

## School Mathematics as a Major Subject for 'Humanity Education'

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One of the major roots of the value and power of mathematical knowledge is the belief on 'the Pythagorean-Platonic divine mathematicity of the universe' and the 'pre-established harmony between mathematics and physics'. This kind of the nature of mathematical knowledge demands strongly the school mathematics to become a subject for humanity education going beyond the practical usefulness. Here, investigating the roots of the thought of mathematical education, we tried to clarify that the traditional educational ideal which has maintained the theoretical knowledge-centered mathematical education is the education of humanity, and investigate the way today's mathematical pedagogy should first turn to if it should realize this ideal.

### 1. Introduction

One of the major problems of today's school mathematics, which is one of the core subjects in compulsory education, is the issue of 'mathematics for all'. What is the purpose of mathematics education as a national mass education? Why is it that we are now making such an effort to teach mathematics to all students? Without question, mathematics is needed in the development of scientific technology. However, not everyone is going to be a scientist or a technologist, and it is difficult to insist that most of what we teach in mathematics class is needed for leading the present day life.

Mathematics has occupied an important place as a core subject in school education ever since

Greece. This shows that teaching and learning mathematical knowledge has been considered 'educationally' important beyond the practical usefulness of mathematics. Why is mathematics 'educationally' important?

As human beings we live two levels of life; practical level and theoretical level, and the shapes of living and education become varied according to the level on which we put more weight. Attaching great importance to theoretical knowledge is the tradition of the Greek education for the free citizen and the medieval seven liberal arts education led by the Scholastics. This was also true in the traditional Korean noblemen's education which valued the liberal education based on the Four Books and the Three Classics of Ancient China. But unfortunately our ancestors undervalued the arithmetic only as a practical

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tool, and the noblemen had no concern for it. This is a big difference of the civilization of Orient and Occident. The English word 'theoretical' was derived from Greek word 'theoria' which means 'seeing', and 'practical' was derived from Greek word 'praxis' which means 'doing'. The education of theoretical knowledge is the tradition of education for developing the conceptual means or our mind's eye for seeing the phenomena (Lee, 2002).

From the period of Greece until now, if there is any thing that has not changed in school mathematics, that will be the fact that theoretical knowledge which goes beyond practical knowledge and is not very much motivated with practical problem solving, has been continued to be taught. The practice of mathematics teaching is as it used to be, but, especially entering the 20th century the pragmatist point of view that regards mathematics as problem solving tools, which is supported by Dewey's experientialism philosophy, was emphasized, and since 1980s endeavor for teaching practical knowledge such as problem solving and applying mathematics has been continued until now.

According to the examination of Niss (1996), although it is difficult to judge that the practice of education was directed that way, in the history of mathematics education of 20th century we can observe the overall oscillation between two kind of goals of mathematics teaching; one is the disciplinary goal that is to further the development of reasoning and logical thinking power, concentration, persistence, and intellectual

functions; the other is the utilitarian goal. And we can find also the cultural or aesthetic aims such as the appreciation of beauty in the geometrical forms, ideals of perfection as to the precise logical reasoning, and power of abstract thinking. But, the question has to be raised about the extent to which we realize that how much the fundamental goal of education is to cultivate the internal eye 'seeing' the phenomena, virtue, humanity and disposition of the students in the mathematics curriculum of each country of the world. Despite of every endeavor for the reform of curriculum, the practice of mathematics education that is centered around the theoretical knowledge and teacher takes the lead, has been scarcely changed (Bodin & Capponi, 1996). On the whole the goal to develop the ability for solving real life problems does not match well with the character of the subjects that have been taught practically in the mathematics class. There are still few items of practical knowledge which serve to solve efficiently the real life problems in the school mathematics except elementary school level. We mathematics educators should face up the actuality of mathematics education and reexamine the educational value of the theoretical mathematics knowledge. Here, investigating the roots of the thought of mathematical education, we will maintain that one of the major ideals of traditional mathematics education based on the theoretical knowledge centered education is the cultivation of humanity, and explore the way that today's mathematical pedagogy should first turn to in order to realize this ideal.

## II. The Roots of Humanistic Mathematics Education I

### 1. Platonism

In the western civilization which has its roots in ancient Greek civilization, mathematics was traditionally located at the core of knowledge education. If you trace back the western ideological history to Greece, you can infer the real reason for learning mathematics. It is well known that Plato said that those who have not learned mathematics cannot be called educated. Why he said so? Plato's view on the nature of mathematics knowledge and its educational importance can be summarized as follows(Yim, 1998).

Plato(1973) assumed the reality, that is, a metaphysical world called Idea. In his dialogues *The Republic* and *Phaedo*, Plato thought that the world of Idea and the physical world is connected 'up and down' and that the upper part reveals the essence of the lower part in its purist form and that the lower part embodies the essence of the upper part. According to Plato, people cannot lead a humane life because of the body's lust, the sense's limitation and the 'ignorance'. But man is a being for whom it is not good enough to remain in this state of life. He has to overcome it in order to be a real human being.

To Plato, education is a way to make people turn their eyes from the real world like the cave, to the true world, the world of Idea. To him, an educated person is the one who perceives the world of reality by stepping over the shadow of

the real world, someone who has recovered their pure spirit within by freeing themselves from the limitation and confinements of their body.

And, Plato cited arithmetic as the first useful subject to lead the spirit to the reality, and following arithmetic he cited geometry as the second in *The Republic*. Since mathematics studies logically the invisible archetype of the things using the eikon that is incompletely embodying it, it becomes the ladder for climbing to the "upper world" from the "lower world." By giving us the clue to understand the harmony and beauty of the world of Idea which exists on the other side of the visible world, mathematics enables the human being to get ready for leading the mind from 'the sensible world' to 'the intelligible world' and by providing such awareness, it becomes a subject that has educational value.

In this way, to the free citizen of Greece mathematics was the most appropriate subject to make them open their 'eye of the soul' and turn toward the world of reality, and for them learning mathematics was a necessary stage in 'the journey toward truth'. Thus the man who did not know arithmetic and geometry was not considered as seeing the image of truth; his 'eye of soul' for the truth was not opened.

Mathematics is a discipline that started as a study for the real human education.

It is interesting to examine how these conceptions on mathematical education have been pursued in the western secondary school education. I think that, today, as mathematics became a required subject for all free citizens, the ideal of Greek mathematical education surpassed the

dimension of the elite education and became an ideal of public mathematical education.

## 2. The Structure of Knowledge and the 'Reality'

The Greek educational ideal pursuing 'the world of reality' was developed into a new dimension by Bruner who wanted to make students see the 'structure of knowledge' through spiral curriculum and discovery learning in the middle of the 20th century.

According to the structure centered curriculum, the content of a curriculum should be the structure of knowledge implicit in the outer layer of knowledge. Bruner used the structure of knowledge as a synonym with 'general idea that make up the foundation of the subject', 'basic notion' and 'general principle', which means 'the eye to look at matters' or 'the way of thinking' that defines each subject. But it is not a state where we can actually reach or realize, but rather a standard for seeing the matters that we should figure out or try to reach. Like Idea, knowledge is such a thing that is impossible to reach completely such that the harder we try to pursue it we realize that we are ignorant.

We can understand this by considering, for example, how difficult it is to grasp the meaning of the notion of function, functional thinking or group in spite of much discussion.

Bruner(1962) thought that his structure centered view of the school subjects is different from the Dewey's pragmatist view of the school subjects and said that the mistake of Dewey was that he looked at the school subjects mainly from

the practical aspect. It should be noticed that the discovery learning teaching method that Bruner suggests is a theoretical combination of Plato and Piaget by dynamically adapting modern developmental theory combined with classical rationalism(Shulman, 1970). The structure based curriculum shown in Bruner's *The Process of Education* can be seen as an effort to face the 'structure of knowledge' or 'the form of knowledge' by the spiral curriculum and discovery learning, according to the Zeit geist in his day which is called structuralism. We can say that it was an attempt to do successfully the education of the theoretical knowledge in order to cultivate the internal eye 'seeing' the real nature of phenomena, and now such an ideal is being pursued as usual.

## III. The Roots of Humanistic Mathematics Education II

### 1. Practical Mathematics of Archimedes and Nature's Mathematical Order

There is not only one root to the thought of mathematics education, but more than two. The second root is the practical mathematics of Archimedes, Heron or others, and the rise of the scientific investigation of the mathematical laws of nature, as was initiated by Galileo. Practical mathematics has the spirit of inquiry, discovery, experiment, calculation and application, and has become the essential tool for the inquiry of nature since Newton. After that, the basic character of nature which the physical laws are

described by mathematics began to become known.

“The leading figures of the Modern Era Kepler, Galileo, Descartes, Newton and others emphasized in the most elevated terms the Pythagorean idea of the divine mathematicalness of the world. To quote Leibniz : “Cum Deus calculat, fit mundus” (As God calculates so the world is made). ... Hilbert spoke of the “pre established harmony” between mathematics and physics. ... Einstein returned to the thought of this harmony, but this time he spoke of the pre established harmony between physical reality and mathematical structures ... He used the example of conic sections, realized in the orbits of celestial bodies, to explain the sense of this harmony...” (Vizgin, 2004: 265-267)

The awe inspiring nature of the mathematical knowledge which constitutes the order of universe, obviously demands that mathematics become a subject for the moral education of the students going beyond a practical tool for solving real life problems. Froebel was an educational thinker who highly appreciated the value of mathematics for humanity education with the ideological background that the harmony of mathematics and physical world is the proof of the existence of God. He investigated the problem in the nature of human education and made the following assertions(Han, 2000).

Through mathematics a man can conceive the divinity that exists inside the universe, both human and natural. Through studying mathematics, he knows that there is a mathematical order inside the natural world, and that this order is expressed through the law of speculation by

the pure human spirit. Here, he becomes conscious of the divinity inside man and nature, and becomes a valuable person who by realizing the spirit of God, believes in the existence of God and lives by the will of God. So, to Froebel mathematics must be an educational means to perceive the spirit of God in nature and man and it should be a subject that has essential meaning in ‘human education’.

To learn mathematics is to acquire the eye to ‘see’ the world created by God through the ‘form’ of mathematics, and this means that the mind of us changes accordingly and we can be pious human beings. Thus, to Froebel, ‘education without mathematics is weak, imperfect patch-work’(Froebel, 1887, p.208). The nature of mathematical knowledge demands strongly the school mathematics to become a subject for humanity education going beyond the practical usefulness.

## 2. Computational Mathematics and the Sense of Truth

The mathematics developed in the cultural circle of India and Arabia after the decline of Greek mathematics was practical mathematics which centered on computations. With the discovery of the decimal notational system, the mathematics of India and Arabia brought the development of the four arithmetical computation methods which greatly influenced the development of human civilization. Also, the development of algebra, centering on solving equations, and the introduction of the algebraic symbols by Viète, brought the development of modern mathematics.

The formula for computation, represented by letter variables, is an algorithm deduced from basic rules and an amazing mathematical form seeking truth, where the discovery of the answer and its proof is conducted together. If we do not feel 'the computational spirit and the sense of truth(Rechnungsgeist und Wahrheitsinn)' as Pestalozzi said, when we look at the formula of the roots of quadratic equations or the fundamental theorem of calculus, we do not 'see' the true form of mathematics. The two branches of mathematics geometrical mathematics and computational mathematics combined and brought the birth of Descartes' analytic geometry in modern Europe. And this again combined with applied mathematics, leading Newton and Leibniz to invent differential and integral calculus which provided basis to modern scientific civilization. The computational mathematics which attracted Descartes, who sought for the universal method to solve problems, and Leibniz, who said even that 'As God calculates so the world is made', is the most certain method of inquiring truth together with axiomatic mathematics. In the middle of 20th century human beings invented computer and led to information society and moved up even to the dream of 'computopia'.

The spirit of computational mathematics ought to be the base of national education, making students pursue the sense of truth and become a virtuous man. We should teach the students to realize that the method to compute is by no means a mechanical operation and rather the most wonderful method which human beings created to inquire the truth.

#### IV. The Roots of Humanistic Mathematics Education III

One other root of the thought of mathematics education can be found in the formal discipline theory. The basic assumption of this theory is that general mental ability such as inference, memory, imagination and will power, which are 'the muscles of the mind', can be trained by drilling, just as the muscles in our body. Once trained, they can be used for solving many intellectual and practical problems. It is claimed that the way to train such general mental abilities is to study traditionally acknowledged subjects such as classical language, history, mathematics, and science and others(Lee, 1992).

This theory has been denied by Thorndike's Identical Element Theory. According to Thorndike, transfer is not guaranteed by 'general ability', but by the 'identity of elements' between two situations, only to the extent that there is the sameness. Dewey(1960) criticized the point of view that if one learns mathematics, the inherent logical thinking form is acquired and one can think logically, and stated that 'logical form' and 'logical thinking' should be distinguished and that the 'logical' thinking of a child should be gradually trained through his own reflective thinking, rational thinking, and proper thinking.

But, according to Vygotsky(1962) the formal discipline theory is not related to the training of specific skills, but related to the education which activates the dimension of consciousness including awareness, abstraction and self control such as mathematics and classical language education.

And mathematical thinking is the result of the introduction to the mathematical forms of knowledge, and learning mathematics is the means to seeing the world through the forms of mathematical knowledge and to changing the minds of students. The education of theoretical knowledge can have educational value in that through this kind of change of mind people can become righteous human beings.

Though the formal discipline theory has been criticized and denied by Dewey, Thorndike and others, if it is viewed in the light of those who emphasize the form or structure of knowledge, reflective thinking, and the activation of the domain of awareness and consciousness, it is as certain as ever that learning mathematics 'properly' changes his 'way of looking at the world', and cultivates his 'logical and reasonable thinking ability'.

## V. Foundation of Humanistic Mathematical Education

What was shown from the above mentioned thoughts of the traditional mathematics education demands that mathematics education makes students see the image of truth, investigate the order of nature, by means of the computational methods of seeking the truth and intelligent logical thinking. The great educator who set up the mathematics education as human discipline is Pestalozzi. The educational thought of Pestalozzi is called the idea of basic cultivation, and he tried to naturally cultivate the three basic powers of man mental power, moral power and func-

tional power and to realize the development of 'a power of high humanity' through mathematical education. He believed in the educational value of mathematics as the stepping stone to cultivating mind. He made the following claims. To learn mathematics is to do a 'spirit gymnastics(Geistesgymnastik)', To divide the spirit of computation and the sense of truth is to divide what God has combined(Wer Rechnungsgeist und Wahrheitsinn trennt, der trennt, was Gott zusammen gefügt hat). 'To compute and to count is the base to every order of the brain (Zählen und Rechnen ist der Grund aller Ordnung der Kopf),' Mathematics is the operation which raises 'the natural aptitude of reason to the power of reason (die Vernunftanlage des Menschen zur Vernunftkraft),' Mathematics is the 'basic educational gymnastics(Erziehungsgymnastik)'(Kim, 1970).

I think that this thought of mathematical education has been pursued consistently in the West, even though almost implicitly, by mathematics educators who led the reform movement of mathematics education. In the early 20th century, Perry(1902) in his speech "The Teaching of Mathematics" at the British Royal Society, suggests eight kinds of usefulness resulting from the study of mathematics. The first three are producing the high emotion and giving mental pleasure, brain development and producing logical ways of thinking, and the aid given of mathematical weapons in the study of physical science. Let's focus on the order here. It should be noted here that between the intrinsic and extrinsic purpose of education, and between the value of knowledge itself and instrumental value, the first one, the affluent mental life, intellectual

satisfaction and seeking for beauty come first.

It is said that actuality is the parody of ideal. As in other areas, actuality and ideal are different in mathematics education, too. Unlike the Greek ideology that mathematical education leads the eye of reason to reality, mathematical education drifted to formal instruction. Rather than gaining the sense of truth and spiritual happiness, the more mathematics the students learn, didn't they become to loathe mathematics?

## VI. The Danger of Mathematical Education

Look at the preface of the book written by Dewey and McLellan(1895); *The Psychology of Number and its Applications to Methods of Teaching Arithmetic*. It is written; "There is no subject taught that is more dangerous to the pupil (than mathematics) in the way of deadening his mind and arresting its development, if bad methods are used." And, what is the reason that Euclid suffocates the minds of the students as Descartes said? How can we educate mathematics so that the students understand the true meaning and taste of mathematics and their human nature can be cultivated? This will be the most important problem we as mathematics educators must face. Piaget's theory which says thinking is operation and operation is internal reversible action, demands an active learning principle. Polya(1965) demands a teaching atmosphere where guessing, discovery and desirable habit of thinking induced by the questions and suggestions of the teacher as a midwife, are emphasized.

Dewey(1960) sees reflective thinking as having an important role in solving the practical problems or in the development of logical thinking, and gives central role to reflective thinking in education. What does 'doing mathematics' mean? Let's look at Freudenthal's claims(1983). To do mathematics is to observe one's or other's mathematical activity and reflect on it. To him, the main body of mathematising activity is reflective thinking. And what is educable? The following claim by Gattegno(1963) is meaningful.

'Only awareness is educable.' Awareness can be interpreted as realization. Man becomes educated only by realization. Then, how is awareness formed? The conclusion given to us is that it is obtained by activity, discussion and reflection. Let's learn from the following saying from the *Analects of Confucius*(1992). "Mere reading without thinking causes credulity; mere thinking without reading results in perplexities(學而不思則罔 思而不學則殆)." Mathematical education for every student should be conducted as cultural education in the real sense. Mathematics is the most pertinent subject to educate people to lead a life toward the reality by opening one's 'eye of awareness'. Here, actual mathematical activity and reflective thinking play a major role. As Polya(1965) points out, 'What the teacher says is not unimportant but what the student thinks is a thousand times more important.'

## VII. The Didactical Analysis of the Structure of Mathematical knowledge

If we want to make a change in the way



students view the world and the way they live their life, through mathematical education, and if we want to realize human education through mathematical education, first of all, there should be change in the mind of the mathematics teacher and the educational philosophy of the teacher should be established. But, as Polya(1965) emphasized, it is impossible to teach what they don't know. A deep understanding of school mathematics on the part of the mathematics teachers is the alpha and omega of mathematical education. The point of view which emphasizes 'the structure of knowledge' or 'the forms of knowledge' is to prescribe the meaning of the knowledge education in the light of the nature of the knowledge. But, school mathematics is a formal 'closed' knowledge that does not reveal the nature. To 'open' this and realize it into educationally meaningful knowledge through the didactical analysis, that is mathematical, epistemological, historic genetical, psychological, linguistic, practical and educational analysis of the structure of school mathematics may be the most important task for the teaching of mathematics for humanity education. The noteworthy ones here are the historic genetic developments of the school mathematics of Clairaut, Branford, Toeplitz and others, and Didactical Phenomenology of Mathematical Structures of Freudenthal. These are thought to have its goal in trying to open the closed school mathematics to make it become a didactically meaningful knowledge. This task, as Freudenthal insists, may be the starting point to mathematics educational research. The didactic phenomenological analysis of ratio and proportion, group, functional concepts and others that Freu-

dential attempted and the didactical analysis of the probability concept that Kapadia and Borovenik(1991) attempted, show how hard it is to 'see' the deep idea, the structure, the form and nature of mathematical knowledge. How can a teacher who does not see it properly or even does not try to see it, think about teaching it? These researches show how the mathematical knowledge that we are teaching and learning is only at the surface of the knowledge and how we are missing the much more important 'educationally' essential viewpoints. The mathematic educational research, first of all, ought to start from the inquiry of the deep 'structure' of school mathematics. And we also should not undervalue the role of the oral and written language in education. Recently the discovery and constructive approach through the learner centered activities using the concrete materials are emphasized, but it should not be overlooked that the using of language by the teacher in education has a more important role than anything else. The Socratic obstetrics in the Plato's Dialogues, the Analects of Confucius and Polya's modern heuristics are written in the dialogue style centered on teacher. And the sentence which is the record of knowledge may be a kind of residue of the realization as satirized in a Chinese classic, but it is the only one clue which make us guess what is the realization. Thus we can do nothing but try to get the realization through the written sentence.

## VII. Concluding Remarks

Now the time has come to rediscover the

idea of mathematical education as a public education and cultural education, and to recover the original feature of mathematical education.

Today it has become one of the central theme of the mathematical education to develop the student's power of solving the practical problems and applying the mathematics, but generally it does not fit into the nature of the mathematical knowledge which we are actually teaching in the school. Here we noticed this fact and reexamined the idea of the traditional mathematics education centered on the theoretical knowledge.

Education is an endeavor to identify and realize 'the forms of living' trying to approach the supreme good, that is, the human virtue. The great thinkers in the Orient and the Occident almost without exception recognized the world of logic, forms, substance and reality beyond the world of facts, phenomena and mortals. Teaching of the theoretical knowledge is to make the students 'see' the phenomena through such a form and at the same time to implant the belief that it is a valuable life to live seeing the phenomena so that they accept the theoretical posture of living, devoting themselves to the endless inquiry, and eventually conform to seeing 'the forms of living'(Lee, 2002).

Here we ought to notice that coming to the present day the tendency in which the basic physical laws are described in the language of mathematics, became more clear, and the quasi religious belief of the great mathematicians and physicists, such as Hilbert and Einstein, on the mathematical nature of the physical world, became more deep. Mathematics reflects the reality 'the logical cause of the universe' and

we face the image of it when we learn mathematics. The main purpose of teaching and learning knowledge called mathematics is to go beyond practicality and achieve the human education that help people come to realize the existence of the reality that rules the phenomenal world, and cause endless longing for it.

Is this not the idea that the school mathematics as a major subject for humanity education ought to try to attain? If the pendulum of mathematical education seems so far to have swung to the direction of Satan trying to pull to 'formal' meaningless education, we have to continuously push the pendulum to the opposite direction.

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## 인간 교육을 위한 주요교과로서의 학교수학

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수학교육은 이제 국민대중교육, 교양교육으로서의 수학교육의 이념을 재발견하여 수학교육의 본래의 모습을 회복해야 할 시점에 와 있다. 오늘날 우리가 학교에서 가르치는 수학 지식은 어떤 성격의 지식이며 우리는 그러한 지식을 가르침으로써 인간을 어떤 상태로 만들려고 하는가? 본고에서는 수학교육 사상의 뿌리

를 탐색해 봄으로서 수학이라는 지식을 배우는 주요한 목적은 실용성을 넘어 수학이라는 지식을 통하여 인간과 만물 이면에 있는 현상의 세계를 지배하는 실재를 깨달아 가도록 하는 인간교육의 구현에 있음을 밝히고, 그러한 이념을 오늘날 국민교육으로서의 수학교육이 우선적으로 지향해야 할 방향으로 제시하였다.

\* key words : humanity education(인간교육), platonism(플라톤 사상), reality(실재), computational mathematics(계산 수학), didactical analysis(교수학적 분석), reflection thinking(반성적 사고)

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