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An Analysis of the Influences of Psychological and Social Theories on Mathematics Education

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Psychological and social theories have influenced on making sense of teaching and learning of mathematics. This paper analyzes major influences of such theories – behaviorism cognitivism, and situativity – on mathematics education. Instead of reviewing the theories per se, it intends to explicate how different perspectives have shaped our understanding of mathematics education both in theory and in practice. Given that the current mathematics education reform ideas are theoretically based on the constructivist and the sociocultural perspectives, the main focus is given on cognitivism, situativity, and various coordinations between the two. Exploring about psychological and social theories in the context of mathematics education is expected to enrich our understanding of where we have come from and where we are going.

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I. OPENING

Mathematics education recommendations for practice have been influenced by psychological and social theories, ranging from behaviorism through cognitivism to the recent situativity. The particular research paradigms and traditions in each theory have shaped the teaching and learning of school mathematics in a different way. It may not be an overstatement to say that many theoretical constructs and perspectives in mathematics education research originally stem from psychological and social theories.

The influences of psychological and social theories on mathematics education may be interpreted as the history of mathematics educators' effort to explicate the nature of mathematics learning and to enhance mathematics instruction. In behaviorism, learning is to strengthen bonds between stimuli and responses. In cognitivism, learning is to process information in terms of production systems. In situativity, learning is to interact with resources in a situation. Shifts in focus in mathematics education have been accompanied by these different interpretations of learning. In fact, much of the current mathematics education reform movement reflects new cognitive perspectives. This paper analyzes such shifts in mathematics education with regard to the influences of theories of learning. A cautionary note is that this paper does not intend to review the psychological and social theories per se in detail.

II. BEHAVIORISM IN MATHEMATICS ECUATION

Though behaviorism has dominated educational psychology since the beginning of the twentieth century, a more direct influence on mathematics education emerges in the 1960s by Gagné, whose main concern is the behavioral response of the learner following some form of instruction. Gagné regards learning as observable changes in the student's behaviors. He made specific mathematics learning hierarchies, containing a sequence of prerequisite abilities (*e.g.*, Gagné 1962). In the hierarchy frame, students are supposed to acquire these prerequisite abilities in order to master higher order skills. In behaviorism, since most activities in mathematics require definable and observable prerequisite learning, mathematics topics lend themselves to hierarchical analyses.

External conditions of learning in mathematics, the use of drill and practice to build arithmetic skills, understanding problem solving as a skill, and evaluation of mathematical understanding by immediate recall and retention were main influences of the behavioral approach through the 1970s (Kieran 1994). Specifically, the focus on using drill and practice for acquiring accurate skills of four basic operations seemed to fit well with the back-to-basics movement during the 1970s.

The most influential factor was emphasis on measurable performance. Mathematics education research focused on experimental format and on statistical significance. Students were considered as direct receivers of mathematical knowledge, and their understanding of this knowledge was measured by tests. What counted was not the learning process, but the quantification of outcomes. The effectiveness of mathematics teaching was evaluated by how well students mastered specific, precisely defined content material. As a result, specific behavioral terms and knowledge hierarchies formed the basis for much of the school mathematics curricula (Krol 1989).

To be clear, there is an important distinction to be made between the educational practices under the guise of behaviorism and what the theory actually says about learning. For instance, it is usual to attribute to behaviorists the notion of a "passive student" and the educational practices of lecture. But this characterization of behaviorism doesn't much accord with behavioral theory per se (Zimmerman & Schunk 2003). Nevertheless, the influences of behaviorism can be seen today in behavioral objectives, in individualized learning packages, and in more recently some computer-based instructional systems. Drill and practice are still popular. In short, behaviorism provides us with a mechanistic view of learning in which understanding played little part. Learning mathematics means to solve particular kinds of problems accurately.

III. COGNITIVISM IN MATHEMATICS EDUCATION

3. 1. Transition from behaviorism to cognitivism

During the 1970s the learning of mathematics as a cognitive process was presented, in conjunction with critiques about shortcomings of the behavioral perspectives (*e.g.* Wittrock 1974). A main critique was that most studies under the behaviorism paradigm did not describe thought processes involved in learning and understanding mathematics.

A manifest sign of transition from behaviorism to cognitivism in mathematics education can be illustrated by two critiques of a Gagné article in 1983. Gagné addressed a three-phase performance model on the basis of cognitive learning theory to improve student' arithmetic skills:

(a) translating from a problem statement to a mathematical expression,
(b) carrying out an operation on the expression, and
(c) validating the solution.

Specifically, he recommended that mathematics teachers should teach correct rules and make computations automatic by increasing practice. Wachsmuth (1983) pointed out learning with understanding, and relationships between syntactic rules and semantic knowledge. Steffe & Blake (1983) claimed that Gagné distorted the nature of mathematical learning under the guise of application of information-processing theory, only to emphasize computation in a similar way in which early behaviorists such as Thorndike did.

Automaticity of skills for optimal performance at the expense of understanding was problematic in the 1980s. Instead of finding ways of achieving skill automaticity, the mathematics education community focused on students' sense-making in solving problems. In particular, problem solving was recommended as the main theme of school mathematics in relation to the failures of the back-to-basics movement (National Council of Teachers of Mathematics [NCTM] 1980). Practicing skills was left to problem solving situations that required application of the skills.

3. 2. Cognitive science: Information-processing approach

Focus on mental processes involved in learning mathematics in the 1980s reflects the influence of the information-processing approach. Task analyses in the behavioral framework were changed into thinking-process analyses. There is disagreement with regard to the accomplishments in cognitive science (*e.g.*, Ohlsson, Ernest & Rees 1992; Schoenfeld 1987). In fact, there are those who have critiqued cognitive science as reducing mathematical meaningfulness to an algorithmic form. However, the view of cognitive science as addressing conceptual structure is supported by the earnest attempts of cognitive scientists to develop models of meaningful learning.

Research on mathematical problem solving was heavily influenced by the theories and methods of cognitive science in the 1980s. Problem solving research in the 1970s focused on key determinants of problem difficulty in terms of task variables, heuristics training, and correlations of abilities and other variables with performance (Lester 1980). However, the focus was shifted to the strategies used by students, comparison of experts and novices problem solvers, studies of metacognition, and relation of affects/beliefs to problem solving (Charles & Silver 1988; Schoenfeld 1987). Mathematical understanding began to be seen in terms of a spectrum rather than in terms of rightness and wrongness of answers. With this transition, new kinds of research method were necessary to investigate the processes of learning as well as the products. This was accompanied by an increasing movement toward the use of methods of cognitive science such as case studies, interviews, and thinking-aloud protocol analysis (McLeod 1994; Schoenfeld 1994).

Cognitive science has provided a new way for analyzing mathematical knowledge. Because a major goal of mathematics instruction in cognitivism is to help students acquire well-structured knowledge, analyses of mathematical knowledge are necessary (Resnick & Ford 1981). Many models of cognitive structures and processes involved in doing tasks have been developed, including arithmetic computation, elementary word problems, complex algebraic equations, and geometry proof exercise (*e.g.*, Bruer 1993). Students' systematic errors in applying algorithmic procedures were analyzed and computer-based tutorial systems for remediation were developed (Maurer 1987; VanLehn 1982). These kinds of detailed analyses enable mathematics teachers to create prescriptive versions of teaching and to identify cognitive obstacles from which students might suffer while solving problems.

3. 3. Constructivism

Constructivism inspired by Piaget's genetic epistemology is not a part of cognitive science per se. But constructivism has come to play an important role in supporting ideas of cognitive science. Specifically, the view that individuals encode, store, and recall information from memory is related to the constructivist view that individuals build interpretive knowledge structure in their minds through reflection on their actions in the world, through assimilation and accommodation (von Glasersfeld 1995).

The constructivist perspective assumes that learners do not simply add new information to already established knowledge structures. Instead, they connect or construct new relationships among the interpretive structures. Within this perspective, a teacher is very concerned about the possibility that an individual's knowledge structure may be isolated from each other, rather than integrated together. In other words, mathematics education emphasizes the importance of building on individual students' prior knowledge and making connections for their conceptual organization. Because of its intensely personal nature, however, learning and cognition are different for each individual. This raises the question of whether mathematics is invented in terms of an internal construction or discovered (Resnick & Ford 1981). Constructivism also challenges the attempt to establish empirical generalizations between teaching behaviors and students' achievement (Steffe & Kieren 1994).

The constructivist view of mathematics learning has been popular since the early 1980s. In particular, this view served as one of the fundamental theoretical constructs for the current mathematics education reform movement, emphasizing how students learn mathematics as well as what mathematics is. Many researchers have claimed that mathematical learning consists of students constructing mathematical concepts and procedures (*e.g.* Kamii 1990; Steffe & Blake 1983). This view has helped to overturn the view of mathematical teaching as the transmission of the teacher's knowledge and mathematical learning as passive reception.

Although several versions of constructivism are identified (*e.g.*, Confrey 1995; Ernest 1996), the discussion here attempts to capture the fundamental aspects of cognitive constructivism. The constructivist perspective assumes that learning occurs through cognitive conflicts by which the individual's mental structure evolves into more viable structure (von Glasersfeld 1995). Thus, the main concern for teaching in mathematics education is to help students enhance their cognitive structures with respect to specific mathematical content (Cobb & Steffe 1983). Within this, social interaction contributes to the extent it raises cognitive conflict and perturbation leading to cognitive reorganization in the process of individual's sense making (Steffe & Kieren 1994). Consequently, the crucial role of a teacher is to provide a learning environment wherein students can confront the limitations of their current understanding of a specific mathematical concept, which in turn leads to conceptual changes. For this reason, it is important for a teacher to conjecture about a student's previous construction of a mathematical topic and to develop extremely detailed teaching strategies in order to modify the student's thinking (Simon 1995). The teacher continually re-assesses his or her conceptual portrait of the student and the corresponding teaching model based on the effectiveness of the interactions with the student.

Recently, Piaget's reflective abstraction is elaborated as a key concept of explaining mathematics conceptual learning. In fact, Simon and his colleagues (Simon, Tzur, Heinz & Kinzel 2004) shift the focus of creating cognitive conflict to promoting reflective abstraction. Specifically, they propose a lesson design process, specifying students' current knowledge, specifying the pedagogical goal, identifying an activity sequence, and selecting a task.

However, mathematics educators who follow the constructivist approach encounter a dilemma of their role of teaching. The teacher is a facilitator who effectively organizes learning environment for each student's knowledge construction. Since learning is intrinsic to the individual and thus intensely personal, more responsibility for learning is given to the student rather than to the teacher.

IV. SITUATIVITY IN MATHEMATICS EDUCATION

4. 1. Sociocultural and anthropological approach

The sociocultural perspective has revealed the importance of joint activities in social situations in which participants appropriate each other's contributions. Specifically, even during the early 1980s some mathematics researchers espoused Vygotsky's *Zone of Proximal Development* as a useful theoretical and pedagogical construct (*e.g.* Carpenter 1980; Fuson 1980). However, the influence of sociocultural perspectives on mathematics education is relatively recent.

The anthropological approach has explored the relations between cultural activities and cognitive development, specifically the comparisons of children's mathematical thinking in and out of school culture (*e.g.*, Carraher, Carraher & Schliemann 1985; Lave 1988; Saxe 1991). Such an approach has often demonstrated that school mathematical knowledge is noticeably absent in out-of-school settings, suggesting that individual's arithmetical activities are profoundly influenced by their participation in encompassing cultural practices.

The sociocultural and the anthropological perspectives have challenged the view that the individual constructs mathematical knowledge structure in his or her mind, and that school mathematics has to be deliberately de-contextualized for generalization. First of all, this challenge brought about consideration of a much broader context for mathematical problem solving; For example, presenting problem situations that closely resemble real situations in their richness and complexity and asking students to pose problems, to generate conjectures, and to share their mathematical analyses (Charles & Silver 1988).

Apprenticeship model has emerged from anthropological literature. Applying this model to school mathematics, Lave, Smith, and Butler (1988) suggested that activities of school mathematics should provide students with rich contexts in which knowledge is situated through intellectual tasks. The apprenticeship notion implies that the purpose of school mathematics is enculturation into mathematical practices. Learning is characterized as mutual appropriation by which the teacher (or master) and the students (or apprentices) continually coopt each other‘s contributions until the students are engaged in expected practices (Leont’ev 1981). The teacher, serving as a representative of a mathematical community, organizes classroom activity settings in such a way that students experience the authentic nature of mathematical activities including mathematical ways of knowing, communicating, valuing, justifying, agreeing, arguing, etc. (Collins, Brown & Newman, 1989; Lampert, Rittenhouse & Crumbaugh 1996). Mathematical knowledge is related to social interaction with various resources in a given situation, rather than to mental representation of mathematical concepts and procedures. This view reflects a move away from explaining cognition as an individual mental process to understanding the interpersonal context of cognitive growth (Forman, Minick & Stone 1993).

The sociocultural and anthropological perspectives have influenced the current mathematics education reform movement. During the 1980s the series of NCTM yearbooks focused on curricular issues and on the teaching of specific mathematical contents. However, the 1990 yearbook is concerned with diverse issues for reform in the new decade, including analysis of small-group cooperative learning, emphasis on communication, and consideration of contextual factors. These changes are consistent with *Curriculum and Evaluation Standards* and *Principles and Standards* of which central idea is the development of mathematical power for all students (NCTM 1989; 2000). This idea is accompanied by consideration of mathematics as problem solving, communication, reasoning and proof, representations, and connections beyond the view of mathematics as a collection of concepts and skills. Purpose of school mathematics is not only the acquisition of mathematical objects but the situated, collaborative practices of mathematical thinking. Concomitantly, NCTM (1995) focuses on assessment of the process and the individual’s participation. This reflects a substantive shift from outcome and from the individual’s possession as a basis for assessment.

Another influence of situativity is an emphasis on social aspects of classroom micro culture. For example, Bauersfeld and his colleagues (*e.g.*, Bauersfeld, Krummheuer & Voigt 1988) regard mathematician as a social practice and use symbolic interactionism to analyze learning and teaching mathematics. In the same vein, Silver (1988) considers mathematics classrooms places in which situated and collaborative practices occur through socially-distributed problem solving. Lester (1994) says that mathematical problem solving research needs:

(a) attention of the teacher's role as the single most important item;
(b) descriptions of classroom atmosphere, including the teacher's behaviors, teacher-student, and student-student interactions; and
(c) focus on groups and whole classes rather than individuals.

This suggestion is in sharp contrast with the emphasis in the 1980's on the thinking processes used by individuals as they solve problems or as they reflect back on their problem solving efforts.

Reviewing research on affective issues in mathematics education, McLeod (1994) claims that we need to analyze possible sociocultural contexts which might influence students' affect on mathematics, including social organization of schools. Cobb, Wood, and Yackel (1993) regard a classroom as a sociocultural system in which the teacher and students negotiate taken-as-shared mathematical knowledge in terms of mutual appropriation. Students are not merely individual learners but members of a classroom community. However, these perspectives require us to develop appropriate methodological tools for quality of the observations and the interpretive analyses: For example, how to observe, document, and describe interpersonal influences in the process of social construction of mathematical knowledge.

4. 2. Individual cognition and social cognition

The constructivist and the sociocultural perspectives constitute two of the major trends in current mathematics education (Davis 1992; NCTM 2000). Whereas constructivist perspectives account for students' conceptual development, sociocultural perspectives illuminate the nature and effects of their participation in socially shared activities (Cobb 1994). As we see in the studies based on the sociocultural and anthropological approach, there has been much concern for social and interpersonal influences at work in the mathematics classroom. This implies that we need to interpret mathematics learning interpersonally as well as intrapersonally.

Mathematics learning cannot be fully understood intrapersonally because of its social aspects. Alternatively, analysis in terms of only interpersonal constructs will be inadequate, since it is the learner who must understand mathematical meanings. Therefore, it is crucial to recognize the relationship between individual cognition (or constructivist perspectives) and social cognition (or sociocultural/anthropological perspectives). Indeed, Balacheff (1990) recommends characterizing the relationships between situational aspects and students' cognitive behaviors as an important field of investigation for future PME research activities. The following is a brief review of three main attempts in mathematics education with regard to this issue.

A first attempt is to take the perspective of individual cognition as its basis and develop a broader perspective by exploring social contexts for the cognitive processes. Constructivism has a difficulty in explaining intersubjective construction of mathematical knowledge. Social constructivism has grown out of the attempt to solve this problem. Social constructivists incorporate social dimensions of learning such as interactions into a deeper knowledge of the learner's mathematical thinking (Cooney, 1994).

However, social aspects in this approach can serve only as a catalyst for an individual's construction of knowledge, because social constructivism also is based on an individualistic psychology (Cobb, Wood & Yackel 1993). Further, the link between social and individual processes is indirect in that participation in social interaction may not determine learning (Cobb 1994). Therefore, social constructivism can not accept the fundamental implications of the sociocultural perspective, in which the link is direct in that the qualities of students' thinking are generated by or derived from the organizational features of the social activities in which they participate (*e.g.*, Newman, Griffin & Cole 1989). Lerman (1996) argues that adding a sociocultural view to constructivism leads to an incoherent theory of mathematical learning, because the two positions are fundamentally different.

A second attempt is to take the perspective of social cognition as its basis and to develop a more comprehensive perspective by probing detailed contributions of individual cognition in the interaction. Situativity comes to provide us with an insight in understanding mathematical learning in a broad context of personal and social factors, connecting analyses of individual, interpersonal, and community process. However, most of the situated cognition theory has been based on out-of-school situations. Thus how the principles of situated cognition play out in relation to mathematics learning is still in the process of being developed, in conjunction with theorization of the relationship between individual and social aspects of cognition.

A third attempt may deny a kind of priority between the individual and the social cognition, claiming that mathematical learning is both active construction and enculturation. Cobb and his colleagues (*e.g.*, Cobb & Bauersfeld 1995; Cobb & Yackel 1996) take a pragmatic approach in which researchers can take either perspective according to problems and issues at hand under the assumption of a complementarity between individual and social aspects: "We consider that students actively contribute to the evolution of classroom mathematical practices as they reorganize their individual mathematical activities and, conversely, that these reorganizations are enabled and constrained by the students' participation in the mathematical practices" (Cobb & Yackel 1996, p.180). Thus, Cobb and his colleagues relate analyses of individual students' thinking to those of classroom interactions, discourse, and the classroom culture. Their analyses show reflexive relationships between students' mathematical activity and the social relationships, between students' realization of sociomathematical norms and the social situation in which they are developed, and between engaging in argumentation and mathematical learning.

The current mathematics education reform movement has been theoretically supported by the constructivist and the sociocultural perspectives, and the reform-oriented teacher is expected to have instructional goals for both students' construction and their enculturation (NCTM, 2000). However, it is a misrepresentation to assume that such instructional approaches can seamlessly be meshed in day-to-day classroom practices. In reality, the relationship between the two perspectives is seen as reflecting tensions endemic to teaching mathematics. For example, creating classroom practices on the basis of ideas such as understanding, authenticity, and community, Ball (1993) illustrates pedagogical complexities in the form of three dilemmas:

(a) representing contents;
(b) respecting children as mathematical thinkers; and
(c) creating and using community.

Wood, Cobb & Yackel (1995) also report that the classroom teacher participating in their year-long experiment is occasionally to be directive with regard to social norms, while she intends to promote students' development of autonomy and independence.

Such dilemmas and difficulties reflect uncertainty of relationship between individual cognition and social cognition on mathematical learning, leading to difficulties in the transition from reform rhetoric to classroom teaching practices. Given this, a crossdisciplinary approach is suggested in place of a comprehensive unitary approach in which reform teaching is envisioned as a consistent and coherent undertaking (Kirshner 2002). The cross disciplinary framework highlights the unreconciled diversity of the different theories of mathematics learning (*e.g.*, construction and enculturation), and provides teachers with opportunities to purposefully select the instructional goals to which their teaching should aspire. Further research on the relationship between individual and social aspects needs to be encouraged and articulated.

V. CLOSING

This paper explores a succession of psychological and social paradigms – behaviorism, cognitivism, and situativity – that typically inform instructional planning and analysis in school mathematics. The influences of theories of learning on mathematics education show how we have seen and done mathematics education in a different way over a half of the century. The psychological and social theories have shed light on mathematical learning and teaching. We still can not underestimate the influences of behaviorism and cognitivism which are manifest in many mathematics textbooks and educational practices: For example, emphasizing practices with similar problems, evaluating learning only by measurable outcomes, and focusing on the individual's mental representations of specific mathematical contents.

However, there have been some substantive shifts in focus, which reflect the current cognitive theories, from learning outcomes to learning processes, from the learner to the teacher, from learning in the individual's mind to learning in the social situation, and from the individual to the classroom culture. In fact, current mathematics education reflects four interrelated areas:

(a) constructivist interpretations of learning,
(b) applications of situated cognition to school mathematics,
(c) innovative development in classroom research, and
(d) movement from an individual cognitive framework to social interactionist orientation.

The changes in mathematics education according to psychological and social theories reflect different point of view about mathematical knowledge, learning, and teaching. What does it mean to know mathematics? The influences of psychological and social theories on mathematics education suggest the importance of the study of mathematics epistemology. The analysis of theory and practice in mathematics education research with respect to the influences of paradigms of learning will delve into the fundamental basis of understanding where we stand and where we head for.

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