International Journal of Management Science
Vol 10, No 2, November 2004

The Cost Impact of Incorrect Assumptions in a Supply Chain

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(Received Feb. 2004 ; Revised Jul. 2004 ; Accepted Jul. 2004)

ABSTRACT

In this paper, the cost impact of incorrect assumptions about the demand process in a supply chain in which there are two participants, a retailer and a manufacturer, is considered. When participants in the supply chain do not notice serial correlation in the demand process, they would turn to a simple inventory model based on an *i.i.d.* demand assumption. A mathematical model that allows us to quantify the cost incurred by each participant in the supply chain, when they implement inventory policies based on correct or incorrect assumptions about the demand process, is developed. This model enables us to identify how much it differs from the optimal costs.

Keywords: Cost impact, Supply chain, Incorrect assumptions about the demand process

1. INTRODUCTION

Serially correlated demands are a characteristic of most of today's consumer product industry (Lee, *et al.* [7] and Erkip, *et al.* [3]). Since most commonly known inventory models are based on the assumption of *i.i.d.* demands, practitioners in industry tend to use these simple inventory models, even when demands are highly autocorrelated. In some cases, practitioners may not realize that demands exhibit serial correlation. In other cases, even though they know that there is correlation between successive demands and that this correlation is significant, practitioners may still choose to implement a simple inventory policy based on *i.i.d.* demands. This choice may be due to a lack of knowledge regarding the form of optimal policy, a desire to use a simple inventory policy or a desire to use a stationary policy.

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Today retailers and other members of the supply chain now have access to significant amounts of demand information thanks to recent advances in information technology which have reduced the costs of information sharing and the time required to communicate and process this information. These advances include point-of-sale (POS) scanners, bar-coding technology, electronic data interchange (EDI), and online inventory and production control system (Buzzell and Ortmeyer [1]). The use of EDI, in particular, has become widespread. EDI not only allows information to be shared between the various stages of the supply chain, but also allows more frequent ordering and replenishment. One estimate states that the use of EDI can reduce the cost of processing a purchase from $150 to $25 (Verity [10]).

Therefore, the pitfalls resulting from this non-optimal practice, namely, incorrect assumptions on the demand process, under information sharing, need to be quantified, which is the goal of the paper.

The remainder of this paper is organized as follows: Section 2 reviews the previous research related to this paper. Section 3 develops a simple supply chain model and Section 4 shows major findings about the cost impact of incorrect assumptions on the demand process. Final remarks are addressed in Section 5.

2. LITERATURE REVIEW

2.1 Inventory Control Under Correlated Demand

In this subsection, some of the relevant research on inventory control for supply chains with correlated demands is briefly reviewed.

Urban [9] notes that traditional reorder point models assume that the average demand rate changes very little with time. The author analyzes the effect of serially correlated demand on the determination of appropriate reorder levels. In this case, while the expected demand during lead time is equal to the long run expected demand over the expected lead time, the expected demand in the short run is not equal to the long run average. The author examines the determination of accurate reorder levels for first order autoregressive demand processes, which are updated every period, conditional on the most recently observed demand. From a numerical analysis, the author indicates that traditional approaches for determining reorder levels can result in excessive inventories and shortages for high levels of autocorrelation.

In this paper, traditional inventory models are compared with accurate inventory models as well. However, this paper differs from Urban [9] in that a periodic review model rather than a continuous review model is considered and in that the impact of traditional inventory models is compared with accurate inventory models for the entire supply chain rather than for a single stage system.

Zinn, *et al.* [11] conduct extensive experimentation on the effect of autocorrelation on safety stock and find that (i) observed stockouts will be significantly more frequent and larger than expected when demand is serially correlated, (ii) the effect of autocorrelation on the number of stockouts observed is directly related to the variability of customer demand and (iii) the effect of autocorrelated demand on the number of stockouts observed is inversely related to the variability of lead time from suppliers.

Lee, *et al.* [7] consider a two stage supply chain consisting of a single retailer and a single manufacturer. The demand process seen by the retailer is serially correlated and both the retailer and the manufacturer know the exact form of the demand process (*e.g.*, both of them know the mean demand, the variance of the error terms and the correlation parameter). The retailer and the manufacturer follow order-up-to inventory policies based on the most recently observed demand data. The authors find that sharing customer demand information can significantly reduce the costs at the manufacturer, particularly when the serial correlation is quite high.

This paper utilizes similar settings as those addressed in Lee, *et al.* [7]. However, this paper differs from all of this previous research, including Lee, *et al.* [7], in that the assumption that the supply chain members know the exact distribution of demand does not hold.

2.2 The Bullwhip Effect

An important phenomenon in supply chain management, the bullwhip effect, suggests that demand variability increases as one moves up a supply chain. For example, empirical evidence indicates that the orders placed by a retailer tend to be much more variable than the customer demand seen by that retailer. This increase in variability tends to propagate up the supply chain, distorting the pattern of orders received by distributors, manufacturers and suppliers. The bullwhip effect is a major concern of many manufacturers, distributors and even retailers because the increased variability makes the supply chain much more difficult to manage and can lead to increased costs due to overstocking throughout the system and the excessive capacity required due to this overstocking.

Chen, *et al.* [2] quantify the bullwhip effect, *i.e.*, the increase in variability that occurs at each stage of the supply chain, due to demand forecasting and order lead times. They determine tight lower bounds on the variance of the orders placed by the retailer relative to the variance of the customer demand observed by that retailer.

A more detailed discussion on the bullwhip effect has been covered in Lee, *et al.* [5, 6]. These papers discuss the four major causes of the bullwhip effect: demand forecast updating, order batching, price fluctuations and rationing/shortage gaming. They also discuss the cost and managerial impacts of this increase in variability. They suggest several methods for reducing the impact of the bullwhip effect, including information sharing, channel alignment, and operational efficiency.

The research mentioned in this subsection demonstrates that the information structure can have a significant impact on the variability in a supply chain. Therefore, since increased variability usually leads to higher costs in the supply chain, it seems clear that the information structure will have a significant impact on the costs. In this paper, this bullwhip literature is extended to investigate the impact of the information structure on the costs in a supply chain.

3. A SUPPLY CHAIN MODEL

3.1 Preliminaries

In this paper, a simple supply chain consisting of a single retailer and a single manufacturer is considered. External demand occurs at the retailer for a single item. A periodic review system is considered, in which each participant reviews his or her respective inventory level and orders from the upstream participant once per review period.

A summary of the key notation used in this paper is provided at the end of this subsection. In this notation lower case letters are reserved for the retailer whereas upper case letters are reserved for the manufacturer. Let c denote the review period for the retailer and l denote the lead time for the retailer, where l is a nonnegative integer multiple of c (*i.e.*, $l \in \{0, 1\,c, 2\,c, \cdots\}$). Similarly, let C denote the review period for the manufacturer, where C is a positive integer multiple of the retailer's review period (*i.e.*, $C = mc$ where $m \in \{1, 2, 3, \cdots\}$). Let L denote the lead time for the manufacturer, where L is a positive integer multiple of C (i.e., $L = MC = Mmc$ where $M \in \{1, 2, 3, \cdots\}$). Notice that we require $L \geq C \geq c$. Let $l' = c + l$ ($L' = C + L$) be the effective lead time for the retailer (manufacturer). Therefore, $L' = C + L = mc + Mmc = c(m + Mm)$. Finally, let h, p (H, P) denote the unit holding cost per item per period and shortage cost per item for the retailer (manufacturer). We assume that no fixed cost is incurred when placing an order.

The underlying demand process faced by the retailer is an AR(1) process. The approach presented here can be extended to an AR(n), $n \in \{1, 2, 3, \cdots\}$, process. However, the analysis involved becomes much more complex. Since the purpose of this paper is to gain managerial insights, only the AR(1) process is considered. Let d_t be the demand faced by the retailer during period t. Then, d_t can be written as:

$$d_t = \mu + \rho d_{t-1} + \varepsilon_t, \tag{1}$$

where μ is a non-negative constant, ρ is a correlation parameter with $-1 < \rho < 1$, and the error terms, ε_t, are *i.i.d.* normal random variables with mean 0 and variance σ^2. It is also assumed that μ is large enough relative to σ so that the probability of negative demand is negligible. It can be easily shown that, as $t \to \infty$, $E[D_t] = \frac{\mu}{1-\rho}$ and $V[D_t] = \frac{\sigma^2}{1-\rho^2}$. Finally, if $\rho = 0$, Equation (1) implies that demands are *i.i.d.* with mean μ and variance σ^2.

Next, the sequence of events within each review cycle is described. First, consider the events occurring at the retailer. At the start of every review cycle, at period t, where $t \in \{1, 1+c, 1+2c, \cdots\}$, the retailer observes the inventory level and the previous demands and calculates the order-up-to level, $y_{j,t}$, $j = s, n$, from which the retailer determines the order quantity, $q_{j,t}$, $j = s, n$, to place to the manufacturer. Here the subscript "s" refers to the "smart" retailer and the subscript "n" refers to the "naïve" retailer. These terms will be defined more clearly in Subsection 3.2. The subscript t refers to the time period. The retailer will receive the shipment of this order, placed at the start of period t, at the beginning of period $t + l$. Excess demand is backlogged.

Next, at the start of period t, where $t \in \{1, 1+c, 1+2c, \cdots\}$, the manufacturer receives and ships the required order quantity $q_{j,t}$, $j = s, n$, to the retailer. In order to simplify the analysis, it is assumed that the manufacturer can always ship the entire order to the retailer. This assumption requires that, if the manufacturer does not have enough stock on hand to fill the order quantity, the manufacturer can always find an alternative source to borrow from, with some additional cost, P per unit, and that the borrowed items are returned to the source when the next replenishment arrives, as if they were backlogged items. This assumption is identical to that made by Lee, *et al.* [7] and is required to obtain closed form expressions for the expected costs at each stage.

Next, suppose the manufacturer places an order at the start of period t, where $t \in \{1, 1+C, 1+2C, 1+3C, \cdots\}$, right before the retailer orders, based on the inventory level and the demand information shared by the retailer. This order will arrive at the start of period, $t + L$. Finally, the supplier from whom the manufacturer orders is assumed to have infinite capacity, so that the manufacturer's order is always satisfied after the fixed lead time, L.

In this model, it is assumed that the retailer and the manufacturer determine the order quantities in such a way that, they believe, minimizes their own total expected holding and penalty costs over an infinite planning horizon, namely, long run average total inventory costs per period. Thus, the retailer and the manufacturer will use order-up-to inventory policies based on inventory position (on hand plus on order). Since the review cycle is fixed and the per unit per period holding cost and per unit penalty cost are fixed, in order to model the behavior of the retailer and the manufacturer, it is assumed that each participant takes a myopic approach to the infinite planning horizon, based on the perceived demand distribution and the obtained demand information.

Showing the notations to be used in this paper concludes this subsection. First, the notations for the retailer are summarized in Table 1.

Table 1. Notation for Retailer

Notation	Description
c	review period
l	lead time
l'	effective lead time, $l' = l+c$
h	holding cost per unit per unit time
p	penalty cost per unit associated with backlogged demand
z	safety factor
$d_t \mid d_{t-1}$	actual demand in period t, given the most recently observed demand, d_{t-1}
$d_t^{l'} \mid d_{t-1}$	actual total demand over the effective lead time, l', starting in period t, given the most recently observed demand, d_{t-1}
$d_{s,t}^{l'}\ (d_{n,t}^{l'})$	perceived total demand over the effective lead time, l', starting in period t, as perceived by smart (naïve) retailer
$y_{s,t}\ (y_{n,t})$	order-up-to level for period t for smart (naïve) retailer
$z_{s,t}\ (z_{n,t})$	standardized value of order-up-to level for period t for smart (naïve) retailer
$\bar{z}_s\ (\bar{z}_n)$	expected value of $z_{s,t}\ (z_{n,t})$
$\sigma_{z_s}^2\ (\sigma_{z_n}^2)$	variance of $z_{s,t}\ (z_{n,t})$
$q_{s,t}\ (q_{n,t})$	order quantity placed by smart (naïve) retailer at the start of period t
$inv_s\ (inv_n)$	average inventory level per period for smart (naïve) retailer
$g_s\ (g_n)$	long run average total inventory costs per period for smart (naïve) retailer

Note that in Table 1, $d_{j,t}^{l'}$, $j = s$, n, represents the perceived effective lead time demand at the retailer while $d_t^{l'} \mid d_{t-1}$ represents the actual effective lead time demand.

The parameters for the manufacturer are defined in the same way as for the retailer, but with the lower case letters replaced with capital letters. For convenience, only the notation for the case of the manufacturer with smart retailer is shown in Table 2.

Table 2. Notation for Manufacturer

Notation	Description
C	review period
L	lead time
L'	effective lead time, $L' = L + C$
H	holding cost per unit per unit time
P	penalty cost per unit associated with backlogged demand
Z	safety factor
$D_{t\mid s}^{L'}$	actual total demand over the effective lead time, L', starting in period t, faced by the manufacturer
$D_{S,t\mid s}^{L'} \left(D_{N,t\mid s}^{L'}\right)$	perceived total demand over the effective lead time over the effective lead time, L', starting in period t, faced by smart (naïve) manufacturer
$Y_{S,t\mid s}^{L'} \left(Y_{N,t\mid s}^{L'}\right)$	order-up-to level for period t for smart (naïve) manufacturer
$Z_{S,t\mid s}^{L'} \left(Z_{N,t\mid s}^{L'}\right)$	standardized value of order-up-to level for period t for smart (naïve) manufacturer
$\overline{Z}_{S\mid s} (\overline{Z}_{N\mid s})$	expected value of $Z_{S,t\mid s}^{L'} \left(Z_{N,t\mid s}^{L'}\right)$
$\sigma_{Z_{s\mid s}}^2 (\sigma_{Z_{N\mid s}}^2)$	variance of $Z_{S,t\mid s}^{L'} \left(Z_{N,t\mid s}^{L'}\right)$
$INV_{S\mid s} (INV_{N\mid s})$	average inventory level per period for smart (naïve) manufacturer
$G_{S\mid s} (G_{N\mid s})$	long run average total inventory costs per period for smart (naïve) manufacturer

The other cases are defined in a similar manner. For instance, s will be replaced by n if the demand stream faced by the manufacturer with naïve retailer, *e.g.*, $D_{t\mid s}^{L'}$ will become $D_{t\mid n}^{L'}$.

3.2 COST ANALYSIS AT THE RETAILER

Given the model outlined in Subsection 3.1, an approach for determining the long run average total inventory costs per period at the retailer can be described. For the retailer, two cases are considered. First, the retailer may be aware that the demand process follows an AR(1) process and may take advantage of this knowledge to determine the order-up-to level, $y_{s,t}$. This case is referred to as the smart retailer and the subscript $j = s$ is used to denote the smart retailer. Alternatively, the retailer may not be aware that the demand follows an AR(1) process and thus may resort to an inventory model based on the assumption of *i.i.d.* demands to determine the order-up-to level, $y_{n,t}$. This case is referred to as the naïve retailer and the subscript $j = n$ is used to denote the naïve retailer.

First, the actual effective lead time demand seen by the retailer is described. Note the following relationship between d_{t+k} and d_{t-1} from Equation (1):

$$d_{t+1} \mid d_{t-1} = \left(\frac{1-\rho^{k+1}}{1-\rho}\right)\mu + \rho^{k+1} d_{t-1} + \sum_{u=0}^{k} \rho^{k-u} \varepsilon_{t+u},\ k \in \{0, 1, 2, \cdots\}. \tag{2}$$

Equation (2) can be used to show that

$$d_t^{l'} = \sum_{u=0}^{l'-1} d_{t+u} \mid d_{t-1} = \theta(d_{t-1}) + \theta'(\varepsilon_t, \varepsilon_{t+1}, \cdots, \varepsilon_{t+l'-1}), \tag{3}$$

where $\theta(d_{t-1})$ is a linear function of d_{t-1}, the most recent demand observation, and $\theta'(\varepsilon_t, \varepsilon_{t+1}, \cdots, \varepsilon_{t+l'-1})$ is a linear function of future unobserved error terms, $\varepsilon_t, \varepsilon_{t+1}, \cdots, \varepsilon_{t+l'-1}$. From Equation (3), it can be seen that the actual effective lead time demand, $d_t^{l'} \mid d_{t-1}$, follows a normal distribution with mean $\theta(d_{t-1})$ and variance $V[\theta'(\varepsilon_t, \varepsilon_{t+1}, \cdots, \varepsilon_{t+l'-1})]$.

Next, the order-up-to level for period t, denoted by $y_{j,t}$, $j = s, n$, is calculated as $y_{j,t} = E[d_{j,t}^{l'}] + z\sqrt{V[d_{j,t}^{l'}]}$, $j = s, n$, where $d_{j,t}^{l'}, j = s, n$, is the random variable representing the effective lead time demand starting in time period, t, as perceived by the retailer, and z, the safety factor, is a constant chosen to meet a desired service level. Notice that the perceived effective lead time demand is used by the retailer to determine the order-up-to level. The costs incurred by the retailer will be a function of this order-up-to level and the actual effective lead time demand.

Next, long run average total inventory cost per period incurred by the retailer is considered. To calculate the expected holding cost per period, an approximation for the average inventory level, given by Silver and Peterson [8], is used. Let the expected demand during the retailer's review period of length c, starting at $t + l$, given d_{t-1}, be $E[d_{t+l}^{c} \mid d_{t-1}]$ and let the expected demand over the effective lead time, starting at t, given d_{t-1}, be $E[d_t^{l'} \mid d_{t-1}]$. Then, for a periodic review inventory system with order-up-to level $y_{j,t}$, $j = s, n$, the average inventory level over the c periods between $t + l$ and $t + l'$, denoted by $inv^c_{j,t+l}$, $j = s, n$, is approximated as follows:

$$inv^c_{j,t+l} = y_{j,t} - E[d^{l'}_t | d_{t-1}] + \frac{E[d^c_{t+l} | d_{t-1}]}{2}, j = s, n.$$

Therefore, the expected average inventory level per period can be represented as follows:

$$inv_j = E[y_{j,t}] - E[d^{l'}_t] + \frac{E[d^c_{t+l}]}{2}, j = s, n, \tag{4}$$

where $E[d^{l'}_t] = E[E[d^{l'}_t | d_{t-1}]]$ and $E[d^c_{t+l}] = E[E[d^c_{t+l} | d_{t-1}]]$.

To calculate the expected shortage cost per period, an expression for the expected number of stockouts between periods $t + l$ and $t + l'$ for a fixed value of $y_{j,t}$, $j = s, n$, can be written as

$$\int_{y_{j,t}}^{\infty} (d^{l'}_t | d_{t-1} - y_{j,t}) dF(d^{l'}_t | d_{t-1}), j = s, n,$$

where $F(d^{l'}_t | d_{t-1})$ is the cumulative distribution function (cdf) of demand for the l' periods starting at period t given the demand in the previous period, d_{t-1}. Since $d_{t+1} | d_{t-1}$ follows a normal distribution, the above formula can be simplified to:

$$\sqrt{V[d^{l'}_t | d_{t-1}]} \int_{z_{j,t}}^{\infty} (x - z_{j,t}) \phi(x) dx, j = s, n,$$

where x is a standard normal random variable and $z_{j,t} = \frac{y_{j,t} - E[d^{l'}_t | d_{t-1}]}{\sqrt{V[d^{l'}_t | d_{t-1}]}}$, $j = s, n$, is the standardized value of the order-up-to level. Here $\Phi(\bullet)$ is the probability distribution function (pdf) for the standard normal distribution. The mean and variance of the standardized value of the order-up-to level, $\bar{z}_j$ and $\sigma^2_{z_j}$, $j = s, n$, respectively, are defined as follows:

$$\bar{z}_j = E[z_{j,t}], \ \sigma^2_{z_j} = V[z_{j,t}], j = s, n, \tag{5}$$

where the expectation and the variance are taken over d_{t-1}.

According to Zipkin [12], using a standard transformation, the above expression for the expected number of stockouts per review period can be further simplified to:

$$\sqrt{V[d_t^{l'} | d_{t-1}]}\left[\phi(z_{j,t}) - z_{j,t}(1-\Phi(z_{j,t}))\right],\ j = s, n,$$

where $\Phi(\bullet)$ is cdf of standard normal distribution. When using this formula to calculate the expected stockouts, a difficulty occurs when $z_{j,t}$, $j = s$, n, is a random variable due to the dependence of $y_{j,t}$, $j = s$, n, on the previous demand observations. For this model, since $z_{j,t}$, $j = s, n$, follows a normal distribution, we have:

$$E\left[\phi(z_{j,t}) - z_{j,t}(1-\Phi(z_{j,t}))\right] = h(\bar{z}_j)(1+\sigma_{z_j}^2) - \bar{z}_j(1-H(\bar{z}_j)),\ j = s, n,$$

where the expectation is taken over $z_{j,t}$, and $h(\bullet)$ and $H(\bullet)$ are the pdf and cdf for a normal distribution with mean 0 and variance $1+\sigma_{z_j}^2$, $j = s, n$. For derivation, see Kim and Ryan [4].

Since $\sqrt{V[d_t^{l'} | d_{t-1}]}$ is a constant, regardless of the specific observed value of d_{t-1}, the long run average total inventory costs per period, given order-up-to levels, $y_{j,t}$, $j = s, n$, where $t \in \{1, 1+c, 1+2c, \cdots\}$, denoted by g_i, $j = s, n$, can be written as follows:

$$g_j = h\, inv_j + \frac{p}{c}\sqrt{V[d_t^{l'} | d_{t-1}]}\left[h(\bar{z}_j)(1+\sigma_{z_j}^2) - \bar{z}_j(1-H(\bar{z}_j))\right],\ j = s, n. \tag{6}$$

Notice that, in Equation (6), the expected number of shortages during any review period is divided by the length of review period, c, to obtain the expected number of shortages per period. Notice also that, in Equation (6), the terms that differ for $j = s$ and $j = n$ are the average inventory level (inv_j, $j = s$, n) and the mean and variance of the standardized order-up-to level ($\bar{z}_j$ and $\sigma_{z_j}^2$, $j = s$, n). Therefore, in order to evaluate this cost for each case ($j = s$ and $j = n$), it is necessary only to evaluate these three quantities.

Finally, the retailer's order quantity for period t, which becomes the manufacturer's demand for that period, can be written as:

$$q_{j,t} = y_{j,t} - y_{j,t-c} + \sum_{u=0}^{c-1} d_{t-c+u},\ j = s, n. \tag{7}$$

In Equation (7), there exists a final difficulty with this model. The possibility exists that the retailer may not be able to raise the inventory level to the desired point in each review period. In other words, it is possible that $y_{j,t-c} - \sum_{u=0}^{c-1} d_{t-c+u} >$

$y_{j,t}, j = s, n$. To handle this case, like other researchers (e.g., Lee, *et al.* [7] and Chen, *et al.* [2]), it is assumed that negative order quantities are allowed. If the mean of the effective lead time demand is significantly larger than the standard deviation of the effective lead time demand, then the probability that $q_{j,t} \leq 0$, $j = s, n$, is close to 0 for the model presented here, and thus this assumption will have little impact on our results.

In the following, we use this model to calculate the long run average total inventory costs per period for the smart retailer and the naïve retailer.

Smart Retailer In this case, the retailer knows that the customer demands follow an AR(1) process. Therefore, the perceived effective lead time demand at the start of time period *t*, follows the actual effective lead time demand distribution. In other words, $d_{s,t}^{l'} = d_t^{l'} \mid d_{t-1}$. Therefore, the order-up-to level for smart retailer, at the start of time period *t*, *t* = 1, 1+*c*, 1+2*c*, $\cdots$, can then be written as:

$$\begin{aligned} y_{s,t} &= E[d_{s,t}^{l'}] + z\sqrt{V[d_{s,t}^{l'}]} \\ &= E[d_t^{l'} \mid d_{t-1}] + z\sqrt{V[d_t^{l'} \mid d_{t-1}]}. \end{aligned} \tag{8}$$

Note that it is assumed that the retailer has sufficient past demand data to calculate good estimates of μ, ρ and σ, possibly using some standard forecasting technique, and then uses these estimates, along with the most recent demand, to calculate $E[d_{s,t}^{l'}] = E[d_t^{l'} \mid d_{t-1}]$ and $\sqrt{V[d_{s,t}^{l'}]} = \sqrt{V[d_t^{l'} \mid d_{t-1}]}$.

In this case, since the retailer adjusts the order-up-to level according to the most recently observed demand, d_{t-1}, the long run average total inventory costs per period does not depend on specific time period. Also, note that $z_{s,t} = z$. Therefore, $\bar{z}_s = z$ and $\sigma_{z_s}^2 = 0$. In addition, from Equation (4),

$$\begin{aligned} inv_s &= E\left[E[d_{s,t}^{l'}] + z\sqrt{V[d_{s,t}^{l'}]}\right] - E[d_t^{l'}] + \frac{E[d_{t+l}^{c}]}{2} \\ &= z\sqrt{V[d_t^{l'} \mid d_{t-1}]} + \frac{c}{2}\left(\frac{\mu}{1-\rho}\right). \end{aligned}$$

Given the expressions for $\bar{z}_s$, $\sigma_{z_s}^2$ and inv_s, the long run average total inventory costs per period, using Equation (6), can be calculated.

In addition, the order quantity placed by the retailer at the start of period *t*, from Equation (7), can be written as:

$$q_{s,t} = \rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)(d_{t-1} - d_{t-c-1}) + \sum_{u=0}^{c-1} d_{t-c+u}. \tag{9}$$

Finally, as an example of this model, consider the case in which $c = 1$, $l = 0$ and $l' = 1$. Then $y_{s,t} = \mu + \rho d_{t-1} + z\sigma$ and $q_{s,t} = y_{s,t} - y_{s,t-1} + d_{t-1} = (1+\rho) d_{t-1} - \rho d_{t-2}$.

Naïve Retailer A naïve retailer believes that the effective lead time demand is *i.i.d.* from a normal distribution with a mean and a variance that can be estimated from some set of previous demand observations. Therefore, the perceived effective lead time demand, $d^{l'}_{n,t}$, at the start of time period t, follows a normal distribution with mean

$$E[d^{l'}_{n,t}] = E\left[\sum_{u=0}^{l'-1} d_{t+u}\right] = l'\left(\frac{\mu}{1-\rho}\right)$$

and variance

$$V[d^{l'}_{n,t}] = V\left[\sum_{u=0}^{l'-1} d_{t+u}\right] = V[E[d^{l'}_t \mid d_{t-1}]] + V[d^{l'}_t \mid d_{t-1}].$$

Notice that $V[d^{l'}_{n,t}] = V\left[\sum_{u=0}^{l'-1} d_{t+u}\right] = V[E[d^{l'}_{s,t}]] + V[d^{l'}_{s,t}] \geq V[E[d^{l'}_{s,t}]]$. The order-up-to level for naive retailer, at the start of time period t, $t = 1, 1+c, 1+2c, \cdots$, can then be written as:

$$\begin{aligned} y_{n,t} &= E[d^{l'}_{n,t}] + z\sqrt{V[d^{l'}_{n,t}]} \\ &= l'\left(\frac{\mu}{1-\rho}\right) + z\sqrt{V[E[d^{l'}_t \mid d_{t-1}]] + V[d^{l'}_t \mid d_{t-1}]}. \end{aligned} \tag{10}$$

Note that it is not assumed that the retailer in this case knows μ, ρ and σ, and use these parameters to calculate $E[d^{l'}_{n,t}]$ and $V[d^{l'}_{n,t}]$. Instead, given some set of previous demand data, the retailer will calculate the mean and variance for the effective lead time demand, assuming the demands are *i.i.d.*, and using some standard forecasting technique, *e.g.*, the sample mean and variance. Given that the retailer has sufficient data, these estimates of the mean and variance of the effective lead time demand will equal $E[d^{l'}_{n,t}]$ and $V[d^{l'}_{n,t}]$, respectively.

In this case, the retailer's order-up-to level is constant even though the distribution of the actual effective lead time demand changes according to the most recent demand data, d_{n-1}. Therefore, from Equation (5), we have:

$$\bar{z}_n = z\frac{\sqrt{V[d_{n,t}^{l'}]}}{\sqrt{V[d_t^{l'} \mid d_{t-1}]}},$$

$$\sigma_{z_n}^2 = \frac{V[E[d_{n,t}^{l'} \mid d_{t-1}]]}{V[d_{n,t}^{l'} \mid d_{t-1}]}.$$

Also from Equation (4), we have:

$$\begin{aligned} inv_n &= E\left[E[d_{n,t}^{l'}] + z\sqrt{V[d_{n,t}^{l'}]}\right] - E[d_t^{l'}] + \frac{E[d_{t+l}^c]}{2} \\ &= z\sqrt{V[E[d_t^{l'} \mid d_{t-1}]] + V[d_t^{l'} \mid d_{t-1}]} + \frac{c}{2}\left(\frac{\mu}{1-\rho}\right). \end{aligned}$$

Given the expressions for $\bar{z}_n$, $\sigma_{z_n}^2$ and inv_n, the long run average total inventory costs per period, using Equation (6), can be calculated.

Finally, the order quantity placed by the retailer at the start of period t, from Equation (7), can be written as:

$$q_{n,t} = \sum_{u=0}^{c-1} d_{t-c+u}. \tag{11}$$

Notice that, in this case, since the order-up-to level is a constant, the order quantity placed by the retailer is just the sum of the demands over the previous review period.

Finally, as an example of this model, consider the case in which $c = 1$, $l = 0$ and $l' = 1$. Then $y_{n,t} = \frac{\mu}{1-\rho} + z\frac{\sigma}{\sqrt{1-\rho^2}}$ and $q_{n,t} = d_{t-1}$.

3.3 COST ANALYSIS AT THE MANUFACTURER

Given the material in Subsection 3.2, an approach for determining the long run average total inventory costs per period at the manufacturer can now be described. First, notice that whether the retailer is smart or naïve ($j = s, n$) will affect the demand stream faced by the manufacturer. In addition, the effective lead time demand distribution perceived by the manufacturer will be different depending on whether the manufacturer itself is smart or naïve ($J = S, N$). It is also assumed that smart manufacturer knows whether the retailer is smart or naïve, *i.e.*, knows the form of the q_j, $j = s, n$. Therefore, for the manufacturer, four cases ($j = s, n$; $J = S, N$) will be considered.

Next, notice that the actual effective lead time demand faced by the manufacturer at the start of period t, where $t = 1{+}C,\ 1{+}2C,\ 1{+}3C,\ \cdots$, becomes:

$$D^{L'}_{t|j} = \sum_{u=0}^{m+mM-1} q_{j,t+cu} \mid d_{t-1}, d_{t-2}, \cdots, \ j = s, n, \tag{12}$$

where $q_j, j = s, n,$ are as given in Equations (9) and (11).

Next, it is easy to show that $D^{L'}_{t|j}, j = s, n,$ follows a normal distribution, and can be represented as a function of the known previous demand data and the future unknown error terms, as follows:

$$D^{L'}_{t|j} = \Theta_j(d_{t-1}, d_{t-2}, \cdots) + \Theta'_j(\varepsilon_t, \varepsilon_{t-1}, \cdots, \varepsilon_{t+L'-c-1}),\ j = s, n.$$

Therefore, the actual effective lead time demand has mean

$$E[D^{L'}_{t|j}] = \Theta_j(d_{t-1}, d_{t-2}, \cdots),\ j = s, n,$$

and variance

$$V[D^{L'}_{t|j}] = V[\Theta'_j(\varepsilon_t, \varepsilon_{t-1}, \cdots, \varepsilon_{t+L'-c-1})],\ j = s, n.$$

Next, the order-up-to level at the start of period t, $Y^{L'}_{J,t|j}, J = S, N, j = s, n,$ will be calculated as

$$Y^{L'}_{J,t|j} = E[D^{L'}_{J,t|j}] + Z\sqrt{V[D^{L'}_{J,t|j}]},\ J = S, N, j = s, n, \tag{13}$$

where $D^{L'}_{J,t|j}, J = S, N, j = s, n,$ is the random variable representing the effective lead time demand starting at time period, t, as perceived by the manufacturer, and Z is a constant chosen to meet a desired service level.

As we did for the retailer, we can write the long run average total inventory costs per period, given order-up-to levels, $Y^{L'}_{J,t|j}$, where $t \in \{1,\ 1{+}C,\ 1{+}2C,\ \cdots\}$, denoted by $G_{J|j}, J = S, N, j = s, n,$ as follows:

$$G_{J|j} = H\,INV_{J|j} + \frac{P}{C}\sqrt{V[D^{L'}_{t|j}]}\left[h(\overline{Z}_{J|j})(1+\sigma^2_{Z_{J|j}}) - \overline{Z}_{J|j}(1-H(\overline{Z}_{J|j}))\right] J = S, N, j = s, n, \tag{14}$$

where

$$INV_{J|j} = E[Y_{J,t|j} - E[D^{L'}_{t|j}]] + \frac{E[D^{C}_{t+L|j}]}{2},\ \ J = S, N, j = s, n,$$

and

$$\overline{Z}_{J|j} = E[Z_{J,t|j}^{L'}],\ \sigma_{Z_{J|j}}^2 = V[Z_{J,t|j}^{L'}],\ J = S, N, j = s, n.$$

As in Equation (5), $Z_{J,t|j}^{L'}, J = S, N, j = s, n,$ is defined as

$$Z_{J,t|j}^{L'} = \frac{Y_{J,t|j} - E[D_{t|j}^{L'}]}{\sqrt{V[D_{t|j}^{L'}]}}, J = S, N, j = s, n. \tag{15}$$

Now is ready to consider each of the four cases. For each case, expressions for $INV_{J|j}$, $\overline{Z}_{J|j}$ and $\sigma_{Z_J|j}^2, J = S, N, j = s, n,$ will be obtained to calculate the long run average total inventory costs per period.

Smart Retailer, Smart Manufacturer In this case, at the start of period t, where t ∈ {1+C, 1+2C, 1+3C, $\cdots$}, the manufacturer will forecast the effective lead time demand with the available demand information, *i.e.*, d_{t-1}, $d_{t-2}, \cdots$. Therefore, the effective lead time demand, perceived by the manufacturer, using the relationships shown in Equations (9) and (12), is:

$$D_{S,t|s}^{L'} = D_{t|s}^{L'} = \sum_{u=0}^{m+mM-1} q_{s,t+cu} = \rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)(d_{t+L'-c-1} - d_{t-c-1}) + \sum_{u=0}^{L'-1} d_{t-c+u} \mid d_{t-1}.$$

This is because the smart manufacturer knows the form of $q_{s,t}$, t ∈ {1, 1+c, 1+2c, $\cdots$} and demands, d_{t-1}, $d_{t-2}, \cdots$, that the smart manufacturer is able to predict $q_{s,t}$ with no uncertainty. This, in turn, is because $q_{s,t}$ is only a function of d_{t-1}, $d_{t-2}, \cdots$, which have already occurred and have been shared with the manufacturer. Therefore, the lead time demand perceived by the manufacturer follows a normal distribution with mean $E[D_{S,t|s}^{L'}] = E[D_{t|s}^{L'}]$ and variance $V[D_{S,t|s}^{L'}] = V[D_{t|s}^{L'}]$. In this case, we have $INV_{S|s} = Z\sqrt{V[D_{t|s}^{L'}]} + \frac{C}{2}\frac{\mu}{1-\rho}$, $\overline{Z}_{S|s} = Z$ and $\sigma_{Z_{S|s}}^2 = 0$. Given these expressions, we can calculate the long run average total inventory costs per period using Equation (14).

Finally, as an example of this model, consider the case in which $c = 1$, $l = 0$, $l' = 1$, $C = 1$, $L = 1$ and $L' = 2$, then

$$\begin{aligned} D_{S,t|s}^{L'} &= \sum_{u=0}^{1} q_{s,t+cu} = \sum_{u=0}^{1} \left((1-\rho)d_{t+u-1} - \rho d_{t+u-2}\right) \mid d_{t-1} \\ &= (1+\rho)\mu + (1+\rho+\rho^2)d_{t-1} - \rho d_{t-2} + (1+\rho)\varepsilon_t. \end{aligned}$$

Therefore, we have $Y^{L'}_{S,t|s} = (1+\rho)\mu + (1+\rho+\rho^2)d_{t-1} - \rho d_{t-2} + Z\sigma(1+\rho)$.

Naïve Retailer, Smart Manufacturer When information on the actual customer demand is shared between the retailer and the manufacturer, using the relationships shown in Equations (11) and (12), the manufacturer will write the effective lead time demand, given the available information, *i.e.*, $d_{t-1}, d_{t-2}, \cdots,$ as:

$$D^{L'}_{S,t|n} = D^{L'}_{t|n} = \sum_{u=0}^{m+mM-1} q_{n,t+cu} = \sum_{u=0}^{L'-1} d_{t-c+u} \mid d_{t-1}$$

This is because the smart manufacturer knows the form of $q_{n,t}$, $t \in \{1, 1+c, 1+2c, \cdots\}$ and demands, $d_{t-1}, d_{t-2}, \cdots,$ that the smart manufacturer is able to predict $q_{n,t}$ with no uncertainty. This is, in turn, because $q_{n,t}$ is only a function of $d_{t-1}, d_{t-2}, \cdots,$ which have already occurred and have been shared with the manufacturer. Therefore, the lead time demand perceived by the manufacturer follows a normal distribution with mean $E[D^{L'}_{S,t|n}] = E[D^{L'}_{t|n}]$ and variance $V[D^{L'}_{S,t|n}] = V[D^{L'}_{t|n}]$. In this case, we have $INV_{S|n} = Z\sqrt{V[D^{L'}_{t|n}]} + \frac{C}{2}\frac{\mu}{1-\rho}$, $\overline{Z}_{S|n} = Z$, and $\sigma^2_{Z_{S|n}} = 0$. Given these expressions, we can calculate the long run average total inventory costs per period using Equation (14).

Finally, as an example of this model, consider the case in which $c = 1$, $l = 0$, $l' = 1$, $C = 1$, $L = 1$ and $L' = 2$, then

$$\begin{aligned} D^{L'}_{S,t|n} &= \sum_{u=0}^{1} q_{n,t+cu} = \sum_{u=0}^{1} d_{t+u-1} \mid d_{t-1} \\ &= \mu + (1+\rho)d_{t-1} + \varepsilon_t. \end{aligned}$$

Therefore, we have $Y^{L'}_{S,t|n} = \mu + (1+\rho)d_{t-1} + Z\sigma$.

Smart Retailer, Naïve Manufacturer In this case, the manufacturer does not notice the correlation between the customer demands or between the retailer demands. Therefore, the manufacturer will use an inventory policy based on the assumption of *i.i.d.* demands. Therefore, the lead time demand perceived by the manufacturer follows a normal distribution with mean $E[D^{L'}_{N,t|s}] = E[E[D^{L'}_{t|s}]]$ and variance $V[D^{L'}_{N,t|s}] = V[E[D^{L'}_{t|s}]] + V[D^{L'}_{t|s}]$. Here, we have $INV_{N|s} = Z\sqrt{V[D^{L'}_{N,t|s}]} + \frac{C}{2}\frac{\mu}{1-\rho}$,

$\overline{Z}_{N|s} = Z\frac{\sqrt{V[D_{N,t|s}^{L'}]}}{\sqrt{V[D_{t|s}^{L'}]}}$ and $\sigma_{Z_{N|s}}^2 = \frac{V[E[D_{N,t|s}^{L'}]]}{V[D_{t|s}^{L'}]}$. Given these expressions, we can calculate the long run average total inventory costs per period using Equation (14).

Finally, as an example of this model, consider the case in which $c = 1$, $l = 0$, $l' = 1$, $C = 1$, $L = 1$ and $L' = 2$, then

$$D_{N,t|s}^{L'} = \sum_{u=0}^{1} q_{s,t+cu} = \sum_{u=0}^{1} d_{t+u-1} \mid d_{t-1} = (1+\rho)\mu + (1+\rho+\rho^2)d_{t-1} - \rho d_{t-2} + (1+\rho)\varepsilon_t.$$

From this, we have $E[D_{N,t|s}^{L'}] = 2\frac{\mu}{1-\rho}$ and

$$V[D_{N,t|s}^{L'}] = V[E[D_{t|s}^{L'}]] + V[D_{t|s}^{L'}]$$
$$= \left((1+\rho+\rho^2)^2 + \rho^2 - 2(1+\rho+\rho^2)\rho^2\right)\frac{\sigma^2}{1-\rho^2} + (1+\rho)^2\sigma^2.$$

Therefore, $Y_{N,t|s}^{L'} = 2\frac{\mu}{1-\rho} + Z\sigma\sqrt{\left((1+\rho+\rho^2)^2 + \rho^2 - 2(1+\rho+\rho^2)\rho^2\right)\frac{\sigma^2}{1-\rho^2} + (1+\rho)^2}$.

Naïve Retailer, Naïve Manufacturer Following the same reasoning, the lead time demand perceived by the manufacturer follows a normal distribution with mean $E[D_{N,t|n}^{L'}] = E[E[D_{t|n}^{L'}]]$ and variance $V[D_{N,t|n}^{L'}] = V[E[D_{t|n}^{L'}]] + V[D_{t|n}^{L'}]$. Here, we have $INV_{N|n} = Z\sqrt{V[D_{N,t|n}^{L'}]} + \frac{C}{2}\frac{\mu}{1-\rho}$, $\overline{Z}_{N|n} = Z\frac{\sqrt{V[D_{N,t|n}^{L'}]}}{\sqrt{V[D_{t|n}^{L'}]}}$ and $\sigma_{Z_{N|n}}^2 = \frac{V[E[D_{N,t|n}^{L'}]]}{V[D_{t|n}^{L'}]}$. Given these expressions, we can calculate the long run average total inventory costs per period using Equation (14).

Finally, as an example of this model, consider the case in which $c=1$, $l=0$, $l'=1$, $C = 1$, $L = 1$ and $L' = 2$, then $D_{N,t|n}^{L'} = \sum_{u=0}^{1} q_{n,t+cu} = \sum_{u=0}^{1} d_{t+u-1} \mid d_{t-1} = \mu + (1+\rho)d_{t-1} + \varepsilon_t$.

From this, we have

$$E[D_{N,t|n}^{L'}] = 2\frac{\mu}{1-\rho} \text{ and } V[D_{N,t|n}^{L'}] = (1+\rho)^2\frac{\sigma^2}{1-\rho^2} + \sigma^2.$$

Therefore, $Y_{N,t|n}^{L'} = 2\frac{\mu}{1-\rho} + Z\sigma\sqrt{\frac{(1+\rho)^2}{1-\rho^2} + 1}$.

4. BENEFITS OF USING CORRECT DEMAND MODEL

In this section, the model presented in Section 3 is used to analyze the value of utilizing the true nature of the demand process in our simple supply chain model.

First, it is demonstrated that using the correct demand model always leads to lower long run average total inventory costs per period. This seemingly intuitive result is always true for both the retailer and the manufacturer.

Proposition 1. The long run average total inventory costs per period for a naïve retailer will be always larger than or equal to the long run average total inventory costs per period for a smart retailer. Similarly, the same results apply for the manufacturer.

Proof. The proof of Proposition 1 is provided in Appendix A.

Next, it may seem intuitive that the minimum cost for the supply chain as a whole is obtained when both the retailer and the manufacturer are smart. This is not always the case, however. To understand why, note that when the retailer is smart and $\rho > 0$, the manufacturer faces more severe bullwhip effect, *i.e.*, the variance of the orders placed by the smart retailer is greater than the variance of the orders placed by the naïve retailer. Therefore, when $\rho > 0$, the manufacturer is better off with a naïve retailer than with a smart retailer.

To demonstrate this, it is proved first that the variance of the orders placed by a smart retailer is greater than the variance of the orders placed by a naïve retailer when $\rho > 0$. Next, it is proved that, if $\rho > 0$, the long run average total inventory costs per period for the smart manufacturer are lower when the demand stream is from the naïve retailer than when the demand stream is from the smart retailer.

Proposition 2. $V[q_{s,t}] > V[q_{n,t}]$ when $\rho > 0$.

Proof. The proof of Proposition 2 is provided in Appendix B.

From Propositions 1 and 2, we have a following corollary.

Corollary 3. When $\rho > 0$, we have $G_{S|s} > G_{S|n}$.

Proof. proof of Corollary 3 is provided in Appendix C.

5. FINAL REMARKS

The mathematical model developed in this paper has been used to evaluate the value of using the right demand model. Clearly, the model considered in this paper is quite simple. Despite the simplicity of the model, however, the question of how the long run average total inventory costs per period at a retailer and/or manufacturer depends on the choice of customer demand model is an important one. Since very few participants of a supply chain operate under the conditions assumed by most standard inventory models, it is important to understand: (i) how the actual inventory policies used by real retailers and/or manufacturers differ from the optimal policies suggested by most standard inventory models and (ii) how the actual costs at the retailers and/or manufacturers differ from the optimal costs suggested by the optimal policies.

A brief summary of the key managerial insights gleaned in this paper is as follows:

- Knowledge of the actual demand process is always beneficial to the retailer in terms of reducing long run average total inventory costs per period, given that the retailer knows how to use it.
- The retailer's knowledge of the actual demand process may induce the bullwhip effect, thus increasing the long run average total inventory costs per period at the manufacturer. Therefore, it is possible that sometimes a supply chain with a naive retailer and a smart manufacturer has a lower system wide long run average total inventory costs than a supply chain with a smart retailer and a smart manufacturer.
- Shared demand data is beneficial to the manufacturer, especially when the manufacturer understands the true nature of demand process and uses the demand data accordingly.

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APPENDIX A: Proof of Proposition 1 in Section 4

First, from the expressions for $\bar{z}_n$ and $\sigma^2_{z_n}$ in Subsection 3.2 and the fact that $V[d^{l'}_{n,t}] = V[E[d^{l'}_t | d_{t-1}]] + V[d^{l'}_t | d_{t-1}]$, we have:

$$\frac{\bar{z}_n}{\sqrt{1+\sigma^2_{z_n}}} = \frac{z\dfrac{\sqrt{V[d^{l'}_{n,t}]}}{\sqrt{V[d^{l'}_t | d_{t-1}]}}}{\sqrt{1+\dfrac{V[E[d^{l'}_t | d_{t-1}]]}{V[d^{l'}_t | d_{t-1}]}}} = z. \tag{16}$$

Next, we will show that:

$$\left[h(\bar{z}_n)(1+\sigma_{z_n}^2)-\bar{z}_n(1-H(\bar{z}_n))\right]=\sqrt{1+\sigma_{z_n}^2}\int_z^{\infty}(x-z)\phi(x)dx.$$

First, from the facts that:

$$\begin{aligned} h(\bar{z}_n)(1+\sigma_{z_n}^2) &= (1+\sigma_{z_n}^2)\frac{1}{\sqrt{2\pi}\sqrt{1+\sigma_{z_n}^2}}\exp\left\{-\frac{1}{2}\left(\frac{\bar{z}_n^2}{1+\sigma_{z_n}^2}\right)\right\} \\ &= \int_{\bar{z}_n}^{\infty} t\frac{1}{\sqrt{2\pi}\sqrt{1+\sigma_{z_n}^2}}\exp\left\{-\frac{1}{2}\left(\frac{t^2}{1+\sigma_{z_n}^2}\right)\right\}dt \\ &= \int_{\bar{z}_n}^{\infty} th(t)dt, \end{aligned}$$

and that:

$$\bar{z}_n(1-H(\bar{z}_n))=\bar{z}_n\int_{\bar{z}_n}^{\infty}h(t)dt,$$

we have:

$$\left[h(\bar{z}_n)(1+\sigma_{z_n}^2)-\bar{z}_n(1-H(\bar{z}_n))\right]=\int_{\bar{z}_n}^{\infty}(t-\bar{z}_n)h(t)dt.$$

Notice that, from Zipkin [12], $\int_{\bar{z}_n}^{\infty}(t-\bar{z}_n)h(t)dt$ can also be written as follows:

$$\sqrt{1+\sigma_{z_n}^2}\int_{\frac{\bar{z}_n}{\sqrt{1+\sigma_{z_n}^2}}}^{\infty}\left(x-\frac{\bar{z}_n}{\sqrt{1+\sigma_{z_n}^2}}\right)\phi(x)dx.$$

Therefore, we have:

$$\left[h(\bar{z}_n)(1+\sigma_{z_n}^2)-\bar{z}_n(1-H(\bar{z}_n))\right]=\sqrt{1+\sigma_{z_n}^2}\int_{\frac{\bar{z}_n}{\sqrt{1+\sigma_{z_n}^2}}}^{\infty}\left(x-\frac{\bar{z}_n}{\sqrt{1+\sigma_{z_n}^2}}\right)\phi(x)dx. \tag{17}$$

Finally, from Equations (16) and (17), we have:

$$\left[h(\bar{z}_n)(1+\sigma_{z_n}^2)-\bar{z}_n(1-H(\bar{z}_n))\right]=\sqrt{1+\sigma_{z_n}^2}\int_z^{\infty}(x-z)\,\phi(x)dx.$$

In order to prove the desired result, note that the long run average total inventory costs per period for the smart retailer can be written as:

$$\begin{aligned} g_s &= h\,inv_s+\frac{p}{c}\left(\sqrt{V[d_t^{l'}\mid d_{t-1}]}\left[h(\bar{z}_s)(1+\sigma_{z_s}^2)-\bar{z}_s(1-H(\bar{z}_s))\right]\right) \\ &= h\left(z\sqrt{V[d_t^{l'}\mid d_{t-1}]}+\frac{c}{2}\left(\frac{\mu}{1-\rho}\right)\right)+\frac{p}{c}\sqrt{V[d_t^{l'}\mid d_{t-1}]}\int_z^{\infty}(x-z)\,\phi\,(x)dx, \end{aligned}$$

while the long run average total inventory costs per period for the naïve retailer can be written as:

$$g_n = h\,inv_n + \frac{p}{c}\left(\sqrt{V[d_t^{l'} \mid d_{t-1}]}\left[h(\bar{z}_n)(1+\sigma_{z_n}^2) - \bar{z}_s(1-H(\bar{z}_n))\right]\right)$$
$$= h\left(z\sqrt{V[E[d_t^{l'} \mid d_{t-1}]] + V[d_t^{l'} \mid d_{t-1}]} + \frac{c}{2}\left(\frac{\mu}{1-\rho}\right)\right)$$
$$+ \frac{p}{c}\sqrt{V[d_t^{l'} \mid d_{t-1}]}\sqrt{1+\sigma_{z_n}^2}\int_z^\infty (x-z)\,\phi(x)dx,$$

Since $\sqrt{V[E[d_t^{l'} \mid d_{t-1}]] + V[d_t^{l'} \mid d_{t-1}]} \geq \sqrt{V[d_t^{l'} \mid d_{t-1}]}$ and $\sigma_{z_n}^2 \geq 0$, we have $g_n \geq g_s$. The same method can be used to prove the result for the case of the manufacturer.

APPENDIX B: Proof of Proposition 2 in Section 4

First note that, from Equations (9) and (11), $q_{s,t} = \rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)(d_{t-1} - d_{t-c-1}) + \sum_{u=0}^{c-1} d_{t-c+u}$ and $q_{n,t} = \sum_{u=0}^{c-1} d_{t-c+u}$, respectively.

Therefore, we can write

$$V[q_{s,t}] = V\left[\rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)(d_{t-1} - d_{t-c-1}) + \sum_{u=0}^{c-1} d_{t-c+u}\right]$$
$$= V\left[\rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)d_{t-1}\right] + V\left[\rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)d_{t-c-1}\right] + V\left[\sum_{u=0}^{c-1} d_{t-c+u}\right]$$
$$- 2Cov\left(\rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)d_{t-1}, \rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)d_{t-c-1}\right) + 2Cov\left(\rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)d_{t-1}, \sum_{u=0}^{c-1} d_{t-c+u}\right)$$
$$- 2Cov\left(\rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)d_{t-c-1}, \sum_{u=0}^{c-1} d_{t-c+u}\right)$$
$$= \left[2\left(\rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)\right)^2(1-\rho^c) + 2(1-\rho)\rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)\left(\frac{1-\rho^c}{1-\rho}\right)\right]\frac{\sigma^2}{1-\rho^2} + V\left[\sum_{u=0}^{c-1} d_{t-c+u}\right],$$

and

$$V[q_{n,t}] = V\left[\sum_{u=0}^{c-1} d_{t-c+u}\right].$$

Finally, from above, $V[q_{s,t}]-V[q_{n,t}]$ can be written as:

$$\left[2\left(\rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)\right)^2(1-\rho^c)+2(1-\rho)\rho\left(\frac{1-\rho^{l'}}{1-\rho}\right)\left(\frac{1-\rho^c}{1-\rho}\right)\right]\frac{\sigma^2}{1-\rho^2}.$$

Therefore, when $\rho>0$, the above implies $V[q_{s,t}]>V[q_{n,t}]$. Note that the reverse is not always true.

APPENDIX C: Proof of Corollary 3 in Section 4

First, from Appendix A, note that:

$$G_{S|s}=H\left(Z\sqrt{V[D_{t|s}^{L'}]}+\frac{E[E[D_{t|s}^{L'}]]}{2}\right)+\frac{P}{C}\sqrt{V[D_{t|s}^{L'}]}\int_Z^\infty(X-Z)\phi(X)dX,$$

and that:

$$G_{S|n}=H\left(Z\sqrt{V[D_{t|n}^{L'}]}+\frac{E[E[D_{t|n}^{L'}]]}{2}\right)+\frac{P}{C}\sqrt{V[D_{t|n}^{L'}]}\int_Z^\infty(X-Z)\phi(X)dX.$$

Next, from the fact that $E[E[D_{t|s}^{L'}]]=E[E[D_{t|n}^{L'}]]$, we can write $G_{S|s}-G_{S|n}$ as:

$$\left(\sqrt{V[D_{t|s}^{L'}]}-\sqrt{V[D_{t|n}^{L'}]}\right)\left(HZ+\frac{P}{C}\int_Z^\infty(X-Z)\phi(X)dX\right).$$

Since, from Proposition 2, when $\rho>0$, $\sqrt{V[D_{t|s}^{L'}]}-\sqrt{V[D_{t|n}^{L'}]}>0$ and the term in the second bracket is always positive, we have $G_{S|s}>G_{S|n}$.