Synchronization in Complex Systems

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Abstract—In this paper, we introduce a complex systems synchronization method using hyper-chaos circuit consist of State-Controlled Cellular Neural Network (SC-CNN). We make a complex systems using SC-CNN with the n-double scroll. A complex system is created by applying identical n-double scroll or non-identical n-double scroll and Chua's oscillator with weak coupled method to each cell. Complex systems synchronization were achieved using GS(Generalized Synchronization) method between the transmitter and receiver about each state variable in the SC-CNN.

Index Terms—Chaos, Complex System, Generalized Synchronization, Nonlinear Dynamics.

I. INTRODUCTION

Recently, there has been interest in studying the behavior of chaotic dynamics and complex systems. Chaotic systems and complex systems are characterized by sensitive dependence on initial conditions, making long term prediction impossible, self-similarity, and a continuous broad-band power spectrum, etc. Chaotic systems and complex systems have a variety of applications, including synchronization and secure communication and crypto communication [1-6].

Chaos synchronization and secure communication has been a topic of intense research in the past decade. However, secure communication or cryptographic using chaos has several problems [7]. First, almost all chaos-based secure communication or cryptographic algorithms use dynamical systems defined on the set of real number, and therefore are difficult for practical realization and circuit implementation.

Second, security and performance of almost all proposed chaos-based methods are not analyzed in terms of the techniques developed in cryptography. Moreover, most of the proposed methods generate cryptographically weak and slow algorithms.

To address these problems, we need a complex system to increase the complexity in secure communication or cryptographic communication. In this paper, we introduce a new complex systems synchronization method called GS (Generalized synchronization) using State-Controlled Cellular Neural Network (SC-CNN) as a complex system. We make a complex system using SC-CNN with the n-double scroll [8] and Chua's oscillator.

In order to make a complex system, we used identical n-double scroll or non-identical n-double scroll and Chua's oscillator with weak coupled method to each cell. Then we accomplished a complex system synchronization using GS method between the transmitter and receiver about each state in the SC-CNN.

II. COMOLEX SYSTEMS

To create a complex system, we used to the n-double scroll or non-identical n-double scroll and Chua's oscillator using the weak coupling method [8].

A. n-Double scroll circuit

In order to synthesize a complex system, we first consider Chua's circuit modified to an n-double scroll attractor. The electrical circuit for obtaining n-double scroll, according to the implementation of Arena et al. [12] is given by

$$\begin{aligned} \dot{x} &= \alpha\left[y - h(x)\right] \\ \dot{y} &= x - y - z \\ \dot{z} &= -\beta y \end{aligned} \tag{1}$$

with a piecewise linear characteristic

$$h(x) = m_{2n-1}x + \frac{1}{2}\sum_{i=1}^{2n-1}(m_{i-1} - m_i)(|x + c_i| - |x - c_i|) \tag{2}$$

consisting of 2(2n-1) breakpoints, where n is a natural number. In order to generate n double scrolls one takes $\alpha = 9$ and $\beta = 14.286$. Some special cases are:

1-double scroll

$m_0 = -\frac{1}{7}, m_1 = \frac{2}{7}, c_1 = 1$

2-double scroll

$m_0 = -\frac{1}{7}, m_1 = \frac{2}{7}, m_2 = -\frac{4}{7}, m_3 = m_1,\ c_1 = 1, c_2 = 2.15, c_3 = 3.6$

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3-double scroll

$$m_0=-\frac{1}{7},m_1=\frac{2}{7},m_2=-\frac{4}{7},$$
$$m_3=m_1,m_4=m_2,m_5=m_3,$$
$$c_1=1,c_2=2.15,c_3=3.6,c_4=8.2,c_5=13$$

The 2-double scroll attractor, 3-double scroll attractors and enlargement attractor of center area of 3-double scroll attractor by using PSpice are shown in Fig. 1, Fig. 2 and Fig. 3 respectively.

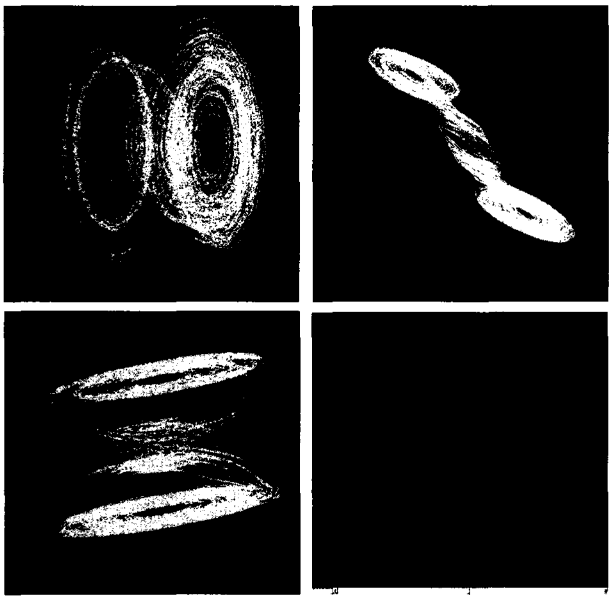


Fig. 1 2-double scroll attractor

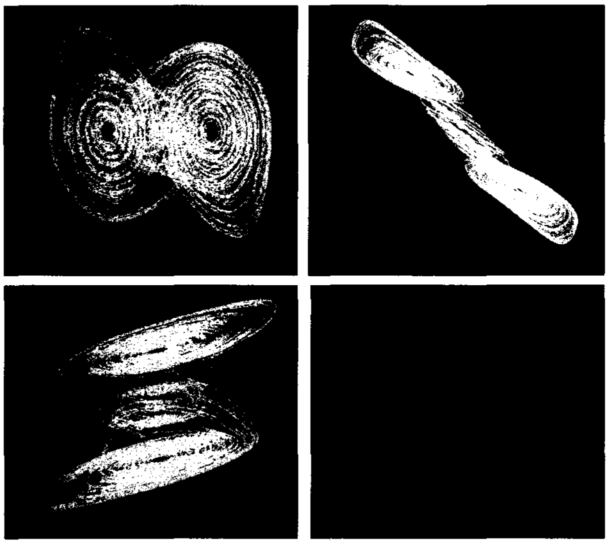
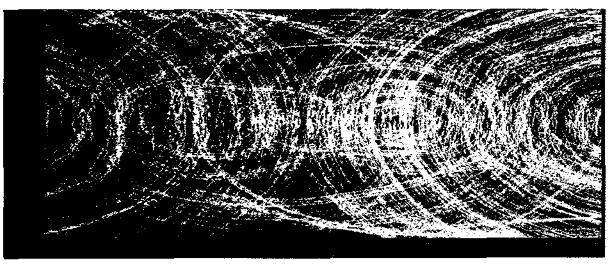


Fig. 2 3-double scroll attractor



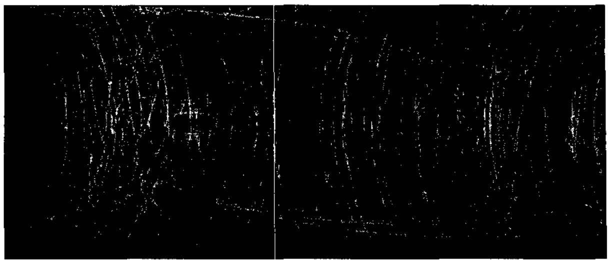


Fig. 3 Enlargement attractor of 3-double scroll attractor

B. SC-CNN model [12,13]

In [12, 13], the follow generalized cell was introduced:

$$\dot{x}_j = x_j + a_j y_j + G_o + G_s + i_j \quad (3)$$

where j is the cell index, x_j the state variable, y_j the cell output given as

$$y_j = 0.5(|x_j+1| - |x_j-1|) \quad (4)$$

where, a_j a constant parameter and i_j a threshold value. In equation (4), G_o is linear combination of the outputs and G_s is state variable of the connected cells.

Generalizing the output nonlinearity (4), the following new output PWL equation is considered

$$y_j = \frac{1}{2}\sum_{k=1}^{2n-1} n_k(|x+b_k| - |x-b_k|) \quad (5)$$

where b_k are the break point and the coefficients n_k are related to the slopes of segments.

SC-CNN cells required to generate the n-double scroll in accordance with the state equation (3) and output equation (6) are given by

$$\begin{aligned}
\dot{x}_1 &= -x_1 + a_1 y_1 + a_{12} y_2 + a_{13} y_3 + \sum_{k=1}^{3} s_{1k} x_k + i_1 \\
\dot{x}_2 &= -x_2 + a_{21} y_1 + a_2 y_2 + a_{23} y_3 + \sum_{k=1}^{3} s_{2k} x_k + i_2 \\
\dot{x}_3 &= -x_3 + a_{31} y_1 + a_{32} y_2 + a_3 y_3 + \sum_{k=1}^{3} s_{3k} x_k + i_3
\end{aligned} \quad (6)$$

where x_1, x_2, x_3 are state variables and y_1, y_2, y_3 are corresponding outputs. More details about the SC-CNN are given in reference [12, 13]

C. Complex system

To synthesize a complex system, we second consider one-dimension cellular neural network (CNN) with n-double scroll cell [8]. The following equations describe a one-dimensional CNN consisting of identical n-double cell with diffusive coupling as

$$\begin{aligned}
\dot{x}^{(j)} &= \alpha\left[y^{(j)} - h(x^{(j)})\right] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\
\dot{y}^{(j)} &= x^{(j)} - y^{(j)} - z^{(j)} \\
\dot{z}^{(j)} &= -\beta\, y^{(j)} \quad j = 1,2,\ldots,L
\end{aligned} \quad (7)$$

or

$$\dot{x}^{(j)} = \alpha\left[y^{(j)} - h(x^{(j)})\right]$$
$$\dot{y}^{(j)} = x^{(j)} - y^{(j)} - z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)})$$
$$\dot{z}^{(j)} = -\beta\, y^{(j)} \quad j = 1,2,...,L \tag{8}$$

where L denotes the number of cells. We impose the condition that $x^{(0)} = x^{(L)}$, $x^{(L+1)} = x^{(1)}$ for equation (7) and (8).

For the coupling constants, $K_0 = 0, K_j = K(j = 1,...,L-1)$ and positive diffusion coefficients D_x, D_y are chosen base on stability theory.

The complex system attractors are shown in Fig. 4.

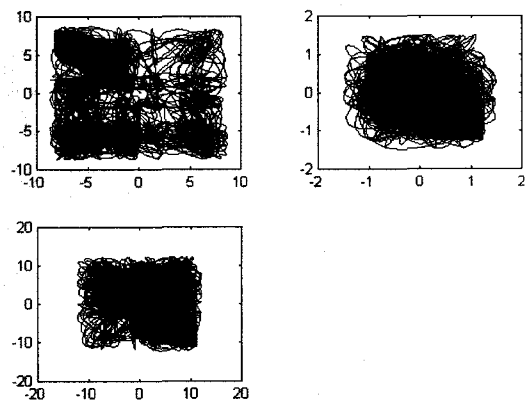


Fig. 3 Complex system attractor

III. THE SYNCHRONIZATION OF COMPXEX SYSTEM USING GENERALIZED SYNCHRONIZATION

In order to apply to generalized synchronization theory in the complex system, we compromised to state equation of dimensionless type of SC-CNN is written as follows:

The state equation of transmitter

$$\dot{x} = Ax + g(x),$$
$$g(x) = [g(x_1),0,0,g(x_4),0,0]^T \tag{9}$$
$$\dot{x}' = Ax' + g'(x') + F(x,x')$$

The state equation of receiver

$$\dot{y} = Ay + g(y),$$
$$g(y) = [g(y_1),0,0,g(y_7),0,0]^T \tag{10}$$
$$\dot{y}' = A'x' + g'(y') + F(y,y')$$

where, $x = [x_1, ...x_6]^T$, $y = [y_1, ...y_6]^T$ are state variable of 2-double scroll circuit, and $x = [x'_1, ...x'_6]^T$, $y = [y'_1, ...y'_6]^T$ are Chua's oscillator, $g(x)$ represented as equation (9) is nonlinear element. The Matrix A and A' have the following structures:

$$A = \begin{bmatrix} -\alpha & \alpha & 0 & 0 & 0 & 0 \\ 1 & -1-K & 1 & 0 & K & 0 \\ 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha & \alpha & 0 \\ 0 & K & 0 & 1 & -1-K & 1 \\ 0 & 0 & 0 & 0 & -\beta & 0 \end{bmatrix} \tag{11}$$

$$A' = \begin{bmatrix} -\alpha' & \alpha' & 0 & 0 & 0 & 0 \\ 1 & -1-K' & 1 & 0 & K' & 0 \\ 0 & -\beta' & -\gamma' & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha' & \alpha' & 0 \\ 0 & K' & 0 & 1 & -1-K' & 1 \\ 0 & 0 & 0 & 0 & -\beta' & -\gamma' \end{bmatrix} \tag{12}$$

The function vector $F(x,x')$ is used to assure the GS [14] between the 2-double scroll and Chua's oscillator by means of a proper linear feedback action. This action is characterized by the difference between selected state variables of 2-double scroll and Chua's oscillator and by a feedback gain M.

$$F(x,x') = [M(x_1 + x_4 - x'_1),0,0,$$
$$M(x_1 + x_4 - x'_1 - x'_4),0,0]^T \tag{13}$$

$$F(y,y') = [M(y_1 + y_4 - y'_1),0,0,$$
$$M(y_1 + y_4 - y'_1 - y'_4),0,0]^T \tag{14}$$

where in order to solve M, we consider equation (9) and (10), have different initial conditions. Following the auxiliary system approach, the time evolution of the differences

$$d_i = x'_i - x''_i, i = 1,2,...6 \tag{15}$$

are described by equations.

$$\dot{d}_1 = \alpha'(d_2 - d_1) - \alpha'[g(x'_1) - g(x''_1)] - Md_1$$
$$\dot{d}_2 = d_1 - d_2 + d_3 + K'(d_5 - d_2)$$
$$\dot{d}_3 = -\beta d_2 - \gamma d_3$$
$$\dot{d}_4 = \alpha'(d_5 - d_4) - \alpha'[g(x'_4) - g(x''_4)] - Md_1 - Md_4 \tag{16}$$
$$\dot{d}_5 = d_4 - d_5 + d_6 + K'(d_2 - d_5)$$
$$\dot{d}_6 = -\beta' d_5 - \gamma' d_6$$

The asymptotical stability of the response system (10) occurs if the dynamical system (16) possess a stable fixed point at the origin d=0, where $d = [d_1, d_2, d_3, d_4, d_5, d_6]^T$

After choosing the following positive definite Lyapunov function as [15]

$$V(d) = d_1^2 + \alpha' d_2^2 + \frac{\alpha'}{\beta'} d_3^2 + d_4^2 + \alpha' d_5^2 + \frac{\alpha'}{\beta'} d_6^2 \tag{17}$$

Thus the derivative of $V(d)$ along the system trajectory can be expressed as:

$$\begin{aligned}\dot{V}(d) \le\ & (-\alpha' - M + \alpha'\max\{|a|,|b|\})d_1^2 \\ & + 2\alpha' d_1 d_2 - \alpha' d_2^2 - \gamma'\frac{\alpha'}{\beta'}d_3^2 \\ & + (-\alpha' - M + \alpha'\max\{|a|,|b|\})d_2^4 \\ & + 2\alpha' d_4 d_5 - \alpha' d_5^2 - \gamma'\frac{\alpha'}{\beta'}d_6^2 - M d_1 d_4 \\ & - \alpha' K' d_2^2 - 2\alpha' K' d_2 d_5 + \alpha' K' d_5^2\end{aligned} \tag{18}$$

By considering the worst case, equation (18) can be rewritten as a quadratic form [16]

$$\dot{V}(d) = d^T \Psi d \tag{19}$$

where $\Psi \in \Re^{6\times 6}$ is a symmetric matrix given by:

$$\Psi = \begin{bmatrix} M-p & -\alpha' & 0 & 0 & 0 & 0 \\ -\alpha' & \alpha'(k'+1) & 0 & 0 & -\alpha' K' & 0 \\ 0 & 0 & -\gamma'\frac{\alpha'}{\beta'} & 0 & 0 & 0 \\ M & 0 & 0 & M-p & -\alpha' & 0 \\ 0 & -\alpha' K' & 0 & -\alpha' & \alpha'(K'+1) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\gamma'\frac{\alpha'}{\beta'} \end{bmatrix} \tag{20}$$

with $p = -\alpha' + \alpha'\max\{|a|,|b|\}$.

Equation (20) can be rewritten as:

$$\dot{V}(d) = -d_r^T \psi' d_r - \gamma'\frac{\alpha'}{\beta'}d_3^2 - \gamma'\frac{\alpha'}{\beta'}d_6^2 \tag{21}$$

Where $d_r = [d_1, d_2, d_4, d_5]^T$ is a reduced difference vector and $\Psi^1 \in \Re^{4\times 4}$ is the symmetric submatrix obtained from Ψ by deleting the third and the sixth rows and the third and sixth columns, respectively.

By imposing that the matrix Ψ^1 be positive definitive, i.e. that all principal minors be strictly positive [17], the parameter M can be derived. Finally, it can be concluded that $\dot{V}(d)$ is strictly negative for every $d_i (i = 1,2...6)$ when

$$M > \frac{\alpha'}{K'+1} + \alpha'\max\{|a|,|b|\} \tag{22}$$

The block diagram of the proposed hyper-chaos synchronization system is shown in Fig. 5 and the result of hyper-chaos synchronization is shown in Fig. 6 respectively.

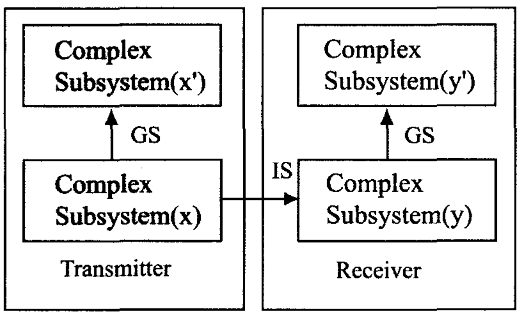
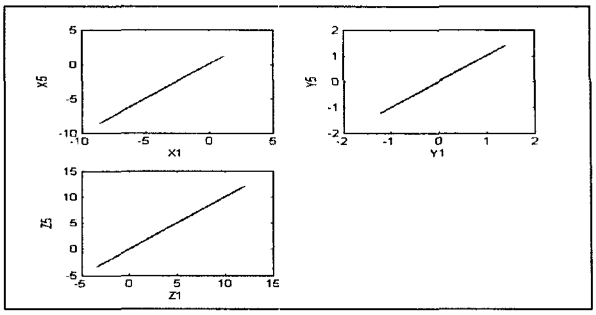
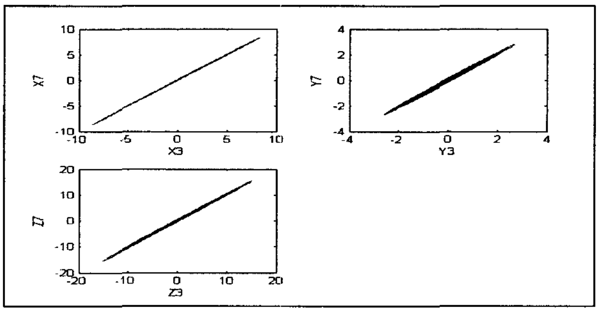


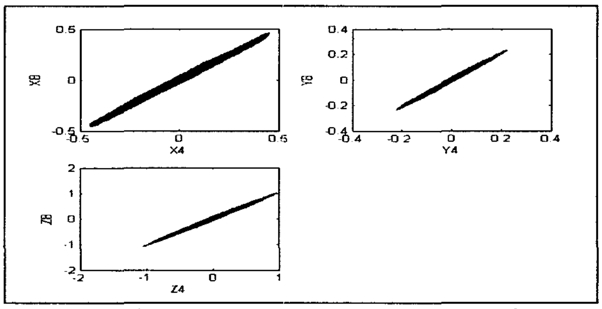
Fig. 4 The Block diagram of complex system synchronization



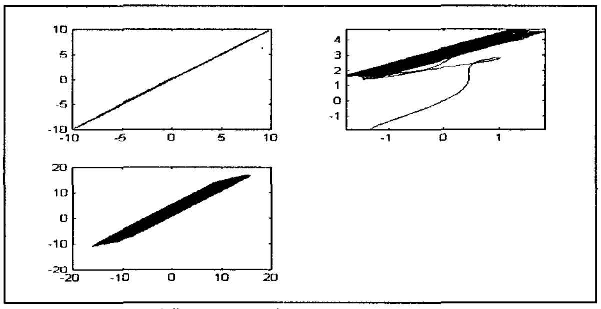
(a) Master subsystem $(x(x_1, x_2, x_3))$ of transmitter vs. master subsystem $(y(y_1, y_2, y_3))$ of receiver



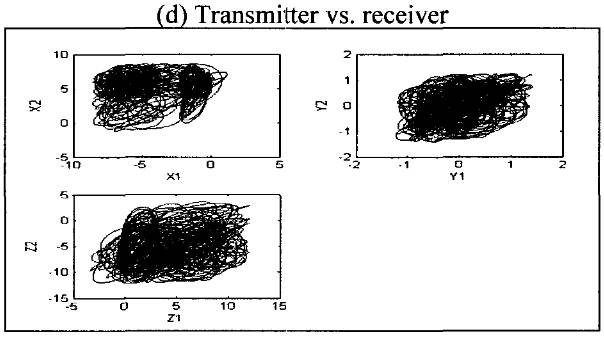
(b) Slave subsystem $(x'(x'_1, x'_2, x'_3))$ of transmitter vs. slave subsystem $(y'(y'_1, y'_2, y'_3))$ of receiver



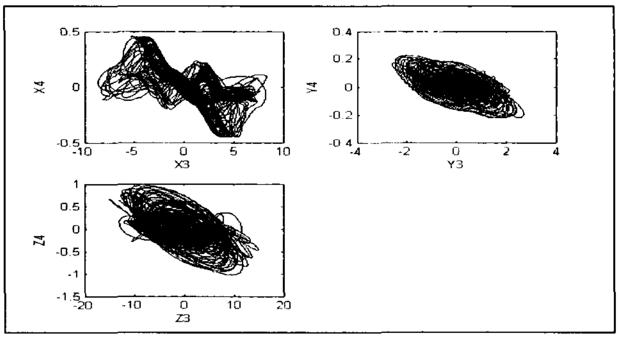
(c) Slave subsystem $(x'(x'_4, x'_5, x'_6))$ of transmitter vs. slave subsystem of $(y'(y'_4, y'_5, y'_6))$ receiver



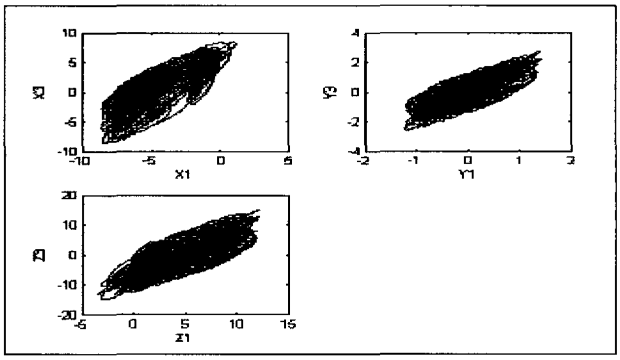
(d) Transmitter vs. receiver



(e) Phase plane of master subsystems (x_1, x_2, x_3) vs. $(y_1, y_2, y_3))$



(f) Phase plane of slave subsystem of transmitter (x'_1, x'_2, x'_3) vs. (y'_1, y'_2, y'_3)



(g) Phase plane of master subsystem (x_1, x_2, x_3) vs. slave subsystem (y'_1, y'_2, y'_3)

Fig. 5 The synchronization result

In Fig. 5, we confirmed that effective synchronization result between the transmitter and receiver in the complex system.

IV. CONCLUSIONS

In this paper, we introduced a new complex system method which is called GS(Generalized Synchronization) method using SC-Cellular Neural Network (SC-CNN) as a complex system. As a computer simulation result, we confirm GS by compare to phase plane in the transmitter and receiver about each other.

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REFERENCES

[1] L. O. Chua "Chua's circuit 10 Years Later", *Int. J. Circuit Theory and Application, vol. 22*, pp 79-305, 1994

[2] M. Itoh, H. Murakami and L. O. Chua, "Communication System Via Chaotic Modulations" *IEICE. Trans. Fundamenrtals. vol. E77-A, no. 6*, pp. 1000-1005, 1994.

[3] L. O. Chua, M. Itoh, L. Kocarev, and K. Eckert, "Chaos Synchronization in Chua's Circuit" *J. Circuit. Systems and computers, vol. 3, no. 1*, pp. 93-108, 1993.

[4] M. Itoh, K. Komeyama, A. Ikeda and L. O. Chua, "Chaos Synchronization in Coupled Chua Circuits", *IEICE. NLP. 92-51*. pp. 33-40. 1992.

[5] K. M. Short, "Unmasking a modulated chaotic communications scheme", *Int. J. Bifurcation and Chaos, vol. 6, no. 2*, pp. 367-375, 1996.

[6] K. M. Cuomo, "Synthesizing Self - Synchronizing Chaotic Arrays", *Int. J.Bifurcation and Chaos, vol. 4, no. 3*, pp. 727-736, 1993.

[7] L. Kocarev, "Chaos-based cryptography: A brief overview", *IEEE, Vol.* pp. 7-21. 2001.

[8] J.A.K.Suykens, "n-Double Scroll Hypercubes in 1-D CNNs" *Int. J. Bifurcation and Chaos, vol. 7, no. 8*, pp. 1873-1885, 1997.

[9] L. M. Pecora and T. L. Carroll "Synchronization in Chaotic System" *Phy. Rev. Lett., vol. 64, no. 8*, pp. 821-824, 1990.

[10] L. Kocarev, K. S. Halle, K. Eckert and L. O. Chua, "Expermental Demonstration of Secure Communication via Chaotic Synchronization" *Int. J. Bifurcation and Chaos, vol. 2, no. 3*, pp. 709-713, 1992.

[11] K. S. Halle, C. W. Wu, M. Itoh and L. O. Chua, "Spread Spectrum communication through modulation of chaos" *Int. J. Bifurcation and Chaos, vol. 3, no. 2*, pp. 469-477, 1993.

[12] P.Arena, P.Baglio, F.Fortuna & G.Manganaro, "Generation of n-double scrolls via cellular neural networks", Int. J. Circuit Theory Appl, 24, 241-252, 1996.

[13] P. Arena, S. Baglio, L. Fortuna and G. Maganaro, "Chua's circuit can be generated by CNN cell", *IEEE Trans. Circuit and Systems I, CAS-42*, pp. 123-125. 1995.

[14] L. Kocarav, L & U. Parlitz, "Generalized synchronization, predictability and equivalence of unidirectionally coupled dynamical systems", *Phys. Rev. Lett, vol. 76, no.11*, pp. 1816-1819, 1996.

[15] M. Brucoli, D. Cafagna, L. Carnimeo & G. Grassi, "An efficient technique for signal masking using synchronized hyperchaos circuits", *Proc. 5th Int. workshop on Nonlinear Dynamics of Electronic Systems(NDES '97)*, Moscow, Russia, June 26-27, pp. 229-232, 1997.

[16] J.A.k. Suyken, P.F. Curran & L.O. Chua, "Master-slave synchronization using dynamic output feedback", *Int. J. Bifurcation and Chaos, vol. 7, no. 3*, pp. 671-679, 1997.

[17] J.J. Slotine & W.Li, *"Applied Nonlinear Control"*, Prentice-Hall, NJ, 1991.

[18] Y.C. Bae, J.W. Kim,H.H. Song,Y.H. Kim, "A study on generalized synchronization in hyper-chaos with SC-CNN", Int. J. Maritime Information and Communication Science, vol. 1, no. 4, pp. 217-222, 2003.

[19] Y.C. Bae, J.W. Kim,H.H. Song,Y.H. Kim, "A study on secure communication in hyper-chaos with SC-CNN using embedding method", Int. J. Maritime Information and Communication Science vol. 1, no. 4, pp. 223-228, 2003.

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