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Uncertain Rule-based Fuzzy Technique: Nonsingleton Fuzzy Logic System for Corrupted Time Series Analysis

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Abstract

In this paper, we present the modeling of time series data which are corrupted by noise via nonsingleton fuzzy logic system. Nonsingleton fuzzy logic system (NFLS) is useful in cases where the available data are corrupted by noise. NFLS is a fuzzy system whose inputs are modeled as fuzzy number. The abilities of NFLS to approximate arbitrary functions, and to effectively deal with noise and uncertainty, are used to analyze corrupted time series data. In the simulation results, we compare the results of the NFLS approach with the results of using only a traditional fuzzy logic system.

Key words: uncertainty, rule-based model, nonsingleton fuzzy system, corrupted time series data

1. Introduction

Time series data analysis is an important tool for forecasting the future in terms of past history. A time series is a sequence of values measured over time, in discrete or continuous time units. By studying many related variables together, a better understanding is often obtained. Robust forecasting must rely on how well the time series is designed. Many techniques for time series analysis have been developed assuming linear relationships among the series variables. Unfortunately, many real world applications involve nonlinearities between environmental variables. Assuming simple relationships among time series variables can produce poor results regarding the ability to predict the future. In many cases, such inaccuracies can produce major problems [4].

During the past few years, fuzzy modeling techniques have become an active research area due to their successful applications to complex, ill-defined and uncertain systems in which conventional mathematical model fails to give satisfactory results. Many researchers have proposed many different techniques. Fuzzy relation model [5], the neural-network-based fuzzy model [6], Takagi and Sugeno's fuzzy linear functional model [7], the fuzzy basis function based model [8], and fuzzy neural integrated system [9].

The most widely used fuzzifier is the singleton fuzzifier [10] mainly because of its simplicity and lower computational requirements. However singleton fuzzifier may not always be adequate, especially in cases where noise is present in the data. In other words, if there is some kind of abstraction, uncertainty, or noise present in the input data, the popular singleton fuzzifier used in singleton FLS may not be adequate. Under these circumstances, a different approach is necessary to account for uncertainty in the data. Nonsingleton fuzzifiers have been used successfully in a variety of applications [11].

In this study, nonsingleton fuzzy logic system [1,2] is applied to model noisy Box-Jenkin's gas furnace data [3]. The Box-Jenkin's gas furnace data will be demonstrated to show the performance of the NFLS. We also compare the results of the NFLS approach with the results of using only a singleton Mamdani fuzzy logic system (SFLS).

2. Nonsingleton fuzzy logic system

Since the NFLS is applied to Box-Jenkin's gas furnace time series data [3], its fundamentals are briefly explained. The detailed descriptions and formulations of the NFLS can be found in [1,2]. The overall structure of the NFLS is shown in Fig. 1. In the figure, crisp inputs are first fuzzified into fuzzy input sets then activate the inference block.

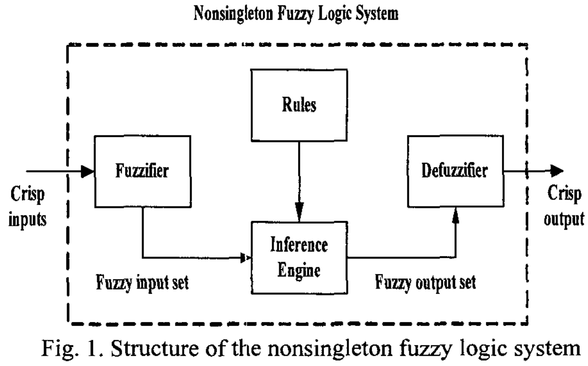

Fig. 1. Structure of the nonsingleton fuzzy logic system

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The NFLS is based on the concept of fuzzy set theory, fuzzy if-then rules, and fuzzy reasoning. Structure of the NFLS is the same as that of the singleton fuzzy logic system. What is different is the fuzzifier, which treats the inputs as fuzzy sets, and the effect of this on the inference block.

Fuzzy sets can be viewed as membership functions μ_X that map each elements x of the universe of discourse, U, to a number $\mu_X(x)$ in the interval [0,1]:

$$\mu_X : U \rightarrow [0,1] \tag{1}$$

A fuzzifier maps a crisp point $x \in U$ into a fuzzy set X, whose domain of support is a subset of U. The nonsingleton fuzzifier maps the point $x \in U$ into a fuzzy set X with support x_i, where μ_X achieves maximum value at x_i=x and decreases while moving away from x_i=x. In other words, measurement x_i=x'_i is mapped into a fuzzy number in nonsingleton fuzzification.

Conceptually, the nonsingleton fuzzifier implies that the given input value x'_i is the most likely value to be the correct one from all the values in its immediate neighborhood, however, because the input is corrupted by noise, neighboring points are also likely to be the correct values, but to a lesser degree. So nonsingleton fuzzification is especially useful in cases where the available data contain any kind of statistical or non-statistical uncertainty. NFLS is not only a generalization of singleton fuzzy logic system but also provide a reconciliation between fuzzy logic techniques and statistical methods for handling uncertainty [1].

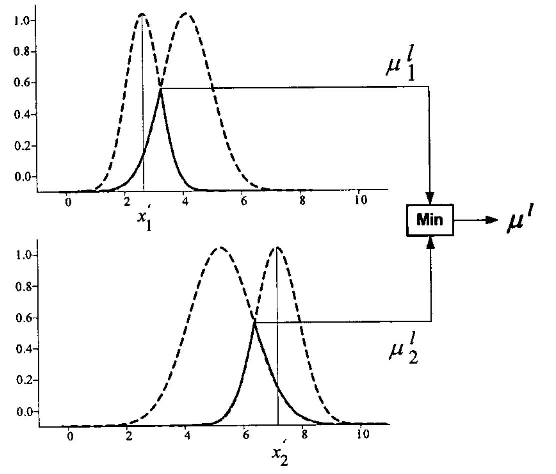
Here, we consider a pictorial description of input and antecedent operations for a nonsingleton fuzzy logic system in Fig. 2. The minimum and product t-norms for a two-antecedent and single-consequent rule are shown in the figure.

Consider a fuzzy logic system with a rule base of M rules, and let the lth rule be denoted by R^l. Let each rule have n antecedents and one consequent, i.e., it is of the general form

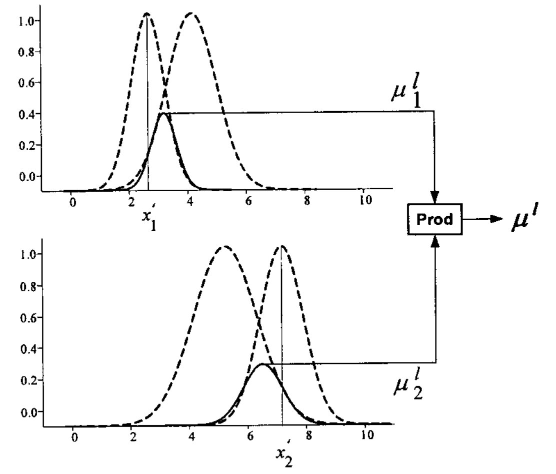
$$\begin{aligned} R^l &: \text{IF } u_1 \text{ is } F_1^l \text{ and } u_2 \text{ is } F_2^l \text{ and} \cdots u_n \text{ is } F_n^l \\ &\text{Then } v \text{ is } G^l \;\; l = 1, \ldots, M \end{aligned} \tag{2}$$

Where u_k, k=1,..., n, and v are the input and output linguistic variables, respectively. Each F_k^l and G^l are subsets of possibly different universes of discourse. Let $F_k^l \subset U_k$ and $G^l \subset V$. Each rule can be viewed as a fuzzy relation R^l from a set U to a set V where U is the Cartesian product $U = U_1 \times \cdots \times U_n$. R^l itself is a subset of the Cartesian product $U \times V = \{(\mathbf{x}, y) : \mathbf{x} \in U, y \in V\}$, where $\mathbf{x} \equiv (x_1, x_2, \cdots, x_n)$, and x_k and y are the points in the universes of discourse U_k and V of u_k and v.

Comparing nonsingleton fuzzy inference engine and singleton fuzzy inference engine, we see that a NFLS first prefilters its input **X**, transforming it to $\mathbf{X}^l_{max}$. This is depicted in Fig. 3. Doing this accounts for the effect of the input measurement uncertainty.



(a) Minimum t-norm



(b) Product t-norm

Fig. 2. Input and antecedent operations for nonsingleton fuzzy logic system

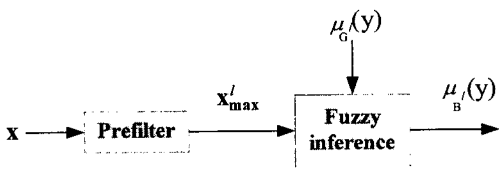


Fig. 3. Prefilter of the nonsingleton fuzzy system

$\mu_{B^l}(y)$ in Fig. 3 is as follows.

$$\begin{aligned} \mu_{B^l}(y) = \mu_{G^l}(y) * \{&[\sup_{x_1 \in X_1} \mu_{X_1}(x_1) * \mu_{F_1^l}(x_1)] * \\ &\cdots * [\sup_{x_n \in X_n} \mu_{X_n}(x_n) * \mu_{F_n^l}(x_n)]\}, \;\; y \in Y \end{aligned} \tag{3}$$

We define

$$\mu_{Q_k^l}(x_k) \equiv \mu_{X_k}(x_k) * \mu_{F_k^l}(x_k) \tag{4}$$

where, k=1,..., n and l=1,..., M.

Finally, (3) can be re-written as:

$$\mu_{B^l}(y) = \mu_{G^l}(y) * [T_{k=1}^n \mu_{Q_k^l}(x_{k,\max}^l)] \tag{5}$$

where T and *are short for a t-norm such as minimum or product.

This is the fundamental equation for a nonsingleton fuzzy logic system. The bracketed term is the firing level for a NFLS.

When the t-norm is the product and all membership functions are Gaussian, then it is straightforward to carryout the (3). The Kth input fuzzy set and the corresponding rule antecedent fuzzy sets are assumed to have the following membership functions.

$$\mu_{X_k}(x_k)=\exp(-\frac{1}{2}\left[\frac{(x_k-m_{X_k})^2}{\sigma_{X_k}}\right]) \tag{6}$$

$$\mu_{F_k^l}(x_k)=\exp(-\frac{1}{2}\left[\frac{(x_k-m_{F_k^l})^2}{\sigma_{F_k^l}}\right]) \tag{7}$$

where k=1,..., n and l=1, ..., M.

Using the rule R^l, NFLS is constructed and its parameters are tuned as follows.

$$y(\mathbf{x}^{(i)})=f_{ns}(\mathbf{x}^{(i)})=\frac{\sum_{l=1}^{M}\bar{y}^l\prod_{k=1}^{n}\mu_{Q_k^l}(x_{k,\max}^{l,(i)})}{\sum_{l=1}^{M}\prod_{k=1}^{n}\mu_{Q_k^l}(x_{k,\max}^{l,(i)})} = \frac{\sum_{l=1}^{M}\bar{y}^l\prod_{k=1}^{n}\exp(-\frac{1}{2}\left[\frac{(x_k^{(i)}-m_{F_k^l})^2}{\sigma_X^2+\sigma_{F_k^l}^2}\right])}{\sum_{l=1}^{M}\prod_{k=1}^{n}\exp(-\frac{1}{2}\left[\frac{(x_k^{(i)}-m_{F_k^l})^2}{\sigma_X^2+\sigma_{F_k^l}^2}\right])} \tag{8}$$

$$m_{F_k^l}(i+1)=m_{F_k^l}(i)-\alpha_m[f_{ns}(\mathbf{x}^{(i)})-y^{(i)}]\times [\bar{y}^{(i)}-f_{ns}(\mathbf{x}^{(i)})]\times[\frac{x_k^{(i)}-m_{F_k^l}(i)}{\sigma_X^2(i)+\sigma_{F_k^l}^2(i)}]\phi_l(\mathbf{x}^{(i)}) \tag{9}$$

$$\bar{y}(i+1)=\bar{y}(i)-\alpha_{\bar{y}}[f_{ns}(\mathbf{x}^{(i)})-y^{(i)}]\phi_l(\mathbf{x}^{(i)}) \tag{10}$$

$$\sigma_{F_k^l}(i+1)=\sigma_{F_k^l}(i)-\alpha_\sigma[f_{ns}(\mathbf{x}^{(i)})-y^{(i)}]\times [\bar{y}^{(i)}-f_{ns}(\mathbf{x}^{(i)})]\times\sigma_{F_k^l}(i)[\frac{x_k^{(i)}-m_{F_k^l}(i)}{\sigma_X^2(i)+\sigma_{F_k^l}^2(i)}]^2\phi_l(\mathbf{x}^{(i)}) \tag{11}$$

$$\sigma_X(i+1)=\sigma_X(i)-\alpha_X[f_{ns}(\mathbf{x}^{(i)})-y^{(i)}]\times [\bar{y}^{(i)}-f_{ns}(\mathbf{x}^{(i)})]\times\sigma_X(i)[\frac{x_k^{(i)}-m_{F_k^l}(i)}{\sigma_X^2(i)+\sigma_{F_k^l}^2(i)}]^2\phi_l(\mathbf{x}^{(i)}) \tag{12}$$

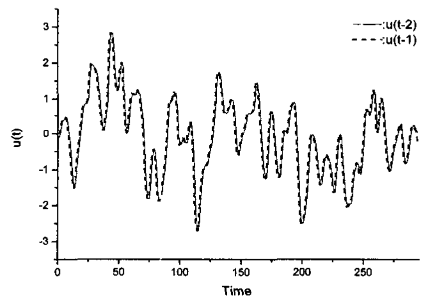
3. Simulation results

We evaluate the performance of NFLS applying it to the modeling of noisy Box-Jenkin's gas furnace time series. In addition, we compare the performance of NFLS with that of singleton FLS. Box and Jenkin's gas furnace is a famous example of system identification. The well-known Box-Jenkins data set consists of 296 input-output observations, where the input u(t) is the rate of gas flow into a furnace and the output y(t) is the CO2 concentration in the outlet gases. The delayed terms of u(t) and y(t) such as u(t-2), u(t-1), y(t-2), and y(t-1)are used as input variables to the NFLS. The actual system output y(t) is used as target output variable for this model. The performance index (PI) is defined as the root mean squared error

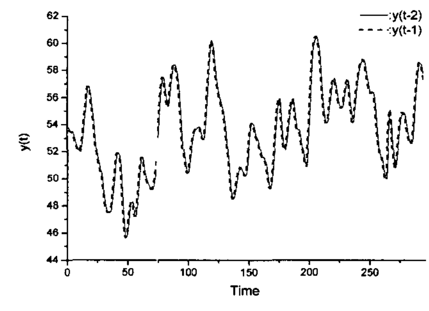
$$PI=\sqrt{\frac{1}{m}\sum_{i=1}^{m}(y_i-\hat{y}_i)^2} \tag{13}$$

where y_i is the actual system output, $\hat{y}_i$ is the estimated output of each node, and m is the number of data.

Simulation of NFLS is conducted in choosing max-product composition, product implication, height defuzzification, and Gaussian membership function.



(a) Noise-free methane gas flow rate: u(t-2) and u(t-1)



(b) Noise-free carbon dioxide density: y(t-2) and y(t-1)

Fig. 4. Noise-free Box-Jenkin's gas furnace time series

In Fig. 4, we present the noise-free data which is four input variables of the Box-Jenkin's gas furnace time series. In the figure, (a) and (b) are the delayed terms of methane gas flow rate u(t) and carbon dioxide density y(t), respectively. We also depict the one realization of 5dB uniformly distributed noise data in Fig. 5. Owing to the noise data shown in Fig. 5, input data will be corrupted as shown in Fig. 6. In Fig. 6, we plot the noise corrupted data of Box-Jenkin's gas furnace time series. The corrupted data is employed as input variables to the NFLS.

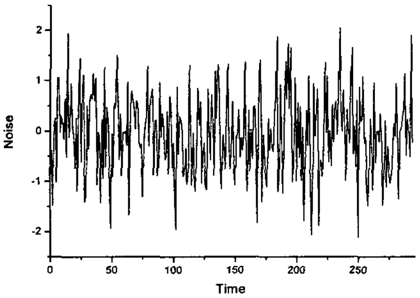
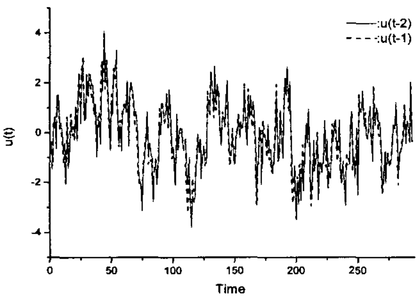
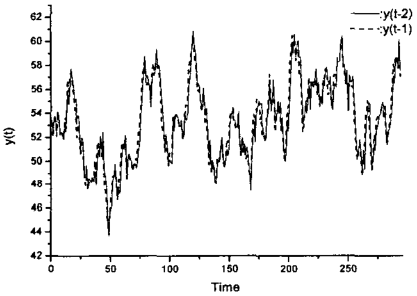


Fig. 5. One realization of 5dB uniformly distributed noise data which will corrupt the noise free signals.



(a) Noise corrupted methane gas flow rate: u(t-2) and u(t-1)



(b) Noise corrupted carbon dioxide density: y(t-2) and y(t-1)
Fig. 6. Noise corrupted Box-Jenkin's gas furnace time series to be employed as input variables for NFLS.

Fig. 7 shows the modeling results of the singleton fuzzy logic system. SFLS contains 36 fuzzy rules with two MFs for gas flow rate and three MFs for carbon dioxide density, which are employed. However, the model outputs do not follow the actual output very well so the SFLS is unable to handle the noise.

Modeling results of the nonsingleton fuzzy logic system are shown in Fig. 8. In the results, NFLS is employed only two fuzzy sets for each of the four antecedents, so the number of rules equals $2^4=16$. In Fig. 9, 36 fuzzy rules are used, here three MFs for gas flow rate and two MFs for carbon dioxide density are considered. The estimated result is shown in Fig. 9. As can be seen from the Fig. 9, the model output follows actual output well. Therefore, the NFLS does a much better job of modeling a noisy time series data than does a SFLS. The value of the performance index of the NFLS is equal to 0.8493.

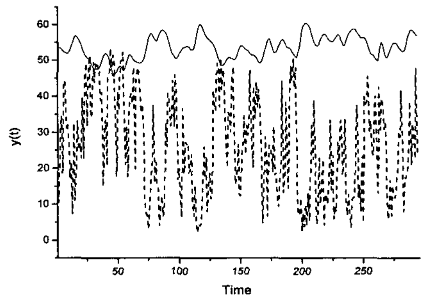


Fig. 7. Modeling results of the singleton fuzzy logic system with 36 fuzzy rules assigned two MFs for gas flow rate u(t) and three MFs for carbon dioxide density y(t) (solid line: actual time series, dotted line: estimated time series)

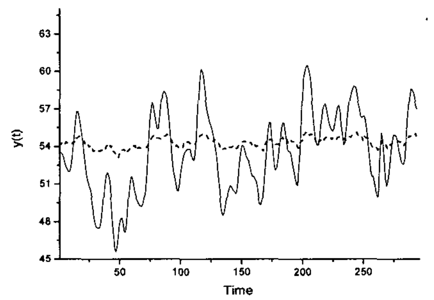


Fig. 8. Modeling results of the nonsingleton fuzzy logic system with 16 fuzzy rules assigned two MFs for each input variable (solid line: actual time series, dotted line: estimated time series)

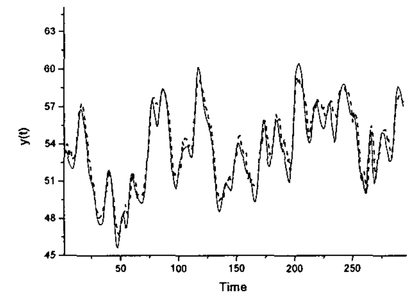


Fig. 9. Modeling results of the nonsingleton fuzzy logic system with 36 fuzzy rules assigned three MFs for gas flow rate u(t) and two MFs for carbon dioxide density y(t) (solid line: actual time series, dotted line: estimated time series)

4. Conclusions

Nonsingleton fuzzy logic system (NFLS) is first applied to model a corrupted Box-Jenkin's gas furnace time series data. The modeling performance of the NFLS is compared to those of the SFLS. The most commonly used fuzzifier is a singleton, but such a fuzzifier is not adequate when data is corrupted by measurement noise.

Simulation results of the Box-Jenkin's gas furnace data was demonstrated to show the performance of the NFLS. As we seen from the simulation results, the NFLS provide a way to handle knowledge uncertainty. Meanwhile, SFLS is unable to directly handle uncertainty. Thus it can be considered NFLS does a much better job of modeling a noisy time series data than does a traditional fuzzy logic system.

References

[1] G.C. Mouzouris, and J.M. Mendel, "Nonsingleton Fuzzy Logic Systems: Theory and Application," *IEEE Trans. Fuzzy Syst*, vol. 5, pp. 56-71, 1997.

[2] D. Kim, S.J. Seo, and G.T. Park, "Modeling Corrupted Time Series Data via Nonsingleton Fuzzy Logic System," *Lecture Notes in Computer Science*, in press 2004.

[3] G.E.P. Box, G.M. Jenkins, and G.C. Reinsel, *Time Series Analysis: Forecasting and Control* 3rd ed. Prentice-Hall, 1994.

[4] Alaa, F.Sheta, and Kenneth De Jong, "Time-series forecasting using GA-tuned radial basis functions," *Inform. Sci*, vol. 133, pp. 221-228, 2001.

[5] J.Q. Chen, and L.J. Chen, "An on-line identification algorithm for fuzzy systems," *Fuzzy Sets and Syst,* vol. 64 pp. 63-72, 1994.

[6] J.R. Jang, "Fuzzy controllers based on temporal back propagation," *IEEE Trans. Neural Networks* vol. 3, pp. 714-723, 1992.

[7] T. Takagi, and M. Sugeno, "Fuzzy Identification of Systems and Its Applications to Modeling and Control," *IEEE Trans. Syst. Man Cybern*, vol. 15, pp. 116-132, 1985.

[8] L.X. Wang, and J.M. Mendel, "Fuzzy basis functions, universal approximation, and orthogonal least-squares learning," *IEEE Trans. Neural Networks*, vol. 3, pp. 807-814, 1992.

[9] M.M. Gupta, and D.H. Rao, "On the principles of fuzzy neural networks," *Fuzzy Sets Syst*, vol. 61, pp. 1-18, 1994.

[10] B. Kosko, "Fuzzy systems as universal approximators," *IEEE Trans. Comput*, vol. 43, pp. 1329-1333, 1994.

[11] Y. Hayashi, J.J. Buckley, and E. Czogala, "Fuzzy neural network with fuzzy signals and weights," *Int. J. Intell. Syst*, vol. 8, pp. 527-537, 1993.

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