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Synthesis and Analysis of Optical Transfer Function of the Modified Triangular Interferometer by Two-pupil Synthesis Method

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We synthesized and analyzed the optical transfer function (OTF) of the modified triangular interferometer (MTI) using two-pupil synthesis method. Also, we presented the optimal MTI, which can obtain any bipolar function by combining two wave plates and a linear polarizer. By using the proposed MTI, we can obtain the complex hologram without bias and conjugate image.

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I. INTRODUCTION

In a conventional incoherent scanning or imaging system, limitations exist on image processing that are due to the resulting nonnegative intensity spread function[point-spread function (PSF)], which, in turn, imposes severe constraints on both the amplitude and the phase of the optical transfer function (OTF) [1]. Such limitations can be circumvented by introducing a two-pupil system that is characterized by a great flexibility in pupil-function specification for a desired synthesized PSF [2,3]. Any bipolar impulse response can be synthesized by using two-pupil methods as long as the pupil function can be arbitrarily specified. Two-pupil systems are usually implemented by separating the responses (i.e., separating the interactive term and the noninteractive term on the basis of the spatial or temporal carriers) [1,4,5,6]. The pupils are created by either amplitude or wave-front divisions [1,3,4,6].

The synthesis methods divide into two important classes which are distinguished by the mathematical structure of the resultant transfer function. There are basically two kinds of syntheses possible: nonpupil interaction synthesis and pupil interaction synthesis [1].

Recently, two-pupil synthesis by the MTI was reported [7]. A simple two-pupil interaction system was implemented by adding two wave plates and a linear polarizer to Cochran's triangular interferometer [8]. However, the principle of proposed system was described, but in the viewpoint of two-pupil synthesis, the analysis of the proposed system was not described. In this paper, we introduce general two-pupil synthesis, and then we demonstrate that removal of bias and conjugate image of the incoherent hologram is possible through OTF synthesis based on two-pupil synthesis.

II. TWO-PUPIL SYNTHESIS OF OTF

1. The OTF of incoherent imaging system

For coherent systems, we may find the complex field in the image plane $U_i(x_i, y_i)$ by convolving the field in the object plane $U_o(x_o, y_o)$ with the system impulse response $h(x_i, y_i; x_o, y_o)$(or the coherent spread function). For a spatially invariant system, we have

$$U_i(x_i,y_i) = \iint U_o(x_o,y_o)h(x_i-x_o,y_i-y_o)dx_o dy_o = U_o * h, \quad (1)$$

where h is the amplitude at image coordinates (x_i, y_i) in response to a point-source object at (x_o, y_o) and * denotes the convolution operation. The image intensity distributions are then obtained by

$$I_i(x_i,y_i) = \langle U_i(x_i,y_i;t)U_i^*(x_i,y_i;t)\rangle, \quad (2)$$

where the angle bracket indicates a time average. Fourier transforming Eq. (1), we have

$$F\{U_i\} = F\{U_o\}F\{h\}. \quad (3)$$

The coherent transfer function of the imaging system is defined on the basis of amplitudes:

$$H(f_x,f_y)=\frac{F\{U_i\}}{F\{U_o\}}=F\{h\} \tag{4}$$

However, $h=F\{P\}$ under the ideal condition (i.e., when the system is properly focused), so that combining Eq. (4) with the definition of H gives

$$H(f_x,f_y)=F\{h\}=F\{F\{P\}\}=P(f_x,f_y). \tag{5}$$

From Eq. (5), we can see that the coherent transfer function is the pupil function of the system. The coherent transfer function H characterizes the performance of the coherent imaging system as it specifies the passband of the spatial frequency.

In the case of an incoherent imaging system, the image intensity distribution is

$$I_i(x_i,y_i)=\iint I_o(x_o,y_o)|h(x_i-x_o,y_i-y_o)|^2\,dx_o dy_o, \tag{6}$$

where is $|h|^2$ the intensity spread function (PSF). Expressing the relationship in the frequency domain, we have

$$F\{I_i\}=F\{I_o\}F\{|h|^2\}. \tag{7}$$

The OTF of the incoherent imaging system is defined on the basis of intensities:

$$OTF=\frac{F\{I_i\}}{F\{I_o\}}=F\{|h|^2\} \tag{8}$$

In terms of the pupil function P, we have, using Eq. (5) and $h=F\{P\}$,

$$OTF=P\otimes P, \tag{9}$$

where $\otimes$ represents the correlation operation. Hence, for incoherent imaging systems the OTF is simply the autocorrelation of the pupil function of the system.

2. Two-pupil transfer function

A single system shown in Fig. 1 serves to illustrate two-pupil synthesis. An extended pupil region between the lenses is divided by a beam splitter into two arms, each arm containing its own pupil transparency and an attenuator. In one arm is a phase shifter. If one arm of the system is blocked off, the system behaves like a normal single-pupil imaging system. With proper alignment of the system, the effective pupil function is given by

$$p(u)=A_1p_1(u)\exp(i\phi)+A_2p_2(u), \tag{10}$$

where A_1 and A_2 are the transmittance factors (assumed positive-real) associated with the two attenuators, and ϕ is the adjustable phase. The associated transfer function is given by, through Eq. (8),

$$\begin{aligned}F(u)=&A_1^2[p_1(u)\otimes p_1(u)]+A_2^2[p_2(u)\otimes p_2(u)]\\&+A_1A_2\{[p_1(u)\otimes p_2(u)]\exp(i\phi)\\&+[p_2(u)\otimes p_1(u)]\exp(-i\phi)\}.\end{aligned} \tag{11}$$

The PSF corresponding to Eq. (11) is

$$\begin{aligned}F(x)=&A_1^2|p_1(x)|^2+A_2^2|p_2(x)|^2+A_1A_2[p_1(x)p_2^*(x)\exp(i\phi)\\&+p_1^*(x)p_2(x)\exp(-i\phi)].\end{aligned} \tag{12}$$

As a first step in our analysis we develop a set of physically obtainable components that span the functional space of all realizable two-pupil synthesis impulse responses. In doing so, we identify $F(x)$ by the notation $F(x;\ A_1,\ A_2,\ \phi)$. The basic set of components is then defined by the following equations:

$$F_1(x)=F(x;1,0,0)=|p_1(x)|^2, \tag{13}$$

$$F_2(x)=F(x;0,1,0)=|p_2(x)|^2, \tag{14}$$

$$\begin{aligned}F_R(x)&=\frac{1}{4}[F(x;1,1,0)-F(x;1,1,\pi)]\\&=\frac{1}{2}[p_1(x)p_2^*(x)+p_1^*(x)p_2(x)]\\&=\mathrm{Re}[p_1(x)p_2^*(x)]=|p_1(x)p_2(x)|\cos[\theta_1(x)-\theta_2(x)],\end{aligned} \tag{15}$$

$$\begin{aligned}F_I(x)&=\frac{1}{4}\{F[x;1,1,-(\pi/2)]-F[x;1,1,(\pi/2)]\}\\&=\frac{i}{2}[p_1(x)p_2^*(x)-p_1^*(x)p_2(x)]\\&=\mathrm{Im}[p_1(x)p_2^*(x)]=|p_1(x)p_2(x)|\sin[\theta_1(x)-\theta_2(x)],\end{aligned} \tag{16}$$

where $\theta(x)=Arg[p(x)]$. Corresponding transfer functions are

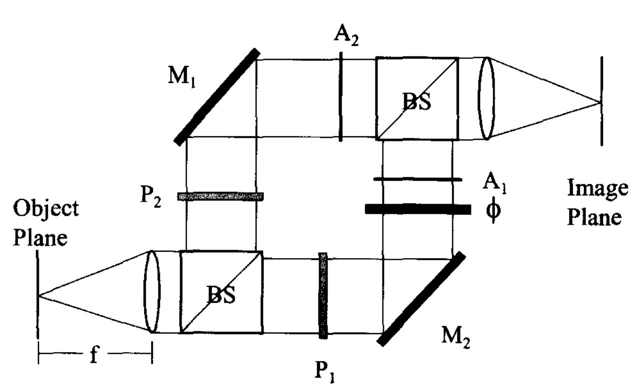

FIG. 1. Example of two-pupil spatial filtering system.

$$F_1(u) = p_1(u) \otimes p_1(u), \tag{17}$$

$$F_2(u) = p_2(u) \otimes p_2(u), \tag{18}$$

$$F_R(u) = \frac{1}{2}[p_1(u) \otimes p_2(u) + p_2(u) \otimes p_1(u)], \tag{19}$$

$$F_I(u) = -\frac{i}{2}[p_1(u) \otimes p_2(u) - p_2(u) \otimes p_1(u)] \tag{20}$$

In choosing these equations, we have deliberately separated pupil interaction components from non-interaction components. It is easily shown that any function of the form given Eq. (12) can be constructed by taking linear combinations of these four components.

III. TWO-PUPIL SYNTHESIS OF OTF IN THE MTI

1. One-pupil synthesis of the MTI

Figure 2 shows the MTI. In Fig. 2, PBS represents a polarizing beam splitter. If a linear polarizer and wave plates are removed and the PBS is replaced with an ordinary beam splitter, the system is the same as Cochran's triangular interferometer. Figure 3 shows double afocal systems of the light that travels clockwise and counterclockwise in the MTI. The polarizations of the light that travels clockwise and counterclockwise are vertical and horizontal, respectively.

Using Jones vector representation [9], the pupil functions of the light that travels clockwise and counter-clockwise in one-pupil system of Fig. 3, respectively, are given by

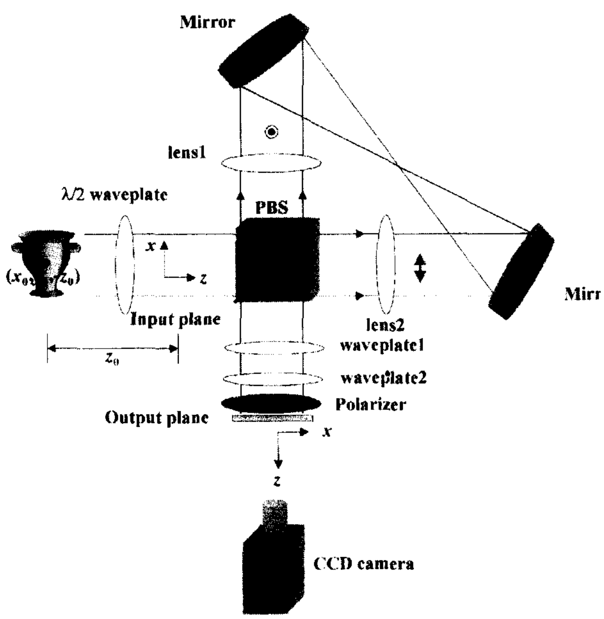

FIG. 2. Modified triangular interferometer.

$$P_{mcw} = P_x R(-\psi) W_2 R(\psi) R(-\psi) W_1 R(\psi) P_{cw} = W_{mcw} P_{cw}, \tag{21}$$

$$P_{mccw} = P_x R(-\psi) W_2 R(\psi) R(-\psi) W_1 R(\psi) P_{ccw} = W_{mccw} P_{ccw}, \tag{22}$$

where P_{mcw} and P_{mccw} denote the pupil functions of the light that travels clockwise and counterclockwise, respectively, and P_x, $R(\psi)$, W_1, and W_2 are defined as

$$P_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$R(\psi) = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix},$$

$$W_1 = e^{-i\phi} \begin{pmatrix} e^{-i\Gamma_1/2} & 0 \\ 0 & e^{i\Gamma_1/2} \end{pmatrix},$$

$$W_2 = e^{-i\phi} \begin{pmatrix} e^{-i\Gamma_2/2} & 0 \\ 0 & e^{i\Gamma_2/2} \end{pmatrix},$$

$$\phi = \frac{1}{2}(n_s + n_f)\frac{wl}{c},$$

where P_x represents Jones matrix of a linear polarizer, ψ represents the azimuth angle of slow axis of wave plate with respect to the x axis. Γ_1 and Γ_2 represent the phase retardations of wave plates 1 and 2, respectively. n_s and n_f represent the refractive index of the slow and fast components of wave plates, respectively, and ω, l, c represent the frequency of the light beam, the thickness of wave plate, and the velocity of the light in the vacuum, respectively. And, the pupil functions of the light that travels clockwise and counter-clockwise in the triangular interferometer, respectively, are given by [7]

$$P_{cw}(x,y) = \frac{ik}{2\sqrt{2}\pi z_0} \exp\left[-i\frac{k}{2z_0}\left\{(\alpha x - x_0)^2 + (\alpha y - y_0)^2\right\}\right], \tag{23}$$

$$P_{ccw}(x,y) = \frac{ik}{2\sqrt{2}\pi z_0} \exp\left[-i\frac{k}{2z_0}\left\{(\beta x - x_0)^2 + (\beta y - y_0)^2\right\}\right], \tag{24}$$

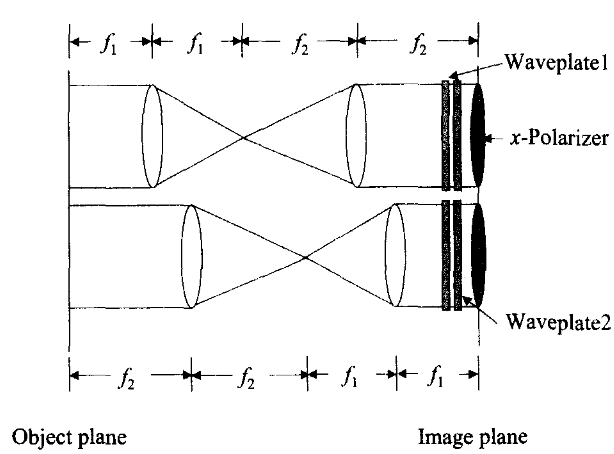

FIG. 3. Double afocal systems for the two beams circulating in opposite directions.

where P_{cw} and P_{ccw} denote the pupil functions of the light that travels clockwise and counterclockwise, respectively, k is the wavenumber, and $\alpha \equiv -f_1/f_2$, $\beta \equiv -f_2/f_1$.

The OTFs corresponding to Eqs. (23) and (24) are given by

$$OTF_{mcw} = P_{mcw}(f_x, f_y) \otimes P_{mcw}(f_x, f_y), \quad (25)$$

$$OTF_{mccw} = P_{mccw}(f_x, f_y) \otimes P_{mccw}(f_x, f_y). \quad (26)$$

From the above equations, we see that any bipolar function cannot be produced by the one-pupil system of the MTI.

2. Two-pupil synthesis of the MTI

We describe the OTF synthesis of the MTI based on OTF synthesis presented in Section 2. The effective pupil function of the MTI in Fig. 2 is given by

$$P(f_x, f_y) = \exp\left(-i\frac{\Gamma_1}{2}\right)\left[\cos\frac{\Gamma_2}{2} P_{ccw}(f_x, f_y) - i\exp(i\Gamma_1)\sin\frac{\Gamma_2}{2} P_{cw}(f_x, f_y)\right]. \quad (27)$$

In the case of the incoherent system, the OTF of Eq. (27) is given by

$$\begin{aligned} OTF = {} & \cos^2\frac{\Gamma_2}{2} P_{ccw}(f_x, f_y) \otimes P_{ccw}(f_x, f_y) + \sin^2\frac{\Gamma_2}{2} P_{cw}(f_x, f_y) \otimes P_{cw}(f_x, f_y) \\ & + i\exp(-i\Gamma_1)\cos\frac{\Gamma_2}{2}\sin\frac{\Gamma_2}{2} P_{ccw}(f_x, f_y) \otimes P_{cw}(f_x, f_y) \\ & - i\exp(-i\Gamma_1)\cos\frac{\Gamma_2}{2}\sin\frac{\Gamma_2}{2} P_{cw}(f_x, f_y) \otimes P_{ccw}(f_x, f_y). \quad (28) \end{aligned}$$

The PSF corresponding to Eq. (28) is given by

$$\begin{aligned} h(x, y; \Gamma_1, \Gamma_2) = {} & \cos^2\frac{\Gamma_2}{2}|P_{ccw}(x, y)|^2 + \sin^2\frac{\Gamma_2}{2}|P_{cw}(x, y)|^2 \\ & + i\exp(-i\Gamma_1)\cos\frac{\Gamma_2}{2}\sin\frac{\Gamma_2}{2} P_{ccw}(x, y) P^*_{cw}(x, y) \\ & - i\exp(-i\Gamma_1)\cos\frac{\Gamma_2}{2}\sin\frac{\Gamma_2}{2} P_{cw}(x, y) P^*_{ccw}(x, y). \quad (29) \end{aligned}$$

We have to synthesize the OTFs to produce the complex hologram without bias and the conjugate image. Through the OTF synthesis using the combination of the phase retardation of wave plates, we can obtain the cosine and sine functions.

First, in the case in which $\Gamma_1 = \pi/2$ and $\Gamma_2 = \pm\pi/2$, through the OTF synthesis using Eq. (29), we can obtain the cosine component.

$$\begin{aligned} h_r(x, y) &= \frac{1}{2}\left\{h\left(x, y; \frac{\pi}{2}, \frac{\pi}{2}\right) - h\left(x, y; \frac{\pi}{2}, -\frac{\pi}{2}\right)\right\} \\ &= |P_{cw}(x, y)P_{ccw}(x, y)|\cos\{\theta_{cw}(x, y) - \theta_{ccw}(x, y)\}. \quad (30) \end{aligned}$$

And, in the case in which $\Gamma_1 = 0$ and $\Gamma_2 = \pm\pi/2$, through the OTF synthesis using Eq. (29), we can obtain the sine component.

$$\begin{aligned} h_r(x, y) &= \frac{1}{2}\left\{h\left(x, y; 0, -\frac{\pi}{2}\right) - h\left(x, y; 0, \frac{\pi}{2}\right)\right\} \\ &= |P_{cw}(x, y)P_{ccw}(x, y)|\sin\{\theta_{cw}(x, y) - \theta_{ccw}(x, y)\}. \quad (31) \end{aligned}$$

We can obtain the complex hologram without bias and the conjugate image using Eqs. (30) and (31). The derived complex hologram using the OTF synthesis is equivalent to the previous result [7] and the complex hologram without bias and without conjugate image is described as follows.

$$\begin{aligned} H(x, y) &= |P_{cw}(x, y)P_{ccw}(x, y)|[\cos\{\theta_{cw}(x, y) - \theta_{ccw}(x, y)\} \pm i\sin\{\theta_{cw}(x, y) - \theta_{ccw}(x, y)\}] \\ &= |P_{cw}(x, y)P_{ccw}(x, y)|\exp[\pm i\{\theta_{cw}(x, y) - \theta_{ccw}(x, y)\}]. \quad (32) \end{aligned}$$

Using the analysis based on the OTF synthesis using two-pupil method, we present another combination of phase retardation of wave plates that can obtain the cosine and sine functions. Examples are described in the following.

First, in the case in which $\Gamma_1 = \pm\pi/2$ and $\Gamma_2 = \pi/2$, through the OTF synthesis using Eq. (29), we can obtain the cosine component.

$$\begin{aligned} h_r(x, y) &= \frac{1}{2}\left\{h\left(x, y; \frac{\pi}{2}, \frac{\pi}{2}\right) - h\left(x, y; -\frac{\pi}{2}, \frac{\pi}{2}\right)\right\} \\ &= |P_{cw}(x, y)P_{ccw}(x, y)|\cos\{\theta_{cw}(x, y) - \theta_{ccw}(x, y)\}. \quad (33) \end{aligned}$$

And, in the case in which $\Gamma_1 = \pi$, 0 and $\Gamma_2 = \pi/2$, through the OTF synthesis using Eq. (29), we can obtain the sine component.

$$\begin{aligned} h_r(x, y) &= \frac{1}{2}\left\{h\left(x, y; \pi, \frac{\pi}{2}\right) - h\left(x, y; 0, \frac{\pi}{2}\right)\right\} \\ &= |P_{cw}(x, y)P_{ccw}(x, y)|\sin\{\theta_{cw}(x, y) - \theta_{ccw}(x, y)\}. \quad (34) \end{aligned}$$

We can obtain the complex hologram without bias and conjugate image using Eqs. (33) and (34).

From the above results, we can see that there are several kinds of the OTF synthesis to obtain the complex hologram without bias and conjugate image. Therefore, we can see that there are several kinds of pupil functions to implement the complex hologram in the MTI.

3. The optimization of the MTI

In Section 2, we can obtain the complex hologram without bias and without conjugate image by changing the pupil function by the combination of the phase retardation of wave plates in the MTI. But to obtain the sine function, we must obtain the intensity patterns in the MTI without wave plate 1. This means that we need two operation modes to obtain the complex hologram without bias and conjugate image in the MTI. Two operation modes are inconvenient in the case of using the MTI. For example, in obtaining the complex hologram, we must use the MTI with two wave plates to obtain the cosine function, and use the MTI without one wave plate to obtain the sine function. Accordingly, to solve the problem, we must obtain the sine function in the MTI with two wave plates. We describe the method of obtaining the sine function in the MTI with two wave plates in the following.

First, the effective pupil function of the MTI with a linear polarizer oriented at an azimuth of 45 degrees in Fig. 2 is given by

$$P_{45}(f_x,f_y)=\frac{1}{2}\exp\left(-i\frac{\Gamma_1}{2}\right)\left\{\left[\left(\cos\frac{\Gamma_2}{2}-i\sin\frac{\Gamma_2}{2}\right)P_{ccw}(f_x,f_y)+\exp(i\Gamma_1)\left(\cos\frac{\Gamma_2}{2}-i\sin\frac{\Gamma_2}{2}\right)P_{cw}(f_x,f_y)\right]\hat{x}+\left[\left(\cos\frac{\Gamma_2}{2}-i\sin\frac{\Gamma_2}{2}\right)P_{ccw}(f_x,f_y)+\exp(i\Gamma_1)\left(\cos\frac{\Gamma_2}{2}-i\sin\frac{\Gamma_2}{2}\right)P_{cw}(f_x,f_y)\right]\hat{y}\right\}. \quad (35)$$

In the case of the incoherent system, the OTF of Eq. (35) is given by

$$OTF=\frac{1}{2}\left(\cos^2\frac{\Gamma_2}{2}+\sin^2\frac{\Gamma_2}{2}\right)\{P_{ccw}(f_x,f_y)\otimes P_{ccw}(f_x,f_y)+P_{cw}(f_x,f_y)\otimes P_{cw}(f_x,f_y)+\exp(-i\Gamma_1)P_{ccw}(f_x,f_y)\otimes P_{cw}(f_x,f_y)+\exp(i\Gamma_1)P_{cw}(f_x,f_y)\otimes P_{ccw}(f_x,f_y)\}. \quad (36)$$

The PSF corresponding to Eq. (36) is given by

$$h_{45}(x,y;\Gamma_1,\Gamma_2)=\frac{1}{2}\left(\cos^2\frac{\Gamma_2}{2}+i\sin^2\frac{\Gamma_2}{2}\right)\{|P_{ccw}(x,y)|^2+|P_{cw}(x,y)|^2+\exp(-i\Gamma_1)P_{ccw}(x,y)P^*_{cw}(x,y)+\exp(i\Gamma_1)P_{cw}(x,y)P^*_{ccw}(x,y)\}. \quad (37)$$

Second, the effective pupil function of the MTI with a linear polarizer oriented at an azimuth of -45 degree in Fig. 2 is given by

$$P_{-45}(f_x,f_y)=\frac{1}{2}\exp\left(-i\frac{\Gamma_1}{2}\right)\left\{\left[\left(\cos\frac{\Gamma_2}{2}+i\sin\frac{\Gamma_2}{2}\right)P_{ccw}(f_x,f_y)-\exp(i\Gamma_1)\left(\cos\frac{\Gamma_2}{2}+i\sin\frac{\Gamma_2}{2}\right)P_{cw}(f_x,f_y)\right]\hat{x}+\left[-\left(\cos\frac{\Gamma_2}{2}+i\sin\frac{\Gamma_2}{2}\right)P_{ccw}(f_x,f_y)+\exp(i\Gamma_1)\left(\cos\frac{\Gamma_2}{2}+i\sin\frac{\Gamma_2}{2}\right)P_{cw}(f_x,f_y)\right]\hat{y}\right\}. \quad (38)$$

In the case of the incoherent system, the OTF of Eq. (38) is given by

$$OTF=\frac{1}{2}\left(\cos^2\frac{\Gamma^2}{2}+\sin^2\frac{\Gamma^2}{2}\right)\{P_{ccw}(f_x,f_y)\otimes P_{ccw}(f_x,f_y)+P_{cw}(f_x,f_y)\otimes P_{cw}(f_x,f_y)-\exp(-i\Gamma_1)P_{ccw}(f_x,f_y)\otimes P_{cw}(f_x,f_y)-\exp(i\Gamma_1)P_{cw}(f_x,f_y)\otimes P_{ccw}(f_x,f_y)\}. \quad (39)$$

The PSF corresponding to Eq. (39) is given by

$$h_{-45}(x,y;\Gamma_1,\Gamma_2)=\frac{1}{2}\left(\cos^2\frac{\Gamma_2}{2}+i\sin^2\frac{\Gamma_2}{2}\right)\{|P_{ccw}(x,y)|^2+|P_{ccw}(x,y)|^2-\exp(-i\Gamma_1)P_{ccw}(x,y)P^*_{cw}(x,y)-\exp(i\Gamma_1)P_{cw}(x,y)P^*_{ccw}(x,y)\}. \quad (40)$$

Then, in the case in which $\Gamma_1=\pi/2$ and $\Gamma_2=\pi/2$, through the OTF synthesis using Eqs. (37) and (40), we can obtain sine component.

$$h_i(x,y)=\frac{1}{2}\left\{h_{45}\left(x,y;\frac{\pi}{2},\frac{\pi}{2}\right)-h_{-45}\left(x,y;\frac{\pi}{2},\frac{\pi}{2}\right)\right\}=|P_{cw}(x,y)P_{ccw}(x,y)|\sin\{\theta_{cw}(x,y)-\theta_{ccw}(x,y)\}. \quad (41)$$

TABLE 1. Intensity patterns by combination of two wave plates and a linear polarizer.

azimuth angle of a linear polarizer (degree)	phase retardation of wave plates	PSF
0	$\Gamma_1=\pi/2$ and $\Gamma_2=\pi/2$	$\frac{1}{2}[1+\cos\{\theta_{cw}(x,y)-\theta_{ccw}(x,y)\}]$
	$\Gamma_1=\pi/2$ and $\Gamma_2=-\pi/2$	$\frac{1}{2}[1-\cos\{\theta_{cw}(x,y)-\theta_{ccw}(x,y)\}]$
45	$\Gamma_1=\pi/2$ and $\Gamma_2=\pi/2$	$\frac{1}{2}[1+\sin\{\theta_{cw}(x,y)-\theta_{ccw}(x,y)\}]$
-45	$\Gamma_1=\pi/2$ and $\Gamma_2=\pi/2$	$\frac{1}{2}[1-\sin\{\theta_{cw}(x,y)-\theta_{ccw}(x,y)\}]$

From Eq. (41), we can see that we obtain the sine function using two intensity patterns obtained by rotating a linear polarizer by 45 degrees and -45 degrees, respectively, with respect to x axis in the MTI with two wave plates. We can obtain the complex hologram without bias and conjugate image by the electronic combination of Eqs. (33) and (41). Table 1 shows the PSFs by the combination of the azimuth angle of a linear polarizer and the phase retardation of two wave plates in the MTI. We can see that the complex hologram without bias and without conjugate image is obtained by combining four PSFs in Table 1 electronically.

IV. CONCLUSION

In this paper, we introduced the general OTF synthesis for spatial filtering in an incoherent imaging system and applied the OTF synthesis to the MTI.

Based on the OTF synthesis, we derived and analyzed the OTFs of the MTI. Because we can obtain only the one OTF in one-pupil system of the MTI, we cannot remove bias and conjugate image in the reconstructed image. Also, we cannot obtain any bipolar function in one-pupil system of the MTI. But, in the MTI with two-pupils, we can remove bias and conjugate image in the reconstructed image, and can obtain two bipolar functions, cosine and sine functions. Also, through the OTF synthesis, we can see that there are several kinds of the combination of the phase retardation of wave plates for removing bias and conjugate image.

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