An Efficient Detection Algorithm for Quasi-Orthogonal Space-Time Block Code with Four Transmit Antennas

Minh-Tuan Le, Van-Su Pham, Mai Linh and **Gi-Wan Yoon** *Member, KIMICS*

Abstract—This paper proposes an efficient detection algorithm, which is composed of an interference nulling- and cancelling-based detection algorithm and a maximum likelihood (ML) detection algorithm having reduced numbers of signal points to be tested, for the quasi-orthogonal space-time code with four transmit antennas. When high-level modulation schemes are employed, the algorithm enables the quasi-orthogonal code to achieve near ML performance with a significant reduction in the computational load.

I. INTRODUCTION

Antenna diversity is a practical and effective technique for combating multipath fading in scattering environment [1]. Classically, antenna diversity is accomplished through the use of multiple antennas together with appropriate combining schemes at the receiver. This, however, will result in large and expensive mobile terminals. Therefore, it is of much economy to apply antenna diversity to the base stations, *i.e.*, transmit diversity, since they often serve a large number of subscribers.

One attractive method for realizing transmit diversity is to employ space-time block codes (STBCs) based on orthogonal designs [2]-[3] due to their full diversity and their simple ML decoding algorithm. Unfortunately, one cannot achieve full rate and, at the same time, full diversity with STBCs as the numbers of transmit antennas greater than 2 [3]. Recently, a quasi-orthogonal space-time block code was developed for 4 transmit antennas in [4]-[5] and independently devised in [6]. The code was shown to provide full rate with half of the maximum possible diversity. By applying the phase shifting to the constellations of the transmitted symbols, it is possible for the quasi-orthogonal space-time code to achieve full diversity [7]. Nonetheless, one drawback associated with the quasi-orthogonal code is that its decoder has to process pairs of transmitted symbols instead of single symbols. Consequently, the detection complexity of the code will noticeably increase as high-level modulation schemes, such as 16-quadrature amplitude modulation (QAM), 64-QAM, and 256-QAM, are utilized due to the exhaustive search over large signal constellations.

In this paper, we present an enhanced decoding algorithm for the quasi-orthogonal code, which allows the tradeoff between performance and detection complexity of the quasi-orthogonal code to be flexibly made for any modulation technique, whereby making the quasi-orthogonal code more practical. The proposed algorithm is performed in two steps. In the first step, the code is detected by an interference nulling- and cancelling based decoder such as the zero-forcing (ZF) –based decoder [8]-[9] or QR-decomposition-based decoder [9]-[10]. Nevertheless, due to the influence of the interference nulling and cancelling processes, there is degradation in the performance of the code as compared to the case it is detected by a ML decoder. In order to further enhance the system performance after the first step, ML decoding is carried out in the second step. Unlike the conventional ML decoding in [6] and [7] where the pairs of transmitted signals that minimize ML decision metrics are found among all possible signal pairs, *i.e.*, exhaustive search over the transmitted signal constellations, in the proposed algorithm the pairs of transmitted symbols, which minimize the ML decision metrics, are searched over given numbers of signal points chosen from the transmitted signal constellations by using the results obtained in the first step and the discrete property of the transmitted signal constellations. As a consequence, the quasi-orthogonal code is able to achieve near ML performance for high-level modulation schemes with a significant decrease in the computational load.

The rest of the paper is organized as follows. Section II presents the system model and the ML decision decoders for the quasi-orthogonal code. Section III proposes the enhanced detection algorithm. Simulation results and discussions are presented in Section IV. The final section will be our conclusions.

II. SYSTEM MODEL AND THE QUASI-ORTHOGONAL CODE'S ML DECODERS

A. System Model

It is assumed that there are n_T transmit and n_R receive antennas. A space-time block code encodes N transmitted symbols, $(s_1, s_2, \dots, s_N)$, into a transmission matrix $\mathbf{S}$ of size $n_T \times L$, where L is the block length. Thus, the transmission rate is $R = N/L$. At time t, $t = 1,2,\dots,L$, the nth element of the tth column of $\mathbf{S}$, $s_{n,t}$, is transmitted from antenna n, $n = 1,2,\dots,n_T$.

The channel is assumed to be quasi-static, *i.e.*, it remains constant during some block of arbitrary length

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and changes from one block to the other. The path gain from transmit antenna n to the receive antenna m, $h_{n,m}$, is modeled as the sample of a zero-mean independent complex Gaussian random variable with equal variance of 0.5 per complex dimension.

At time t, the signal received at antenna m, $r_{t,m}$, is given by:

$$r_{t,m} = \sum_{n=1}^{n_T} h_{n,m} s_{n,t} + w_{t,m} \tag{1}$$

where $w_{t,m}$ are the noise samples modeled as independent samples of a zero-mean complex Gaussian random variable with noise variance σ^2.

The relationship between transmitted and received signals can also be expressed in the following linear form:

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w} \tag{2}$$

where $\mathbf{r}$ is the $M \times 1$ equivalent received signal vector, $\mathbf{H}$ the $M \times N$ equivalent channel matrix, $\mathbf{s}$ the $N \times 1$ vector containing N transmitted symbols encoded in the transmission matrix $\mathbf{S}$, $\mathbf{w}$ the $M \times 1$ equivalent noise vector at the receiver, and $M = n_R L$.

Let us assume that perfect channel state information is available, the receiver choose $(\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_N)$ from the transmission constellation that minimize the following decision metric:

$$\sum_{m=1}^{n_R} \sum_{t=1}^{L} d^2\left(r_{t,m}, \sum_{n=1}^{n_T} h_{n,m} \hat{s}_{n,t} \right) \tag{3}$$

The minimization of (3) results in a maximum-likelihood decoding, which can alternatively be represented by:

$$(\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_N) = \underset{(\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_N) \in C}{\arg\min} \sum_{m=1}^{n_R} \sum_{t=1}^{L} d^2\left(r_{t,m}, \sum_{n=1}^{n_T} h_{n,m} \hat{s}_{n,t} \right) \tag{4}$$

where C denotes the transmission constellation, $d^2(a,b)$ is the squared Euclidean distance between signals a and b calculated by:

$$d^2(a,b) = |a-b|^2 = (a-b)(a^* - b^*) \tag{5}$$

B. Quasi-orthogonal Space-Time Code for Four Transmit Antennas and Its ML Decoders

As shown in [6], there exist a number of structures for the quasi-orthogonal space-time code with four transmit antennas. In the paper we consider the quasi-orthogonal code given by the following transmission matrix:

$$\mathbf{S} = \begin{bmatrix} s_1 & -s_2^* & s_3 & -s_4^* \\ s_2 & s_1^* & s_4 & s_3^* \\ s_3 & -s_4^* & s_1 & -s_2^* \\ s_4 & s_3^* & s_2 & s_1^* \end{bmatrix} \tag{6}$$

Here, the vertical and horizontal axes respectively represent the space and time domains of the quasi-orthogonal code.

For the quasi-orthogonal code, the ML decoder described by (4) reduces to the two independent ML decoders, which jointly detect two pairs of transmitted symbols, (s_1, s_3) and (s_2, s_4), given by:

$$(\hat{s}_1, \hat{s}_3) = \underset{(\hat{s}_1, \hat{s}_3) \in C}{\arg\min} \left[A_{13}\left(|\hat{s}_1|^2 + |\hat{s}_3|^2\right) + \mathrm{Re}(B_{13}\hat{s}_1) + \mathrm{Re}(C_{13}\hat{s}_3) + D_{13}\,\mathrm{Re}\left(\hat{s}_1 \hat{s}_3^*\right) \right] \tag{7}$$

$$(\hat{s}_2, \hat{s}_4) = \underset{(\hat{s}_2, \hat{s}_4) \in C}{\arg\min} \left[A_{24}\left(|\hat{s}_2|^2 + |\hat{s}_4|^2\right) + \mathrm{Re}(B_{24}\hat{s}_2) + \mathrm{Re}(C_{24}\hat{s}_4) + D_{24}\,\mathrm{Re}\left(\hat{s}_2 \hat{s}_4^*\right) \right] \tag{8}$$

where,

$$A_{13} = A_{24} = \sum_{m=1}^{n_R} \sum_{n=1}^{4} |h_{n,m}|^2 \tag{9}$$

$$B_{13} = \sum_{m=1}^{n_R} 2\left(-h_{1,m} r_{1,m}^* - h_{2,m}^* r_{2,m} - h_{3,m} r_{3,m}^* - h_{4,m}^* r_{4,m} \right) \tag{10}$$

$$B_{24} = \sum_{m=1}^{n_R} 2\left(-h_{2,m} r_{1,m}^* + h_{1,m}^* r_{2,m} - h_{4,m} r_{3,m}^* + h_{3,m}^* r_{4,m} \right) \tag{11}$$

$$C_{13} = \sum_{m=1}^{n_R} 2\left(-h_{3,m} r_{1,m}^* - h_{4,m}^* r_{2,m} - h_{1,m} r_{3,m}^* - h_{2,m}^* r_{4,m} \right) \tag{12}$$

$$C_{24} = \sum_{m=1}^{n_R} 2\left(-h_{4,m} r_{1,m}^* + h_{3,m}^* r_{2,m} - h_{2,m} r_{3,m}^* + h_{1,m}^* r_{4,m} \right) \tag{13}$$

$$D_{13} = D_{24} = \sum_{m=1}^{n_R} 4\,\mathrm{Re}\left(h_{1,m} h_{3,m}^* + h_{2,m} h_{4,m}^* \right) \tag{14}$$

and $\mathrm{Re}(a)$ is the real part of a.

III. ENHANCED DETECTION ALGORITHM FOR THE QUASI-ORTHOGONAL SPACE-TIME CODE

Assuming that a 64-QAM modulation scheme with the constellation illustrated in Fig. 1 is adopted. As shown in Equations (7) and (8), the two pairs of transmitted symbols are detected through exhaustive search over the signal modulation constellation. Consequently, the detection complexity can be very high, in the order of 16×64^2 real multiplications. This section presents a detection method that can significantly reduce the detection complexity of the ML decoders in (7) and (8) without noticeably sacrificing the system performance. Moreover, the proposed method can offer a flexible compromise between performance improvement and complexity for any transmission constellation. The main idea behind the proposed algorithm is to decrease the number of signal points to be tested in the ML decoders given by (7) and (8) via the use of a QR-decomposition-based decoder and the discrete nature of the transmission constellations. The proposed detection algorithm is described below.

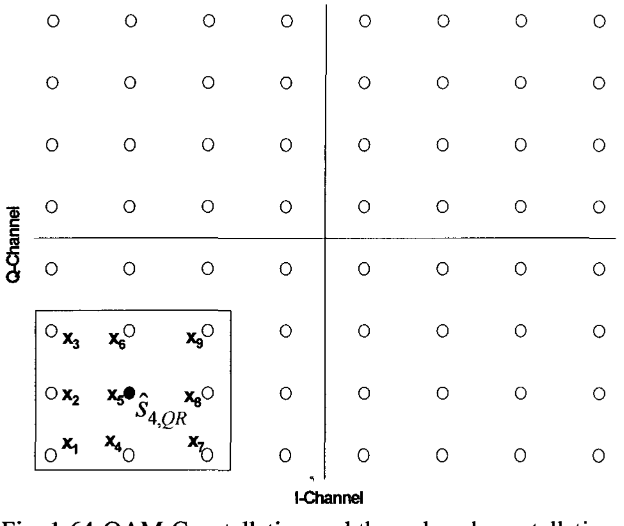

Fig. 1 64-QAM Constellation and the reduced constellation with constraint size $S_c = 9$ signal points

At first, from the linear relationship between the transmitted and received signals as in (2), QR decomposition is applied to detect the transmitted signals. In this step, we are able to obtain 4 estimates of the 4 transmitted symbols, $(\hat{s}_{1,QR}, \hat{s}_{2,QR}, \hat{s}_{3,QR}, \hat{s}_{4,QR})$, with low complexity. The well-known consequences of using interference nulling and cancelling techniques for signal detection are the reduction of diversity orders of the first detected symbols and the error propagation. Hence it is of high possibility that the four estimates $(\hat{s}_{1,QR}, \hat{s}_{2,QR}, \hat{s}_{3,QR}, \hat{s}_{4,QR})$ are not the desired transmitted symbols. In other words, there is a high probability of incorrect detection when QR decomposition is employed. In order to further improve system performance, ML decoding with reduced constellations is used. Illustrated in Fig. 1 is the determination of the reduced constellation for transmitted symbol s_4 based on the estimate $\hat{s}_{4,QR}$. Assume that $\hat{s}_{4,QR}$ is an incorrect estimate of s_4, then the correct one must lie somewhere close to $\hat{s}_{4,QR}$ in terms of Euclidean distance. The reason for this argument is that due to fading and noise the signal points close to $\hat{s}_{4,QR}$, including the desired signal point, will have high probability of being detected as $\hat{s}_{4,QR}$. Therefore, instead of searching over the entire constellation, the ML decoders of the quasi-orthogonal code now have to search only in a given number of signal points closest to $\hat{s}_{4,QR}$. The number of signal points to be tested is called the "*constraint size*" and denoted by S_c. In addition, the signal points determined by the constraint size define the reduced constellation. The reduced constellation for the transmitted symbol s_i, $i = 1,2,3,4$, is mathematically defined as:

$$C_i = \{x_k, k = 1,2,\ldots,S_c : d^2(x_k, \hat{s}_{i,QR}) \text{ are of smallest values}\} \quad (15)$$

The proposed detection algorithm can be summarized as follows.

1. *Using the QR-decomposition-based detection algorithm to obtain the estimates* $(\hat{s}_{1,QR}, \hat{s}_{2,QR}, \hat{s}_{3,QR}, \hat{s}_{4,QR})$
2. *Determining the constraint size* S_c
3. *Determining the reduced constellations* C_i, $i = 1,2,3,4$, *based on the estimates* $(\hat{s}_{1,QR}, \hat{s}_{2,QR}, \hat{s}_{3,QR}, \hat{s}_{4,QR})$ *as in Equation (15)*
4. *Performing ML decoding to retrieve the transmitted symbols as follows:*

$$(\hat{s}_1, \hat{s}_3) = \arg\min_{\hat{s}_1 \in C_1, \hat{s}_3 \in C_3} \left[A_{13}\left(|\hat{s}_1|^2 + |\hat{s}_3|^2\right) + \mathrm{Re}(B_{13}\hat{s}_1) + \mathrm{Re}(C_{13}\hat{s}_3) + D_{13}\,\mathrm{Re}(\hat{s}_1\hat{s}_3^*)\right]$$

$$(\hat{s}_2, \hat{s}_4) = \arg\min_{\hat{s}_2 \in C_2, \hat{s}_4 \in C_4} \left[A_{24}\left(|\hat{s}_2|^2 + |\hat{s}_4|^2\right) + \mathrm{Re}(B_{24}\hat{s}_2) + \mathrm{Re}(C_{24}\hat{s}_4) + D_{24}\,\mathrm{Re}(\hat{s}_2\hat{s}_4^*)\right]$$

IV. SIMULATION RESULTS AND DISCUSSIONS

As shown in [6]-[7], the quasi-orthogonal code does not achieve full diversity unless the constellations of the symbols forming the code are phase-shifted. Analogous to [7], in this paper we apply the rotation to the constellations of the transmitted symbols s_3 and s_4. The optimum rotation angle for 16-QAM is equal to $\phi_{opt,16QAM} = \pi/6$ [7]. Fig. 2 provides simulation results when the quasi-orthogonal code with 16-QAM-modulated symbols is detected by a QR-decomposition-based algorithm, the ML decoders with exhaustive search given in (7) and (8), and the proposed algorithm with different values of S_c. One can observe that the proposed algorithm significantly outperforms the QR-decomposition-based algorithm for both values of the constraint size. In addition, with $S_c = 9$there is almost no difference between the performance of the proposed algorithm and that of the ML decoders with exhaustive search.

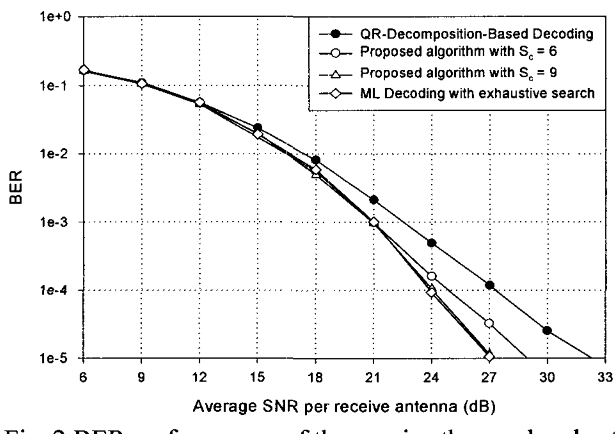

Fig. 2 BER performances of the quasi-orthogonal code at 4bits/s/Hz when different detection algorithms are employed, $n_R = 1$

When 64-QAM and 256-QAM modulation schemes are utilized, using the method of minimum distance in [7] to find the optimum rotation angles can be very time consuming. In the paper, optimum rotation angles for the two modulation schemes are found by consider the bit error rates as functions of the rotation angles shown in Fig. 3. The proposed algorithm with a constraint size of 9 is used for decoding. The average SNR per receive antenna is fixed at 30dB and 36dB for 64-QAM and 256-QAM, respectively.

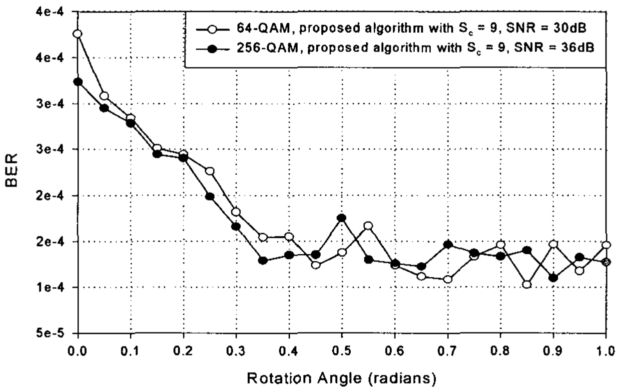

Fig. 3 BER versus rotation angle for the quasi-orthogonal code with 64-QAM- and 256-QAM-modulated symbols, $n_R = 1$

From Fig. 3, the optimum rotation angles for 64-QAM and 256-QAM are chosen to be $\phi_{opt,64QAM} = 0.85$ and $\phi_{opt,256QAM} = 0.9$. With those optimum rotation angles, the BER-versus-SNR curves for the quasi-orthogonal code employing 64-QAM and 256-QAM modulation schemes detected by the proposed algorithm are respectively shown in Fig. 4 and Fig. 5.

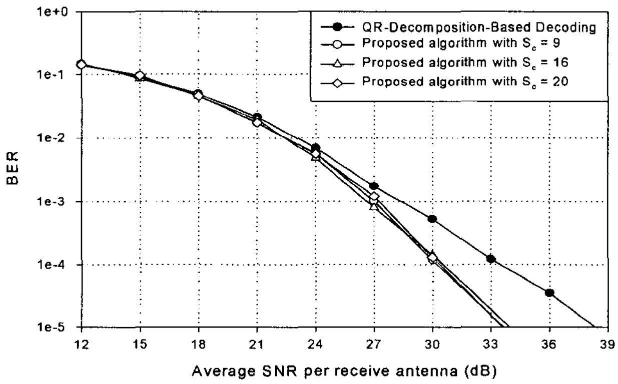

Fig. 4 BER performance of the quasi-orthogonal code at 6bits/s/Hz when detected by QR decomposition and the proposed algorithm with different constraint sizes, $n_R = 1$

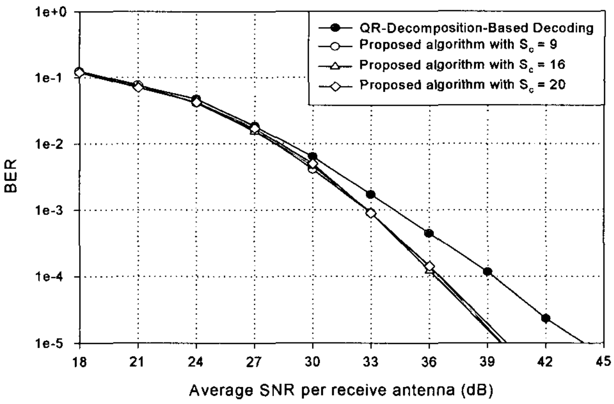

Fig. 5 BER performance of the quasi-orthogonal code at 8bits/s/Hz when detected by QR decomposition and the proposed algorithm with different constraint sizes, $n_R = 1$

From Fig. 4 and Fig. 5, we can witness that the proposed algorithm offer a remarkable improvement in the performance of the quasi-orthogonal code as compared to the QR-decomposition-based algorithm (around 4dB improvement at BER of 10^{-5}). Furthermore, a little performance improvement can be gained as the constraint size of the proposed algorithm is greater than 9. Thus, for the three modulation techniques, namely, 16-QAM, 64-QAM and 256-QAM, $S_c = 9$ seems to be the most appropriate value for the proposed algorithm to achieve high performance while maintaining reasonable complexity.

The complexities of the proposed algorithm, the QR-decomposition-based algorithm, and the ML decoding algorithm with exhaustive search are roughly compared in terms of number of real multiplications and additions in Table 1 when 4 transmit and 1 receive antenna are deployed. Here, we have assumed that a real division and a square root are respectively equal to 8 and 30 real multiplications.

Table 1 shows that QR-decomposition-based algorithm has the least complexity, and obviously lowest performance While having comparable performance, our proposed algorithm offers noticeably lower complexity than does the ML decoding algorithm with exhaustive search. The complexity of our proposed algorithm can be further reduced by decreasing the constraint size S_c, yet at the cost of performance degradation.

V. CONCLUSIONS

This paper proposes an enhanced detection algorithm for the quasi-orthogonal code with 4 transmit antennas, which enables the quasi-orthogonal code to achieve near

Table 1 Complexity comparison between the three decoding algorithms under consideration, $n_T = 4$, $n_R = 1$, burst length of 4 symbol durations

Additions/ Multiplications	QR-decomposition-based algorithm	ML decoding algorithm with exhaustive search	Proposed algorithm, $S_c = 9$
16-QAM	204/332	3430/4161	1359/1693
64-QAM	236/332	53350/65601	1391/1693
256-QAM	300/332	852070/1048641	1455/1693

ML performance with moderate complexity for any modulation scheme. Moreover, it is feasible for the proposed algorithm to make a compromise between performance and complexity by varying the constraint size. Consequently, our proposed algorithm is more practical for implementation as compared to the conventional ML decoding algorithm with exhaustive search.

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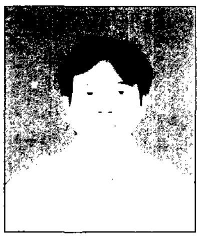

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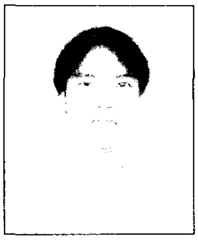

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