with a Wollaston prism is presented, and the feasibility of the proposed method is theoretically demonstrated by Jones vector.

II. SHEAROGRAPHY WITH A WOLLASTON PRISM

Shearography with a Wollaston prism obtains the shearing image using a Wollaston prism instead of the Michelson interferometer. Figure 1 shows the operation of a Wollaston prism, which has a net effect of splitting a ray polarized at 45 degree into two rays that are out of phase, separated from each other, and orthogonally polarized to each other.

Figure 2 shows the schematic setup of a conventional shearography with a Wollaston prism. The application of a Wollaston prism yields a pair of laterally sheared images of the investigated object that is observed in the image plane; i.e., the point P_1 on the object's surface is separated into two points P_1' and P_1'' in the image plane, and point P_2 on the objects's surface is divided into two points P_2' and P_2'' in the image plane as well. The rays from two points in an adjusted distance on the object's surface meet in one point P_1' and P_2'' on the image plane. Light waves belonging to points P_1 and P_2 interfere in this position on the image plane. Using the interference of each point in the image plane yields a speckle interferogram. δx is the shearing distance in the object's plane; $\delta x'$ is the shearing distance in the image plane. Since the polarizations of the optical waves that are reflected from points P_1 and P_2, and then pass through the Wollaston prism are orthogonal, two rays orthogonally polarized to each other do not interfere in the image plane. For fringes to appear, a polarizer, whose axis is at 45 degree to the polarization axes of the Wollaston prism, must be placed at the image plane.

In the absence of the linear polarizer, if the object is coherently illuminated, the optical waves that are reflected from points P_1 and P_2, and then pass through a Wollaston prism arrive at two points P_1' and P_2''. The optical waves reflected from the points P_1 and P_2 can

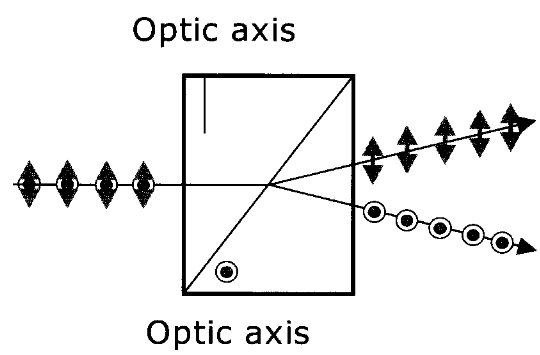

FIG. 1. Operation of a Wollaston prism.

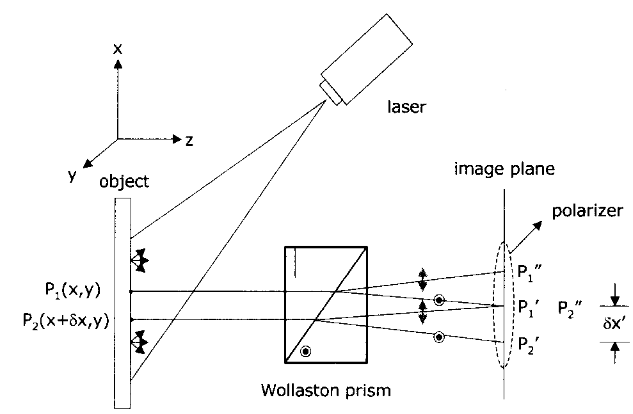

FIG. 2. Shearographic system with a Wollaston prism.

be described by the complex exponential functions

$$U(x,y)=\begin{pmatrix} U_1 \\ U_2 \end{pmatrix}=\begin{pmatrix} a_1 e^{-i\theta(x,y)} \\ a_2 e^{-i\theta(x+\delta x,y)} \end{pmatrix}, \tag{1}$$

where $\theta(x,y)$ and $\theta(x+\delta x,y)$ represent the random phase relation of the light from points $P_1(x,y)$ and $P_2(x+\delta x,y)$, respectively, and a_1 and a_2 are the light amplitudes, which are assumed equal for the two neighboring points. Since the optical waves, U_1 and U_2, have orthogonal polarization, they do not interfere with each other in the image plane. A linear polarizer, whose axis is at 45 degree to the polarization axes of the Wollaston prism, must be placed in the image plane so that the two sheared images can interfere. In the presence of the linear polarizer, Eq. (1) results in

$$U(x,y)=\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \end{pmatrix}=\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a_1 e^{-i\theta_1} \\ a_2 e^{-i\theta_2} \end{pmatrix}$$
$$=\frac{1}{2}\begin{pmatrix} a_1 e^{-i\theta_1}+a_2 e^{-i\theta_2} \\ a_1 e^{-i\theta_1}+a_2 e^{-i\theta_2} \end{pmatrix}. \tag{2}$$

The intensity corresponding to Eq. (2) is given by

$$I=UU^*=(a_1^2+a_2^2)/2+a_1 a_2\cos\phi$$
$$=I_0(1+\gamma\cos\phi), \tag{3}$$

where $I_0=(a_1^2+a_2^2)/2$ is the mean value of the intensity (background brightness I_0), $\gamma=2a_1a_2/(a_1^2+a_2^2)$ is the modulation of the interference term, and $\phi=\theta_1-\theta_2$ is the random phase difference. A visible fringe pattern depicting the deformation derivative does appear if two speckle patterns corresponding to two object states (before and after deformation) are recorded and a comparison between them is made.

This is the basic principle of the shearography with a Wollaston prism. However, it is impossible to obtain more accurate information because the shearographic system with a Wollaston prism does not generate a phase shift.

III. POLARIZATION PHASE-SHIFTING TECHNIQUE USING WAVEPLATES AND A LINEAR POLARIZER

In this paper, we propose a new system to generate a phase shift using passive devices in shearography with a Wollaston prism. Figure 3 shows the proposed system, which consists of two waveplates and one linear polarizer. In this figure, WP1 and WP2 are waveplates, the slow axis of WP1 coincides with the -x axis and the slow axis of WP2 is rotated by ±45 degree with respect to the -x axis. In this paper, we use Jones vector [13] to demonstrate that the proposed

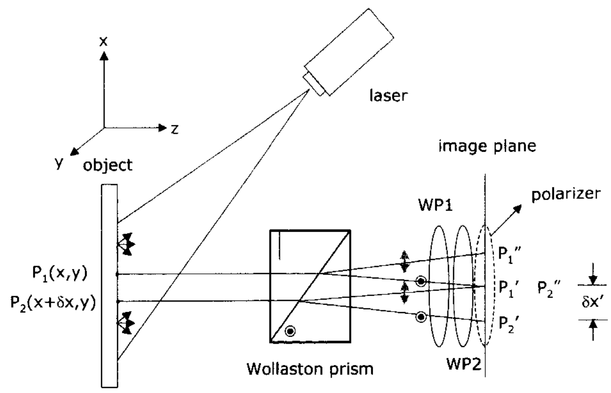

FIG. 3. Shearographic system consisting of a Wollaston prism and two waveplates, and a linear polarizer.

system can generate a phase shift.

After passing through two waveplates, optical waves reflected from points P_1 and P_2 of Fig. 3 are given by

$$U=\begin{pmatrix} \cos\frac{\Gamma_2}{2} & \mp i\sin\frac{\Gamma_2}{2} \\ \mp i\sin\frac{\Gamma_2}{2} & \cos\frac{\Gamma_2}{2} \end{pmatrix}\begin{pmatrix} e^{-i\frac{\Gamma_1}{2}} & 0 \\ 0 & e^{i\frac{\Gamma_1}{2}} \end{pmatrix}\begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$
$$=e^{-i\frac{\Gamma_1}{2}}\begin{pmatrix} \cos\frac{\Gamma_2}{2}U_1 \mp ie^{i\Gamma_1}\sin\frac{\Gamma_2}{2}U_2 \\ \mp i\sin\frac{\Gamma_2}{2}U_1+e^{i\Gamma_1}\cos\frac{\Gamma_2}{2}U_2 \end{pmatrix}, \tag{4}$$

where the minus and plus signs correspond to the cases in which the slow axis of WP2 is rotated by 45 degrees and -45 degrees, respectively, with respect to the -x axis, and Γ_1 and Γ_2 are phase retardations of WP1 and WP2, respectively. After passing through a linear polarizer, the complex amplitude of optical waves in the image plane is given by

$$U_{wp2\pm45}=e^{-i\frac{\Gamma_1}{2}}\left\{\cos\frac{\Gamma_2}{2}U_1\mp ie^{i\Gamma_1}\sin\frac{\Gamma_2}{2}U_2\right\}, \tag{5}$$

where U_{wp2+45} and U_{wp2-45} correspond to the cases in which the slow axis of WP2 is rotated by 45 degree and -45 degree, respectively, with respect to the -x axis. First, in the case of using two $\lambda/4$ plates, since the phase retardation is $\Gamma_1=\Gamma_2=\pi/2$, Eq. (5) results in

$$U_{wp2\pm45}=\frac{\sqrt{2}}{2}e^{-i\frac{\pi}{4}}\{U_1\pm U_2\}. \tag{6}$$

The intensities corresponding to Eq. (6) are given by

$$I_1=U_{wp2+45}U_{wp2+45}^*$$
$$=1/2[(a_1^2+a_2^2)+2a_1a_2\cos\phi]$$
$$=I_0[1+\gamma\cos\phi], \tag{7}$$

$$I_3=U_{wp2-45}U_{wp2-45}^*$$
$$=1/2[(a_1^2+a_2^2)-2a_1a_2\cos\phi]$$
$$=I_0[1-\gamma\cos\phi]=I_0[1+\gamma\cos(\phi+180)]. \tag{8}$$

Second, when the slow axes of WP1 and WP2 are rotated at 0 and 45 degrees(or -45 degree) with respect to the -x axis, respectively, the complex amplitude of optical waves after passing through a linear polarizer (which is rotated at 45 or -45 degree) are given by, respectively,

$$U_{p+45}=\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}\begin{pmatrix}\cos\frac{\Gamma_2}{2}&-i\sin\frac{\Gamma_2}{2}\\-i\sin\frac{\Gamma_2}{2}&\cos\frac{\Gamma_2}{2}\end{pmatrix}\times\begin{pmatrix}e^{-i\frac{\Gamma_1}{2}}&0\\0&e^{i\frac{\Gamma_1}{2}}\end{pmatrix}\begin{pmatrix}U_1\\U_2\end{pmatrix}=\frac{1}{2}e^{-i\frac{\Gamma_1}{2}}\left(\cos\frac{\Gamma_2}{2}-i\sin\frac{\Gamma_2}{2}\right)\begin{pmatrix}U_1+e^{i\Gamma_1}U_2\\U_1+e^{i\Gamma_1}U_2\end{pmatrix}, \quad (9)$$

$$U_{p-45}=\frac{1}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}\begin{pmatrix}\cos\frac{\Gamma_2}{2}&-i\sin\frac{\Gamma_2}{2}\\-i\sin\frac{\Gamma_2}{2}&\cos\frac{\Gamma_2}{2}\end{pmatrix}\times\begin{pmatrix}e^{-i\frac{\Gamma_1}{2}}&0\\0&e^{i\frac{\Gamma_1}{2}}\end{pmatrix}\begin{pmatrix}U_1\\U_2\end{pmatrix}=\frac{1}{2}e^{-i\frac{\Gamma_1}{2}}\left(\cos\frac{\Gamma_2}{2}+i\sin\frac{\Gamma_2}{2}\right)\begin{pmatrix}U_1-e^{i\Gamma_1}U_2\\-U_1+e^{i\Gamma_1}U_2\end{pmatrix}, \quad (10)$$

where U_{p+45} and U_{p-45} correspond to the cases in which the polarization axis of a linear polarizer is rotated by 45 degrees and -45 degrees, respectively, with respect to the -x axis. In which case in which $\Gamma_1=\pi/2$, $\Gamma_2=\pi/2$, the intensities corresponding to Eqs. (9) and (10) are given by

$$I_2=U_{p+45}U_{p+45}^*=1/2[(a_1^2+a_2^2)-2a_1a_2\sin\phi]=I_0[1-\gamma\sin\phi]=I_0[1+\gamma\cos(\phi+90)], \quad (11)$$

$$I_4=U_{p-45}U_{p-45}^*=1/2[(a_1^2+a_2^2)+2a_1a_2\sin\phi]=I_0[1+\gamma\sin\phi]=I_0[1+\gamma\cos(\phi+270)]. \quad (12)$$

From Eqs. (7), (8), (11), and (12), we can obtain four intensity patterns with relative phase shift of $\pi/2$ using two waveplates and a linear polarizer. Using combination of $\lambda/2$ plate and $\lambda/4$ plate, we can also obtain four intensity patterns with relative phase shift of $\pi/2$. In the case in which $\Gamma_1=\pi$, $\Gamma_2=\pi/2$, the intensity corresponding to Eq. (5) results in

$$U_{wp2\pm45}=-\frac{\sqrt{2}}{2}i\{U_1\pm iU_2\}. \quad (13)$$

The intensities corresponding to Eq. (13) are given by

$$I_2=U_{wp2+45}U_{wp2+45}^*=1/2[(a_1^2+a_2^2)-2a_1a_2\sin\phi]=I_0[1-\gamma\sin\phi]=I_0[1+\gamma\cos(\phi+90)], \quad (14)$$

$$I_4=U_{wp2-45}U_{wp2-45}^*=1/2[(a_1^2+a_2^2)+2a_1a_2\sin\phi]=I_0[1+\gamma\sin\phi]=I_0[1+\gamma\cos(\phi+270)]. \quad (15)$$

Second, when the slow axes of WP1 and WP2 are rotated at 0 and 45 degrees (or -45 degrees) with respect to -x axis, respectively, the complex amplitudes of optical waves after passing through a linear polarizer (which is rotated at 45 or -45 degrees) are given by Eqs. (9) and (10), respectively. In which case in which $\Gamma_1=\pi$, $\Gamma_2=\pi/2$, the intensities corresponding to Eqs. (9) and (10) are given by

$$I_3=U_{p+45}U_{p+45}^*=1/2[(a_1^2+a_2^2)-2a_1a_2\cos\phi]=I_0[1-\gamma\cos\phi]=I_0[1+\gamma\cos(\phi+180)], \quad (16)$$

$$I_1=U_{p-45}U_{p-45}^*=1/2[(a_1^2+a_2^2)+2a_1a_2\cos\phi]=I_0[1+\gamma\cos\phi]. \quad (17)$$

By adjusting the relative phase between two optical waves using the two waveplates and a linear polarizer, we can obtain four intensity patterns in Table 1 in the

TABLE 1. Intensity patterns by combination of two waveplates and a linear polarizer.

phase retardation of waveplates	azimuth angle of a linear polarizer (degree)	azimuth angle of waveplates		intensity pattern
		WP1(degree)	WP2(degree)	
$\Gamma_1=\pi/2$, $\Gamma_2=\pi/2$	0	0	45	$I_0[1+\gamma\cos\phi]$
	45	0	45(or -45)	$I_0[1+\gamma\cos(\phi+90)]$
	0	0	-45	$I_0[1+\gamma\cos(\phi+180)]$
	-45	0	45(or -45)	$I_0[1+\gamma\cos(\phi+270)]$
$\Gamma_1=\pi$, $\Gamma_2=\pi/2$	-45	0	45(or -45)	$I_0[1+\gamma\cos\phi]$
	0	0	45	$I_0[1+\gamma\cos(\phi+90)]$
	45	0	45(or -45)	$I_0[1+\gamma\cos(\phi+180)]$
	0	0	-45	$I_0[1+\gamma\cos(\phi+270)]$

image plane.

In Table 1, we see that we can obtain four speckle patterns with relative phase shift of $\pi/2$ by combination of two $\lambda/4$ plates and a linear polarizer, or $\lambda/2$ and $\lambda/4$ plates, and a linear polarizer. From four intensity patterns, we can calculate the phase at each point of the speckle interferogram, as follows:

$$\phi = \arctan \frac{(I_4 - I_2)}{(I_1 - I_3)}. \quad (18)$$

Phase shifting interferometry can be carried out using a variety of arrangements if we measure the intensity in the interference plane for different orientations of the polarization components. However, the errors in the retardation of the phase plates and their azimuth angle errors will influence the accuracy of the phase measurement. The influence of the errors of the retardation and azimuth angle of the polarization components of a polarization phase shifter was reported, on the phase measurement, in the phase shifting interferometry [14,15].

IV. CONCLUSION

In this paper, we presented the method to obtain four speckle interferograms with relative phase shift of $\pi/2$ by passive devices such as two waveplates and a linear polarizer, calculated the phase at each point of the speckle interferogram in shearography with a Wollaston prism, and theoretically demonstrated the feasibility of the proposed method by Jones vector.

In Table 1, we see that we can obtain four speckle patterns with relative phase shift of $\pi/2$ by combination of two $\lambda/4$ plates and a linear polarizer, or $\lambda/2$ and $\lambda/4$ plates, and a linear polarizer.

After four speckle patterns corresponding to two object states (before and after deformation) are recorded and then the phase at each point of the speckle pattern is calculated, subtracting the phase distribution after deformation from the phase distribution before deformation yields a phase map of the relative phase difference, which depicts the deformation derivatives. The proposed system can generate a phase shift by adding passive devices such as waveplates in the conventional shearographic system with a Wollaston prism. Also, to observe the shearogram in real time, the proposed system can be adapted to instantaneous phase measuring interferometry by introducing the phase shifts in parallel channels.

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*Corresponding author : sgkim@office.hoseo.ac.kr

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