

## Linear Combination Analysis Using GPS Data

Un-Yong PARK\*, Jae-One LEE\*\*, Dong-Rak LEE\*\*\* and Jung-Soo HONG\*\*\*\*

---

### Abstract

We can process and compute the position, velocity and time by satellite signals of GPS. The signals are used to compute positioning of three dimensions and timing offset of the receiver clock when we can track the four satellite signals at least. One of the specified aims is to use less expensive single frequency code/carrier phase GPS receivers, which are typically around half the price of dual frequency receivers. In the study, the author analyzed the accuracy and applicability of frequency linear combination using triangulation points evaluated distance limitation.

*Keywords* : Single frequency, Dual frequency, Linear combination analysis

---

### 1. Introduction

The Global Positioning System is a one-way ranging system. The GPS satellites emit signals-complex modulated radio frequency-which propagate through space to receivers on or near the earth's surface. From the signals it intercepts, a receiver measures the ranges between its nature of the GPS signals.

Each GPS satellite transmits signals centered on two microwave radio frequencies, 1547.42 MHz, referred to as Link 1 or simply  $L_1$ , and 1227.60 MHz, referred to as  $L_2$ . It also transmit an  $L_3$  signal at 1381.05 MHz associated with their dual role as a nuclear burst detection satellite as well as S-band telemetry signals. These channels lie in a band of frequencies known as the L-band (1 to 2 GHz). Within the L-band, the International Telecommunications Union, the radio regulation arm of the United Nations, has set aside special sub-bands for satellite-based positioning system. The  $L_1$  and  $L_2$  frequencies lie within these bands (Kay, S. M. 1993).

One of the specified aims is to use less expensive single frequency code/carrier phase GPS receivers, which are typically around half the price of dual

frequency receivers.

However, regardless of size of a region, the more expensive, dual frequency receiver is being used to obtain accuracy position information and a strict data processing process to obtain results through the most suitable baseline interpretation and network adjustment has not been established.(Feit, L. ; Domanico, P. 1981)

There fore, this study was conducted using triangulation points scattered all over korea and through comparison analyses of the  $L_1$  Only method which processes only single frequency, the  $L_1$ & $L_2$  method which processes dual frequency, the  $L_1$ & $L_{2C}$  method which processes long-baseline, the accuracy and applicability of frequency linear combination and its distance limitations were evaluated.

### 2. Data Processing Modes

#### 2.1 Single-frequency processing : $L_1$ MODE

The classic GPS processing mode uses  $L_1$  as the observable. It is primarily used to process shorter baselines where there is little or no difference on ionospheric delays between the sites, In the  $L_1$  mode,

---

\*Member, Professor, Dep of Civil Engineering, DongA University, Busan, Korea (E-mail : uypark@daunnet.donga.ac.kr)

\*\*Member, Seniority research worker, Korean Association of Surveying & Mapping (E-mail : jolee@kasm.or.kr)

\*\*\*Member, Professor, Dep of Construction & Environment, Changsin College, Masan, Korea (E-mail : drlee@csc.ac.kr)

\*\*\*\*Dr. Course, Dep of Civil Engineering, DongA University, Busan, Korea (E-mail : jungsoo0515@hanmail.net)

sequentially :

1. The triple-difference solution brings the baseline vector estimate to within a meter of uncertainty. This solution helps the automatic cycle slip fixer identify cycle slips and outlying measurements.
2. The float double-difference solution attempts to correct cycle slips, fixes ambiguities (biases) to non-integer values, and brings the baseline vector estimate to within 20 centimeters.
3. The integer fixed double-difference (formerly ambiguity resolution) solution attempts to fix the ambiguities to integer values, and computes the accuracy confidence level is over 95%, the biases are usually fixed properly.
4. The fixed double-difference solution uses the fixed integer ambiguities and provides the final solution.

### 2.2 Dual-frequency processing

Dual-frequency observations can be used to measure the ionospheric delay. The difference is compared on signal delay between the two  $L$ -band frequencies,  $L_1$  and  $L_2$ . This delay can then be removed from the measurements by combining the frequencies and providing the  $L_c$  (linear combination or ionosphere-free) solution. Dual-frequency processing can remove much of the effects of solar flares and magnetic storms and improve the solutions for long baselines where the apparent ionosphere is different between receivers. In times of moderate ionospheric disturbances,  $L_1$ -only baselines should be kept to 15 kilometers or under, and the observation time should be increased. In many cases this is sufficient to get reasonable  $L_1$  results. During large disturbances, however, dual-frequency data will most often be required to obtain reasonable baseline resolution. Ionospheric effects appear to be more of a factor in the polar and equatorial regions. Areas on the middle latitudes (most of the United States) are generally least affected by the ionosphere.

### 2.3 Wideline processing : WIDE $L_N$ MODE

Processing a baseline using the wideline technique provides the facility to obtain an ionosphere-free, biases-fixed (ambiguities-fixed) solution. This option is most effective with  $P$ -code receiver with a codeless receiver, the data is processed on a codeless mode.

Processing two  $P$ -code receivers provides for full-wavelength  $L_1$  and half-wavelength  $L_2$  data on codeless receivers. Processing with full-wavelength  $L_1$  and  $L_2$  makes it easier to fix biases. The goal when processing wideline is to produce an  $L_1/L_2$  biases-

fixed, ionosphere-free ( $L_c$ ) combination. To do this, LINECOMP first attempts to resolve the wideline ( $L_1 - L_2$ ) biases. After fixing the wideline biases, LINECOMP attempts to fix the  $L_1$  biases through a combined  $L_1$  and  $L_2$  observable called  $L_c$  (linear combination). For a description of the rationale, consider the following where  $N_x$  is the bias of Eq. (a) given observable :

$$N_{1-2} = N_1 - N_2, \quad N_2 = N_1 - N_{1-2} \quad (a)$$

$$N_{L_c} = N_1 + AN_2 \quad (b)$$

Where  $A$  = frequency ratio for ionosphere removal.

Eq. (b) can be combined with equation (a) as

$$N_{L_c} = N_1 + A(N_1 - N_{1-2}) \quad (c)$$

According to these equations, if the wideline bias (i.e.,  $N_{1-2}$ ) is fixed, then the  $L_1$  bias can be fixed through the ionosphere-free combination.

The wideline bias for full-wavelength  $L_1$  and  $L_2$  data is approximately 86 cm. For full-wavelength  $L_1$  and half-wavelength as longer. Additionally, the codeless observable, by nature, is noisier than a code-aided full-wavelength observable. The noisier the observable, the more difficult it is to fix to the correct bias. Therefore, it is easier to fix  $P$ -code wideline biases because of the longer wavelength and the reduced noise.

### 2.4 Linear combination, ionosphere-free observable

The ionosphere-free observable combines the  $L_1$  and  $L_2$  observables as

$$L_c = L_1 - \alpha L_2 \quad (d)$$

where

$$\alpha = F_2/F_1 : \text{full wavelength observables}$$

$$\alpha = F_2/(2.0 * F_1) : \text{half wavelength } L_2 \text{ observables}$$

$$F_1 = \text{frequency of } L_1 \text{ (1.57542 MHz)}$$

$$F_2 = \text{frequency of } L_2 \text{ (1.22760 MHz)}$$

Thus,

$$\alpha = 0.7792 \text{ full wavelength observables.}$$

$$\alpha = 0.3895 \text{ half wavelength } L_2 \text{ observables.}$$

Because the observables are combined on this manner, an unfixed slip on  $L_2$  will appear on a residual plot as a fraction of a cycle. different combinations of unfixed  $L_1$  and  $L_2$  slips may make it more difficult

to discern between slips on  $L_1$  and  $L_2$ .

### 2.5 $L_{1C}$ processing MODE

Processing  $L_{1C}$  uses the  $L_C$  observable to provide an ionosphere-free solution, But does not attempt to fix biases. It is normally used on longer lines where widelane processing was not able to fix biases.

### 3. Observation and Baseline Analysis

This study preoccupied a triangulation point of national standard for the baseline distance to make a survey of land using the “third order control point” method regulated by National Geographic Information Institute (NGII) in favor of building an optimal network for relative positioning. In other words, it made a statistical analysis by group with the base points more than two point depending on the scope after measuring the baseline distance based on the first triangulation point of Bongrea Mt. in order to search the change between the scope of each baselines. And it made an observation of the hour in the optimal same hours, given the visibility and dilution of precision of a satellite that is simultaneously orbiting.

There are showing the notice results for each

Table 1. Baseline length of triangulation point (m).

Number	Baseline	Length
1	YOUD - JUNS	20,577.632
2	YOUD - BONH	8,639.150
3	YOUD - MILY	42,143.041
4	YOUD - DAEG	95,336.249
5	YOUD - JINJ	92,600.061
6	YOUD - DAEJ	222,729.853
7	JUNS - BONH	17,790.055
8	JUNS - MILY	23,166.884
9	JUNS - DAEG	76,003.976
10	JUNS - JINJ	78,161.771
11	JUNS - DAEJ	202,248.771
12	BONH - MILY	36,277.765
13	BONH - DAEG	89,049.134
14	BONH - JINJ	94,483.226
15	BONH - DAEJ	219,160.135
16	MILY - DAEG	53,211.809
17	MILY - JINJ	78,271.580
18	MILY - DAEJ	184,271.456
19	DAEG - JINJ	91,865.302
20	DAEG - DAEJ	140,108.330
21	JINJ - DAEJ	156,737.116

triangulation points in Table 1. It used indexes, which adopt an experimental value, as criteria in order to sort data as showed in Table 2, 3, 4 and 5. And it regarded data, which meet three conditions as following, as the measured values of high quality to conduct verification for the limit. Fig. 1 shows the data processing flow of

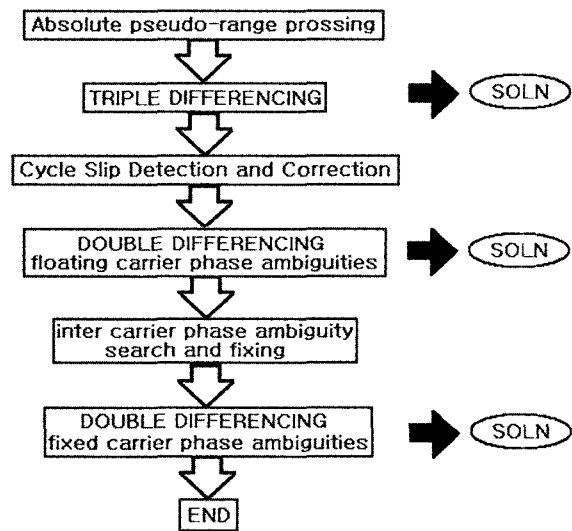


Fig. 1. Data processing flow chat.

Table 2. The analysis of vector (Auto)(m).

Vector	Solution	Legnth	Ratio	RMSE	SD
YOUD-JUNS	Fixed ( $L_1$ )	20,577.607	95	0.024	0.023
YOUD-BONH	Fixed ( $L_1$ )	8,629.137	100	0.013	0.019
YOUD-MILY	Fixed ( $L_1$ & $L_2$ )	42,143.000	96	0.038	0.024
YOUD-DAEG	Fixed ( $L_1$ & $L_2$ )	95,336.175	95	0.144	0.039
YOUD-JINJ	Fixed ( $L_1$ & $L_2$ )	92,599.988	95	0.133	0.037
YOUD-DAEJ	Fixed ( $W L_n$ )	222,729.699	97	0.127	0.068
JUNS-BONH	Fixed ( $L_1$ )	17,790.039	96	0.020	0.018
JUNS-MILY	Fixed ( $L_1$ & $L_2$ )	23,166.857	97	0.032	0.026
JUNS-DAEG	Fixed ( $W L_n$ )	76,003.908	96	0.101	0.024
JUNS-JINJ	Fixed ( $L_1$ & $L_2$ )	78,161.708	97	0.136	0.028
JUNS-DAEJ	Fixed ( $W L_n$ )	202,248.624	97	0.136	0.032
BONH-MILY	Fixed ( $L_1$ & $L_2$ )	36,277.735	95	0.041	0.023
BONH-DAEG	Fixed ( $L_1$ & $L_2$ )	89,049.066	98	0.076	0.029
BONH-JINJ	Fixed ( $L_1$ & $L_2$ )	94,483.142	99	0.102	0.024
BONH-DAEJ	Fixed ( $W L_n$ )	219,159.978	99	0.126	0.023
MILY-DAEG	Fixed ( $L_1$ & $L_2$ )	53,211.746	96	0.044	0.029
MILY-JINJ	Fixed ( $L_1$ & $L_2$ )	78,271.499	96	0.056	0.035
MILY-DAEJ	Fixed ( $W L_n$ )	184,271.321	96	0.125	0.052
DAEG-JINJ	Fixed ( $L_1$ & $L_2$ )	91,865.195	99	0.091	0.040
DAEG-DAEJ	Fixed ( $W L_n$ )	140,108.206	95	0.128	0.041
JINJ-DAEJ	Fixed ( $W L_n$ )	156,736.962	98	0.116	0.069

Table 3. The analysis of vector ( $L_1$  Only)(m).

Vector	Solution	Length	Ratio	RMSE	SD
YOUD-JUNS	Fixed ( $L_1$ )	20,577.589	95	0.024	0.024
YOUD-BONH	Fixed ( $L_1$ )	8,639.137	100	0.013	0.019
YOUD-MILY	Fixed ( $L_1$ )	42,142.956	95	0.031	0.030
YOUD-DAEG	Float ( $L_1$ )	95,336.091	81	0.147	0.046
YOUD-JINJ	Float ( $L_1$ )	92,599.899	89	0.137	0.036
YOUD-DAEJ	Float ( $L_1$ )	222,729.373	59	0.367	0.072
JUNS-BONH	Fixed ( $L_1$ )	17,790.025	95	0.020	0.018
JUNS-MILY	Fixed ( $L_1$ )	23,166.829	95	0.029	0.028
JUNS-DAEG	Float ( $L_1$ )	76,003.802	64	0.169	0.032
JUNS-JINJ	Float ( $L_1$ )	78,161.584	88	0.146	0.030
JUNS-DAEJ	Float ( $L_1$ )	202,248.325	77	0.218	0.042
BONH-MILY	Fixed ( $L_1$ )	36,277.704	96	0.042	0.033
BONH-DAEG	Float ( $L_1$ )	89,048.924	84	0.140	0.030
BONH-JINJ	Float ( $L_1$ )	94,482.827	83	0.114	0.025
BONH-DAEJ	Float ( $L_1$ )	219,159.728	58	0.246	0.023
MILY-DAEG	Fixed ( $L_1$ )	53,211.683	98	0.042	0.030
MILY-JINJ	Float ( $L_1$ )	78,271.402	64	0.156	0.057
MILY-DAEJ	Float ( $L_1$ )	184,271.065	75	0.225	0.052
DAEG-JINJ	Float ( $L_1$ )	91,865.083	89	0.129	0.043
DAEG-DAEJ	Float ( $L_1$ )	140,107.905	75	0.133	0.048
JINJ-DAEJ	Float ( $L_1$ )	156,736.719	67	0.126	0.074

study performance. For the independent observation, RMSE, an average value of residual squares, show the square root type of spread variance.

If a vector acquires a fixed solution, the result value of ratio, or percentage measured by the best and the next best method, show more than 95 as Table below. Also, it shows SD on the coordinate baseline vector and each factors calculated by the data processing. Here, it is unsuitable to use for practical purpose, as it process only random error about a variety of values obtained by software except many other errors that have an influence on values observed by GPS. Many values are obtained by data processing of GPS that combine ambiguity variously, calculating the obtained residual about each phase measurement value.

It treated the obtained GPS data with common software, being able to obtain a variety of data such as the result value of coordinate. The result of baseline analysis improves a positioning accuracy as a processing for pseudo-range from a static engine on program, removing cycle slip by a triple difference. And

Table 4. The analysis of vector ( $L_1$  &  $L_2$ )(m).

Vector	Solution	Legnth	Ratio	RMSE	SD
YOUD-JUNS	Fixed ( $L_1$ & $L_2$ )	20,577.589	95	0.025	0.022
YOUD-BONH	Fixed ( $L_1$ & $L_2$ )	8,639.118	100	0.013	0.019
YOUD-MILY	Fixed ( $L_1$ & $L_2$ )	42,142.956	96	0.038	0.024
YOUD-DAEG	Fixed ( $L_1$ & $L_2$ )	95,336.091	95	0.094	0.035
YOUD-JINJ	Float ( $L_1$ & $L_2$ )	92,599.899	67	0.133	0.037
YOUD-DAEJ	Float ( $L_1$ & $L_2$ )	222,729.373	74	0.325	0.070
JUNS-BONH	Fixed ( $L_1$ & $L_2$ )	17,790.025	95	0.020	0.018
JUNS-MILY	Fixed ( $L_1$ & $L_2$ )	23,166.829	96	0.033	0.025
JUNS-DAEG	Fixed ( $L_1$ & $L_2$ )	76,003.802	96	0.161	0.026
JUNS-JINJ	Float ( $L_1$ & $L_2$ )	78,161.584	84	0.146	0.030
JUNS-DAEJ	Float ( $L_1$ & $L_2$ )	202,248.325	86	0.198	0.033
BONH-MILY	Fixed ( $L_1$ & $L_2$ )	36,277.704	99	0.043	0.025
BONH-DAEG	Fixed ( $L_1$ & $L_2$ )	89,048.924	95	0.076	0.029
BONH-JINJ	Float ( $L_1$ & $L_2$ )	94,482.827	84	0.102	0.024
BONH-DAEJ	Float ( $L_1$ & $L_2$ )	219,159.728	78	0.226	0.023
MILY-DAEG	Fixed ( $L_1$ & $L_2$ )	53,211.683	99	0.044	0.029
MILY-JINJ	Fixed ( $L_1$ & $L_2$ )	78,271.402	97	0.056	0.035
MILY-DAEJ	Float ( $L_1$ & $L_2$ )	184,271.065	76	0.225	0.052
DAEG-JINJ	Fixed ( $L_1$ & $L_2$ )	91,865.083	78	0.091	0.040
DAEG- DAEJ	Float ( $L_1$ & $L_2$ )	140,107.905	76	0.128	0.041
JINJ-DAEJ	Float ( $L_1$ & $L_2$ )	156,736.719	68	0.116	0.069

then, it performs a single difference for the baseline, and the result, which isn't processed here, passes the float double difference solution-the following process. Here, it turns an ambiguity  $N$  into a fixed number to obtain a fixed double difference solution for the carrier phase and code phase of GPS.

For the most general auto processing, it calculate the fixed value to show a stable result value in processing the whole baseline, while for the process by the baseline distance, it apply a variety of the baseline processing methods to show it. For the process of only  $L_1$ , it obtained the similar result value in comparison with that of other baseline distance by showing the fixed value within 53.21 km. But it didn't obtain the fixed solution in the baseline distance more than that. For the  $L_1$  &  $L_2$ , it showed excellent value to the short and middle baseline within 100 km by obtaining the fixed value in the result of 95.336 km. For the  $L_1$  &  $L_2$ , it showed a little higher than value than other process techniques in the baseline distance of 92.6 km, but it mattered little to accuracy in the processing. And it

Table 5. The analysis of vector ( $L_1$  &  $L_{2c}$ )(m).

Vector	Solution	Legnth	Ratio	RMSE	SD
YOUD-JUNS	Fixed ( $L_1$ & $L_2$ )	20,577.534	95	0.027	0.024
YOUD-BONH	Fixed ( $L_1$ & $L_2$ )	8,639.083	100	0.018	0.022
YOUD-MILY	Fixed ( $L_1$ & $L_2$ )	42,142.868	96	0.041	0.027
YOUD-DAEG	Fixed ( $L_1$ & $L_{2c}$ )	95,335.938	95	0.146	0.041
YOUD-JINJ	Float ( $L_1$ & $L_{2c}$ )	92,599.671	67	0.133	0.037
YOUD-DAEJ	Fixed ( $L_1$ & $L_{2c}$ )	222,728.940	96	0.127	0.073
JUNS-BONH	Fixed ( $L_1$ & $L_{2c}$ )	17,789.836	95	0.023	0.018
JUNS-MILY	Fixed ( $L_1$ & $L_{2c}$ )	23,166.774	96	0.035	0.028
JUNS-DAEG	Fixed ( $L_1$ & $L_{2c}$ )	76,003.674	96	0.163	0.024
JUNS-JINJ	Float ( $L_1$ & $L_{2c}$ )	78,161.369	84	0.246	0.030
JUNS-DAEJ	Fixed ( $L_1$ & $L_{2c}$ )	202,248.152	98	0.198	0.033
BONH-MILY	Fixed ( $L_1$ & $L_{2c}$ )	36,277.631	99	0.046	0.023
BONH-DAEG	Fixed ( $L_1$ & $L_{2c}$ )	89,048.786	95	0.076	0.029
BONH-JINJ	Float ( $L_1$ & $L_{2c}$ )	94,482.539	84	0.102	0.024
BONH-DAEJ	Fixed ( $L_1$ & $L_{2c}$ )	219,159.367	95	0.124	0.021
MILY-DAEG	Fixed ( $L_1$ & $L_{2c}$ )	53,211.553	99	0.044	0.029
MILY-JINJ	Fixed ( $L_1$ & $L_{2c}$ )	78,271.149	78	0.056	0.035
MILY-DAEJ	Fixed ( $L_1$ & $L_{2c}$ )	184,270.776	76	0.223	0.050
DAEG-JINJ	Float ( $L_1$ & $L_{2c}$ )	91,864.761	78	0.091	0.040
DAEG-DAEJ	Fixed ( $L_1$ & $L_{2c}$ )	140,107.548	95	0.127	0.037
JINJ-DAEJ	Float ( $L_1$ & $L_{2c}$ )	156,736.287	68	0.113	0.067

turned the value into stability in the baseline distance more than 92.6 km. The following Fig. 2 shows the notice result and residual value for the result value of

each baseline distance.

### 4. Conclusions

In this study, data processing results for each method were estimated by observation of dual-frequency receiver in triangulation point, and its produces comparison of baseline distance. In addition, it considered the accuracy and an efficient of the data processing method which follows in the each linear combination and distance limitations were evaluated.

1. General auto processing, it calculate the fixed value to show a stable result value in processing the whole baseline, while for the process by the baseline distance, it apply a variety of the baseline processing methods to show it. In using  $L_1$  to process baseline data of survey, results within baseline distance of 53.211 km and fixed value showed excellent values with a RMSE of 0.019~0.056 m and SD of 0.013~0.037 m.

2. In using  $L_1$  &  $L_2$  to process baseline 95.336 km distance data, the processing results showed a RMSE of 0.013 m~0.325 m and a SD of 0.019~0.070 m. In this case  $L_1$  &  $L_{2c}$  shows higher residual within baseline distance of 92.600 km, but above distance that it shows the result which is excellent.

3. In addition, in baseline excluding DAEJ, fixed value was yielded. Results of a comparison analysis of the above two sets of data show that in obtaining fixed values in short and mid distance baselines less than 70-80 km, single frequency values were similar to dual

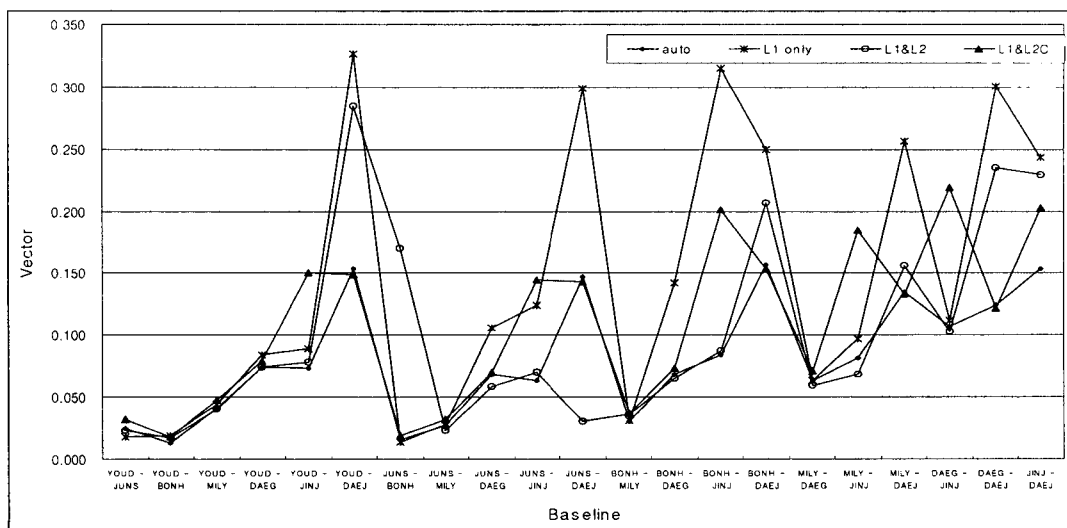


Fig. 2. Residual of Baselines.

frequency values, suggesting that a single frequency is sufficient to obtain accurate results. In addition, in a comparison with the acceptable accuracy of a reference point survey, satisfactory results were obtained in both fixed and float values.

According to the results of this study, a more economic single frequency receiver is sufficient to obtain satisfactory results in short and middle distance survey, and is also beneficial from an economic standpoint. However, in the case of long baselines, the usage of dual frequency will provide better results than single frequency because of uneliminatable errors resulting from ionosphere delay, limitations of receiver and field of vision and mechanical errors, and so it is necessary to think about existing plans and the exact processing method before beginning the survey. Also, further research of the land and an studies regarding the establishment of an international network need to be done.

## Reference

1. Kay, S. M., "Statistical Signal Processing -Estimation Theory", Prentice Hall, Engelwood Cliffs, NJ, 1993.
2. Feit, L., and Domanico, P., "Navigation Signal Random Noise Generator Design and Performance Verification", ITT, EASCON Proceedings, IEEE 81 CH1724-4, 1981.
3. P. Doherty and D. Decker, Boston College : J. Klobuchar, Innovative Solutions International "Observed Ionospheric Dependence on Solar Activity : Implications for a New Single Frequency GPS User Algorithm" The Institute of Navigation, Part 2 of 1, pp. 565-574, 1996.
4. M. Dumaine, Canadian Marconi Company, "High Precision Attitude Using Low Cost GPS Receiver", The Institute of Navigation, Part 2 of 1, pp. 565-574, 1997.
5. Yang Gao, James McLellan, and Pulsearch Navigation Systems, Ins, "Correlation of Dual-Frequency and Accuracy Analysis of Single-Frequency for Positioning Accuracy", The Institute of Navigation, Part 2 of 1, pp. 215-224, 1998.
6. Czopek, F. and Shollenberger, Lt. S. "Description and Performance fo the GPS Block I and II L-Band Antenna and Like Budget", 6th International Technical Meeting, The Institute of Navigation, Sept. 1993.