

Performance analysis of MRC Diversity system for Detection of M-ary Signals on Rician Fading channel

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SUMMARY—The purpose of this paper is to theoretically analysis of performance of MRC diversity for detection of M-ary signals on the Rician fading channels. To do it, the difference between the approximation to bit error probability and the exact bit error rate (BER) in evaluating the performance of MPSK signals is investigated by numerical analyses. In addition, the performance evaluation of MDPSK signals in the independent branch diversity is performed by an integral-form expression in case of when MDPSK signal in MRC system is experiencing the Rician fading.

Key words—MRC, Ricain fading, MPSK, MDPSK

I. INTRODUCTION

The statistical properties of mobile radio environments can be often specified by three propagation effects: 1) short-term fading 2) long-term fading 3) propagation path loss. Especially, the scattering mechanism only results in numerous reflected components under short-term fading. The Rayleigh model is used to characterize the short-term fading in small geographical areas and sometimes does not account for large scale effects like shadowing by building and hills. The Rician model can be obtained from the direct wave and its scattering components [1]. By modeling the channel as a Rician fading channel, a result is obtained that is valid in the limit of the large direct-to-diffuse power ratios for no fading channels and the small direct-to-diffuse power ratios for Rayleigh environments as well as for the general case that neither the direct nor the diffuse components of the signal are negligible. When Rician factor K is 0, the error performances lead to those of Rayleigh fading model in a cellular system.

This paper is evaluated the performance of MPSK signals in a MRC diversity receiving system on Rician fading channels. The evaluated result shows the approximation performance of MPSK over the slow and flat fading channels on the additive white Gaussian noise (AWGN) and an upper bound on the bit error probability of the large values of M signal waveforms. In addition, this

paper shows the effect of increasing M between the approximation of the closed-form and the exact BER of the integral-form. Also this paper presents the reception performance of MRC diversity system for MDPSK signals on the Rician fading channel. The analytical results give useful information to wireless system designer to design a radio system under the Rician fading channel.

II. ERROR RATE ANALYSIS

We assume that there are L diversity branches in the frequency-nonselective and slow fading, carrying the same information-bearing signal. The fading processes among L diversity branches are assumed to be mutually statistically independent. The signal in each channel is corrupted by an additive zero-mean white Gaussian noise. Assuming that each branch has equal fading parameter and average signal-to-noise γ_0 (SNR), the conditional probability density function (PDF) of the instantaneous γ SNR in the Rician fading channel is [3]

$$F(\gamma) = \left(\frac{K+1}{\gamma_0}\right)^{\frac{L+1}{2}} \left(\frac{\gamma}{KL}\right)^{\frac{L-1}{2}} \exp\left[-KL - \frac{(K+1)\gamma}{\gamma_0}\right] \cdot I_{L-1}\left(2\sqrt{\frac{K(K+1)L\gamma}{\gamma_0}}\right), \gamma \geq 0 \quad (1)$$

where $I_{L-1}(\cdot)$ is the (L-1)th-order modified Bessel function of the first kind for statistically identical diversity branches.

The exact SER of coherent MPSK under a no fading channel can be represented as

$$P_{s,exact,MPSK} = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\pi}{M} \exp\left[-\gamma \sin^2\left(\frac{\pi}{M}\right) \sec^2 \theta\right] d\theta \quad (2)$$

The exact integral-expression performance in the fading channels is represented by averaging (2) over underlying fading SNR as follow:

$$P_{s,exact,MPSK,MRC} = \int_0^{\infty} P_{s,exact,MPSK} f(\gamma) d\gamma \quad (3)$$

$$= \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\frac{K+1}{K+1+\gamma_0 \sin^2\left(\frac{\pi}{M}\right) \sec^2 \theta} \right]^L \exp\left[-\frac{KL\gamma_0 \sin^2\left(\frac{\pi}{M}\right) \sec^2 \theta}{K+1+\gamma_0 \sin^2\left(\frac{\pi}{M}\right) \sec^2 \theta}\right] d\theta$$

For the special case of, we can thus find that the result of (3) corresponds to that of [2, Eq. (12)], when there is no diversity branch.

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The approximation performance of coherent MPSK signals on the symbol error probability for larger may be given by]

$$P_{s,MPSK} = \text{erfc}\left(\sqrt{\gamma} \sin \frac{\pi}{M}\right) \tag{4}$$

where $\text{erfc}(\cdot)$ is the error function

Consequently, we can express the closed-form symbol error rate approximation of MPSK signals with MRC diversity reception in a Rician fading channel as

$$P_{s,exact,MPSK,MRC} = \int_0^\infty P_{s,approx,MPSK} f(\gamma) d\gamma \tag{5}$$

$$= \frac{1}{\sqrt{\pi}} e^{-K/L} \sum_{l=0}^\infty \frac{1}{\Gamma(l+1)} (KL)^l \frac{\Gamma(L+l+1/2)}{(1/4)^{L+l}} \frac{\Gamma(L+l+1)}{\Gamma(2L+2l+1)}$$

$$\bullet \beta(K) \left\{ \frac{1}{\beta(K)} - \sum_{l=0}^{L+l-1} \left(\frac{2l}{t} \right) \left[\frac{(1+K)/4}{\mu^2 + 1 + K} \right] \right\}$$

where $\beta(K) = \frac{\mu}{\sqrt{\mu^2 + 1 + K}}$ and $\mu = \sqrt{\gamma_0} \sin\left(\frac{\pi}{M}\right)$

For, we can observe that the result of (5) is equivalent to [6, Eq. (A.5)].

For equiprobable orthogonal signals, the average probability of bit error P_b , is related to the average probability of symbol error P_s , by

$$P_b = \frac{M}{2(M-1)} P_s \tag{6}$$

where the per bit average SNR γ_b is related to the per symbol average SNR by

$$\gamma_b = \frac{L}{\log_2 M} \gamma_0 \tag{7}$$

When MDPSK signals experience no fading, the expression for the probability of symbol error is represented as

$$P_{s,MDPSK} = \frac{\sin \frac{\pi}{M}}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\exp\left[-\gamma(1 - \cos \frac{\pi}{M} \cos \theta)\right]}{1 - \cos \frac{\pi}{M} \cos \theta} d\theta \tag{8}$$

Averaging (8) over underlying fading SNR, we can show that the average SER in receiving MDPSK signals with MRC on Rician fading channels is

$$P_{s,MDPSK,MRC} = \int_0^\infty P_{s,MDPSK} f(\gamma) d\gamma \tag{9}$$

where $P_{s,MDPSK,MRC}$ is the average SER of MDPSK signal under the Rician fading model. Next, substituting (1) and (8) into (9), we find the integral-form symbol error probability under the Rician fading model to be

$$P_{s,MDPSK,MRC} = \frac{\sin \frac{\pi}{M}}{2\pi} \frac{(K+1)^L}{\gamma_0^L} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\left(1 + \frac{1+K}{\gamma_0} - \cos \frac{\pi}{M} \cos \theta\right)^L} 1 - \cos \frac{\pi}{M} \cos \theta \tag{10}$$

$$\bullet \exp \left[-KL + \frac{\gamma_0}{1 + \frac{1+K}{\gamma_0} - \cos \frac{\pi}{M} \cos \theta} \right] d\theta, L \geq 1$$

This can be converted to the BER, using (6) and (7). For the special case of Rician factor $K=0$, we can observe that the result of (10) for DPSK is equivalent to the result of [7, Eq. (B1)] for Nakagami fading index $m=1$, when there is no diversity branch. We can also find that the result of (10) for $L=1$ corresponds to [2, Eq. (5)].

III. NUMERICAL RESULTS

For the particular case when is 1, the results for coherent MPSK using the approximation and the exact BER in the fading channels are plotted with alphabet sizes $M=4, 8$ in Figs. 1 and 2, respectively. Given average SNR per bit, the performance of the exact BER almost becomes close to the approximation without the relation between the fading parameter.

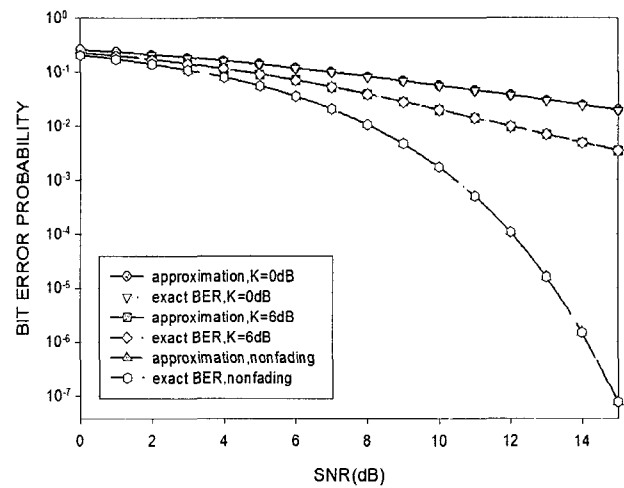


Fig. 1 Coherent MPSK performance comparison of the approximation

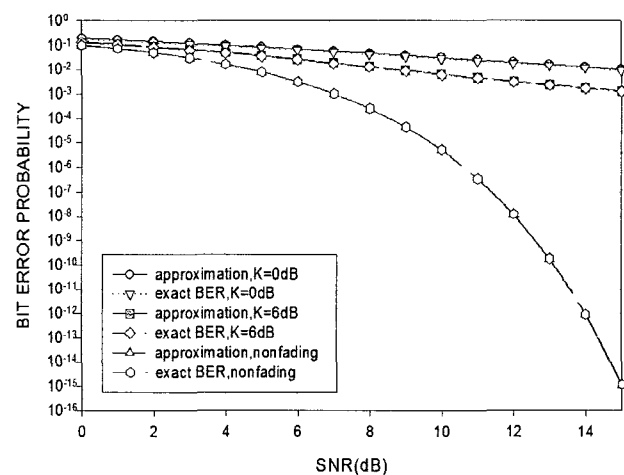


Fig. 2 Coherent MPSK performance comparison of the approximation and the exact BER for $M=8$

Next, the selected numerical results to show the performance of M-ary modulation systems in Rician fading channels with arbitrary fading parameters are presented in the presence of MRC diversity reception.

Let us assume that the performance of coherent MPSK in the fading channels has, first of all, an approximation.

Fig. 3 shows MPSK and MDPSK performance comparison of the Rician channel for $K=6$ dB and $M=8$. Given the bit error probability of M-ary modulation systems, the SNR per bit is more deviated as the number of diversity branches decreases. It is noted that the performance of $L=1$ in MPSK signals is rather better than that of $L=3$ in MDPSK signals in a practical SNR range. Next in Fig. 4, the average SER performances are plotted against the order of diversity with $K=12$ dB and $M=16$. The performance is improved very restrictedly in Rician fading conditions with increasing the diversity branches.

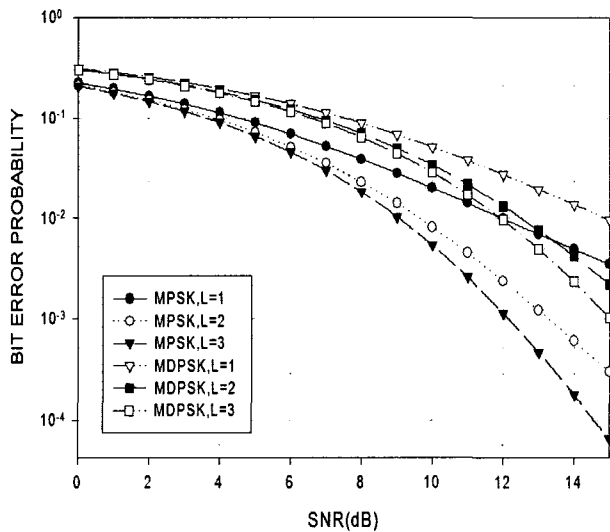


Fig. 3 Coherent MPSK performance comparison of the approximation and the exact BER for $M=8$

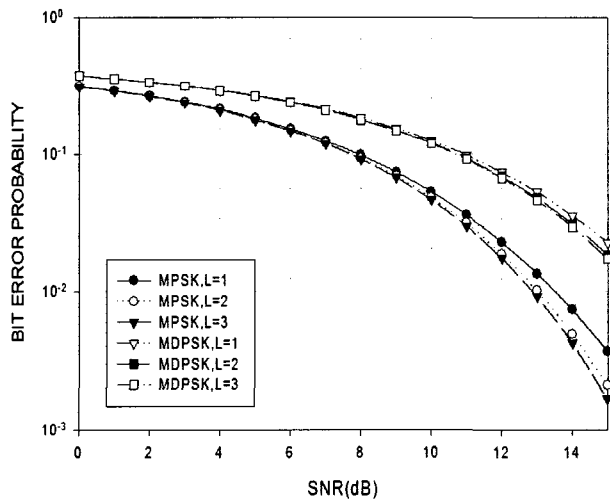


Fig. 4 Error performance comparisons of MPSK and MDPSK signals with MRC diversity receiver structures in Rician fading channels. These parameters for this figure are $M=16$, $L=1,2,3$, and $K=12$ dB

IV. CONCLUSION

The performance for MPSK and MDPSK signals in Rician fading channel environments has been evaluated. The approximations to a bit error probability and the

exact SER for coherent MPSK in the fading channels have been presented, respectively.

The integral-form performances for MDPSK systems employing the multichannel MRC diversity in the presence of Rician-distributed slow and nonselective fading have been analyzed. It is expected result as the diversity branches L increase, the fading depth decreases. This result also shows that the restricted performance gain is achievable with increasing the number of the diversity branches, L . We can predict that the performance improvement saturates as L increases more than 4.

The results of the present works are sufficiently general in offering a convenient method to evaluate the performance of several current M-ary modulation systems that operate on channels with a wide variety of fading conditions in wireless personal communications.

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