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A Study on Proportion-Generating System Based on Dom Hans van der Laan's Proportion Theory

돔 한스 반 데어 란의 비례론에 기초한 비례생성 시스템에 관한 연구

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Abstract

비트루비우스(Vitruvius)에서 벤츄리(Venturi)에 이르기까지 많은 건축사가(建築史家) 또는 건축가(建築家)들이 건축원리에 대한 각자의 규범적 태도를 글로써 밝혀왔다. 이러한 규범적 내용 중 비례이론은 서양건축에 있어 건축미(建築美)의 비밀로 간주되어, 조화로운 건축물이 꼭 갖추어야 할 덕목중의 하나로 인식되었다. 현대건축에서 비교적 근대 비례시스템이라 불릴 수 있는 것으로 르 꼬르뷔지에의 모뒬로르와 반 데어 란의 플라스틱 넘버를 들 수 있다. 모뒬로르는 현재까지 아주 제한된 성공밖에 거두지 못했는데, 이는 피보나치 수열에 기초한 수치들이 커졌을 때 단위의 배수관계가 거의 형성되지 않는다는 사실에 기인한다. 반면에 플라스틱 넘버는 모뒬로르의 결점들을 보완할 수 있는 매력적인 수열을 가지고 있다. 이에 본 연구는 반 데어 란의 비례시스템 분석을 통하여 유도된 수열규칙이 건축디자인과 관련되어 직접적으로 적용되어 질 수 있는 CAAD 시스템을 제안한다. 본 시스템의 초점은 반 데어 란의 비례이론이 어떻게 컴퓨터 언어로 변환 및 CAAD 시스템에 적용되어, 실질적인 건축 실무행위에 있어 컴퓨터가 디자인 도우미로서 역할을 수행할 수 있겠는가하는 것이다. 연구의 결과, 사용자는 이러한 시스템을 사용함으로써 반 데어 란의 비례시스템을 자신의 디자인에 손쉽게 적용할 수 있으며, 이는 복잡한 치수관계로 구성된 비례시스템의 건축실무 활용으로 발전되어질 수 있을 것으로 사료된다.

Keywords : Architectural Proportion, Dom Hans van der Laan, Plastic Number, Harmony, CAAD
주 요 어 : 건축비례, 돔 한스 반 데어 란, 플라스틱 넘버, 조화, CAAD

I. Introduction

"If all elements of a building were harmoniously composed of proportions with each other, would the space applied to them be harmonious?"

Le Corbusier(1887-1966) used the Modulor system that is the proportioning system based on the Fibonacci Series in his designs for Unité d'Habitation at Marseilles, Chapel at Ronchamp, the monastery at La Tourette and the city of Chandigarh, in order to create a harmonious architectural space. He hoped this Modulor system would become an international methodology that would serve as a guide in the design process. However, his hope was little realized because of limitations of the Modulor. For example, Le Corbusier in designing large buildings like the Unité d'Habitation at Marseilles, applied the Modulor only to the restricted range of smaller elements like the interiors of the apartment, and not to the structure as a whole (Padovan, 1994). The actual measurements in this apartment differed considerably from his argument: actual length 140 m, instead of Modulor length 139.01 m; width 24 m, Modulor width 25.07 m; height 56 m, Modulor height 53.10 m. Hence, this Modulor system has been forgotten by architects in the past

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fifty years (Arnheim, 1969; Padovan, 1999; von Meiss, 1990).

If an alternative proportional system overcame the shortcoming of the Modulor, this system could become again an influential measurement in the architectural industry; that is Van der Laan's proportional system "the Plastic Number". Therefore, this paper proposes a CAAD (Computer-Aided Architectural Design) system that generates and evaluates on architectural proportions based on the Plastic Number. This system has been developed as one computational application among many ones which show the way how design grammars of architecture can be computed with aid of computer. Most of these works have focused on analysis and computation of the existed works build by Palladio, Frank Lloyd Wright, De-Stijl, etc. This approach is well described in the work of Georgy Stiny, William J. Mitchell, Ulrich Flemming, Terry W. Knight and others (McCullough *et al.*, 1990; Flemming, 1994; Knight, 1994). However, this paper shows an explicit way how a theoretical design principle can be computed directly during design phases, not through analysis of the existed works by the greatest builders. So, this research can be valuable as an application example to transform a design principle of architecture into a computer-aided design system.

II. Composition Principles and Characteristics of Plastic Number

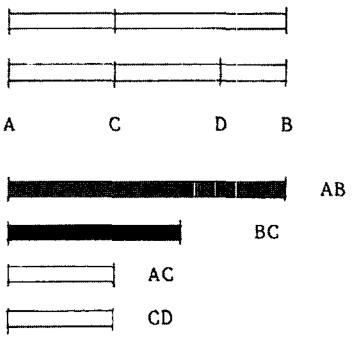
The analysis of van der Laan's proportion theory is an essential prerequisite to wide understanding of this design system. There are only two main books on the Plastic Number: Van der Laan's "Architectonic Space" and Padovan's "Dom Hans van der Laan". So, central arguments in this chapter are based on these references.

1. Composition Principles of Plastic Number

The Plastic Number, discovered by Dom Hans van der Laan (1904-1991) in 1928, differs from all previous systems of architectural proportions. His hypothesis of an architectural proportion comes from the fact that aesthetics of proportion is not concerned with mathematical relationships, but with clarity of perception. For this reason, his basic ratios are determined by the lower and upper limits of our normal ability to perceive differences in size among three-dimensional objects. To seek the visual differences and to derive these limits, thirty-six pebbles were used. For example, in sieving gravel, limits of each grade are set by the size of the mesh. A sieve with an opening of a given size lets through all the gravel of a smaller size, but retains the larger stones. At this point, each opening size of the sieve determines the lower- and the upper-limit.

According to Padovan (Padovan, 1994), the Plastic Number is comparable with the Golden Section. As shown in Fig. 1, the Golden Section used by Le Corbusier binds only three consecutive measurements: the whole and its two parts (AB, BC, AC/CD).

However, the approach by Van der Laan differs from Le Corbusier's. He proposed a proportion between the parts so that when the larger part is again divided, the ratio of the new larger part to the original smaller part equals that between the original parts, as shown in Fig. 2:



$$AB : BC = BC : AC$$

Fig. 1. Single and double bisections of a line AB, according to the Golden Section

According to Van der Laan, the Golden Section

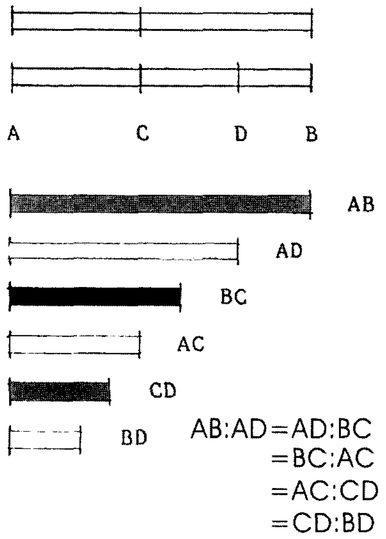


Fig. 2. Single and double bisections of a line AB, according to the Plastic Number

has the great disadvantage that a second division within that ratio produces a measurement identical to the smaller part resulting from the first part. Therefore, it gives no sufficient basis for a series of proportional relationships. So, he devised his own proportional system "the Plastic Number" which can compensate for this disadvantage.

The series of measurements in the Plastic Number are similar to the Fibonacci ones in that an additive progression tends towards the Golden Section, but are different in that the latter largest measurement of any three consecutive measurements equals the sum of the two smallest.

The relationships between these various measurements are illustrated in the following diagram of Fig. 3. In Fig. 3, the eight measurements consist of pairs, by the names whole, part, piece and element each of these has a minor and a major component. Hence, the number 1 in this diagram is called the minor element, 2 the major element, 3 the minor piece, 4 the major piece, 5 the minor part, 6 the major part, 7 the minor whole and 8 the major whole.

Ratios, seven in number, are expressed in terms of the relationship between the smallest measurement and the other seven. In this diagram of Fig. 3, the minor element acts as a unit, so the ratio to it of the other measurements presents itself as a number. For instance, the major and minor parts have the same proportion to the minor element as the major and minor wholes have to the minor piece; therefore, they are expressed by the ratio 4:3 (Van der Laan, 1983). The major whole, being the sum of the major and minor parts, is expressed as the number seven.

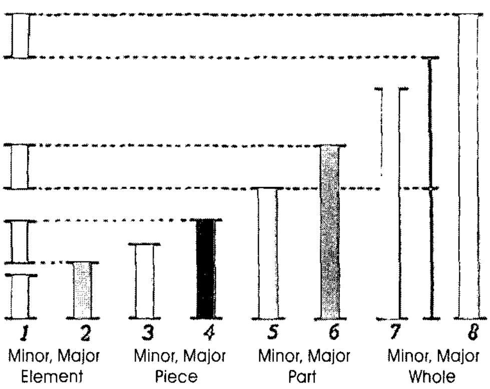


Fig. 3. Plastic Number embraces seven consecutive types contained within eight measurements

Through these relationships, the system's eight measurements are expressed in relation to the minor element, the unit, as the following numbers:

minor element	1	minor part	3
major element	4/3	major part	4
minor piece	7/4	minor whole	16/3
major piece	7/3	major whole	7

Conversely, the measurements can also be regarded as parts of the major whole:

minor element	1/7	minor part	3/7
major element	3/16	major part	4/7
minor piece	1/4	minor whole	3/4
major piece	1/3	major whole	1

2. Characteristics of Plastic Number

When the minor element of system I is given as the value 100, the value for system II and III can be derived from the relationships between the various measurements; the concrete content and relationships of three measurement-systems is well described in Van der Laan's book on pages 88-98 (Van der Laan, 1983).

Table 1. System I, II and III

systems	elements		pieces		parts		wholes	
	minor	major	minor	major	minor	major	minor	major
I	100	132 1/2	175 1/2	232 1/2	308	408	540 1/2	716
II	14	18 1/2	24 1/2	32 1/2	43	57	75 1/2	100
III	2	2 1/2	3 1/2	4 1/2	6	8	10 1/2	14

Numerical values for the measurements of two consecutive derived systems, Ia and II, can be established, taking into account the small quanta from the lower systems. To obtain these values, each authentic measurement is subtracted from its corresponding measurement from the system below:

Table 2. System I, II and Ia

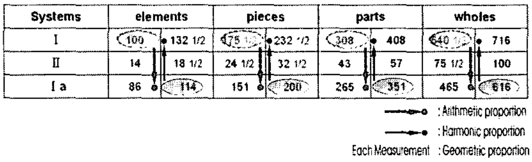
systems	elements		pieces		parts		wholes	
	minor	major	minor	major	minor	major	minor	major
I	100	132 1/2	175 1/2	232 1/2	308	408	540 1/2	716
II	14	18 1/2	24 1/2	32 1/2	43	57	75 1/2	100
I a	86	114	151	200	265	351	465	616

Table 3. System II, III and IIa

systems	elements		pieces		parts		wholes	
	minor	major	minor	major	minor	major	minor	major
II	14	18 1/2	24 1/2	32 1/2	43	57	75 1/2	100
III	2	2 1/2	3 1/2	4 1/2	6	8	10 1/2	14
IIa	12	16	21	28	37	49	65	86

The numerically deduced progressions are, in general, sufficient for the application of the measurement-systems to the linear dimensions of square forms in architecture. If one measurement were to be derived from the original eight authentic systems, eight new derived systems would arise, from which either the breadth or the length could be derived. For instance, the numerical values for a length of 716 are described as above in Table 2. The number 716 represents the composition of the major whole, which we regard as the sum of 700, 14 and 2; in other words, seven times the minor element (7x100) plus the minor element from the system in which 100 is the major whole (14), plus the minor element from the lower system in which 14 is the major whole (2).

The measurements derived from these systems have harmonious relationships with each other. For example, the derived major whole (system Ia) is the harmonic mean between the authentic major and minor wholes and the latter is the arithmetic mean between the derived major and minor whole. Each measurement of the system that is expressed in numbers, such as 716:540 1/2, 540 1/2:408, 408:308, etc., has the geometric proportion, as shown in Fig. 4:



Systems	elements		pieces		parts		wholes	
I	100	132 1/2	175 1/2	232 1/2	308	408	540 1/2	716
II	14	18 1/2	24 1/2	32 1/2	43	57	75 1/2	100
I a	86	114	151	200	265	351	465	616

Fig. 4. Harmonious relationships between the measurements

Finally, these relationships can be extended to a systematic arrangement as a large, two dimensional, right-angled triangle (Fig. 5).

The great merit of Van der Laan's proportional system lies not in the details of its mathematics, but rather in the broad concepts of types and orders of size. For this reason, each measurement in his system is conceived as "about so large": very flexible. They can be given a length in meters, centimeters, millimeters or even feet and inches, according to need.

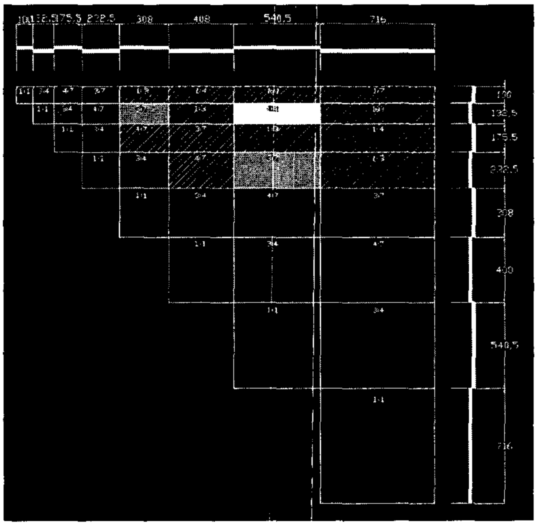


Fig. 5. Two dimensional diagram of measurements of the Plastic Number

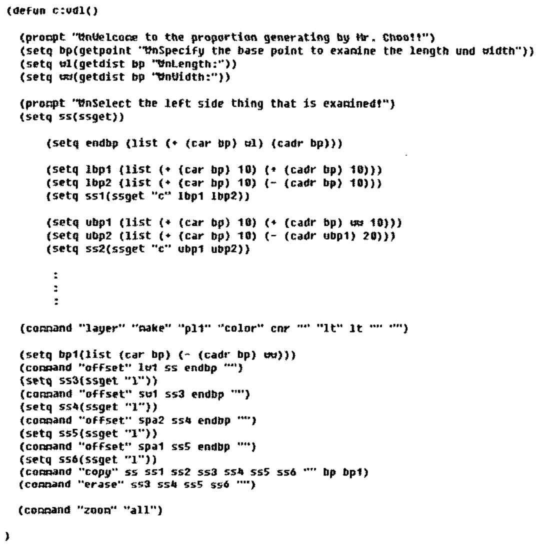
III. Structure and Rule of the Proportion-Generating System

1. Basic Mechanism of the System

The programming language employed in AutoCAD is AutoLISP. As this language retains most of the general LISP (LIST Processing Language)-functions, it is a symbolic manipulation-based, interpreted language that provides a simple mechanism for adding commands to AutoCAD (Sommer, 1999). For example, this interactive programming language in AutoCAD allows users to program external applications, using the AutoCAD drawing generation and manipulation functions for 2D geometry, 3D wire-frame structures and 3D curved surfaces. Therefore, customizing AutoCAD into a more useful tool for a particular application for users can be carried out using AutoLISP programs. The AutoLISP program is normally written by "text editors" and then is saved as "text only" with the ".lsp" extension; i.e. vdl.lsp. An example is shown in the Fig. 6, which is concerned with the Plastic Number and is programed by this author.

For implementation, users must load AutoLISP-files into a computer's memory before using them. There are various ways of loading them. For instance, they can be loaded by typing the instructions in the command prompt in AutoCAD, as follows:

```
COMMAND: (load "c:/LISP/vdl")
C:VDL
```



```
(defun c:vdl()

  (prompt "₩nWelcome to the proportion generating by Mr. Choo!!")
  (setq bp(getpoint "₩nSpecify the base point to examine the length und width"))
  (setq wl(getdist bp "₩nLength:"))
  (setq ww(getdist bp "₩nWidth:"))

  (prompt "₩nSelect the left side thing that is examined!")
  (setq ss(ssget))

      (setq endbp (list (+ (car bp) wl) (cadr bp)))

      (setq lbp1 (list (+ (car bp) 10) (+ (cadr bp) 10)))
      (setq lbp2 (list (+ (car bp) 10) (- (cadr bp) 10)))
      (setq ss1(ssget "c" lbp1 lbp2))

      (setq ubp1 (list (+ (car bp) 10) (+ (cadr bp) ww 10)))
      (setq ubp2 (list (+ (car bp) 10) (- (cadr ubp1) 20)))
      (setq ss2(ssget "c" ubp1 ubp2))

       :
       :
       :

  (command "layer" "make" "pl1" "color" cnr "" "lt" lt "" "")

  (setq bp1(list (car bp) (- (cadr bp) ww)))
  (command "offset" lw1 ss endbp "")
  (setq ss3(ssget "l"))
  (command "offset" sw1 ss3 endbp "")
  (setq ss4(ssget "l"))
  (command "offset" spa2 ss4 endbp "")
  (setq ss5(ssget "l"))
  (command "offset" spa1 ss5 endbp "")
  (setq ss6(ssget "l"))
  (command "copy" ss ss1 ss2 ss3 ss4 ss5 ss6 "" bp bp1)
  (command "erase" ss3 ss4 ss5 ss6 "")

  (command "zoom" "all")

)
```

Fig. 6. Example of AutoLISP-Program

The choice of AutoLISP in this work has been made for several reasons: this author's amount of experience with both AutoLISP and AutoCAD; the reliable graphic control/output in AutoCAD and the worldwide usage of this system in architectural offices. This program can be used as soon as it is needed and simultaneously the user can obtain some design advice from his own computer for a quality assessment of the particular design qualities. The system demonstrated here operates in MS-Windows XP and in the AutoCAD 2000 environment.

2. Rule of Plastic Number in the System

The above-mentioned relationships in Chapter 2 can be translated into superimposed ratios between the measurements of a system.

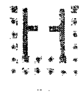
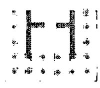
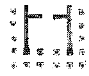
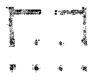
The ancients always based the column-spacings of their temples on fixed ratios. For example, the forms of buildings are described mathematically in terms of numerical relationships, throughout Vitruvius' "The Ten Books". He specified five types with specific ratio of column thickness to spacing between columns, which were measured between faces of columns, not center to center, and to column height (Knell, 1985). For example, in the case of the Pychnostyle, the ratio of the column-interval, measured center-to-center, to the diameter is equal to that between the derived minor part to the minor element, 265:100. The ratio of the Systyle is 308:100, in other words that between the minor part and the minor element. Eustyle is 351:100 (the derived major part to the minor element), Diastyle 408:100 (the major part to the minor element) and Araeostyle 465:100 (the derived minor whole to the minor element). Given a numerical value of 100 for the column interval, the value for the diameters is given within

Table 4. A measurement-system and ratio A (diameter) to B (intercolumniation) of the ancient temple columns

systems	elements		pieces		parts		wholes	
	minor	major	minor	major	minor	major	minor	major
I	100	132½	175½	232½	‹308›	‹408›	540½	716
II	14	18½	24½	32½	43	57	75½	100
I a	86	114	151	200	265›	‹351›	‹465›	616

Systyle Diastyle

Pycnostyle Eustyle Araeostyle

A B =	Pycnostyle		2 3
	Systyle		1 2
	Eustyle		4 9
	Diastyle		1 3
	Araeostyle		1 4

the measurement of the previously mentioned system, as shown in Table 4:

These numerical relationships can be expressed as specific ratios in this table, so that the measurements and the derived ratios can be applied to a CAAD system.

The basic shape, which is defined by this author, for implementation of the proportion-generating system based on the Plastic Number is illustrated in Fig. 7. In Fig. 7, each column-thickness and spacing between columns on a facade is illustrated as A:B:A.

This formula can be calculated automatically through the computer, according to the specific ratios of the Plastic Number.

IV. Testing the System

In practice, it would be a simple matter to translate the specific numerical relationships of the Plastic Number into mechanical operation with the proposed system. This program suggests some optimum relationships between columns and spacing to the user, with respect to the specific harmonious ratios of Van der Laan. Fig. 8 illustrates the original elevation of a building to be examined:

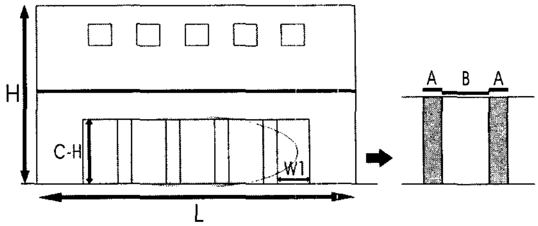


Fig. 7. Basic shape for implementation

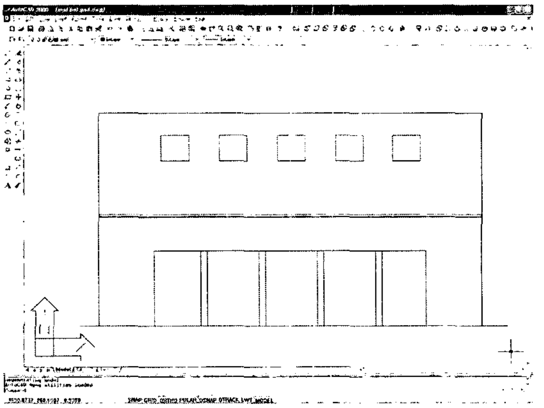


Fig. 8. Original elevation of a building

The column thickness in Fig. 8 seems to be narrow. So, the user decides to get some advice about the column thickness from his own PC. He uses "(vdl)" in the command prompt in the AutoCAD to invoke the AutoLISP program. Using this program, the user can choose his favourite example, which has an appropriate column thickness based on the Plastic Number, from among those that are proposed by the computer on the screen, as shown in the above figure of Fig. 9.

When the user selects one, the program inserts the selected example into the plan, deleting the old one, as shown in the bottom figure of Fig. 9. The modified ratios of column-thickness to space-width are set more harmoniously than the original one.

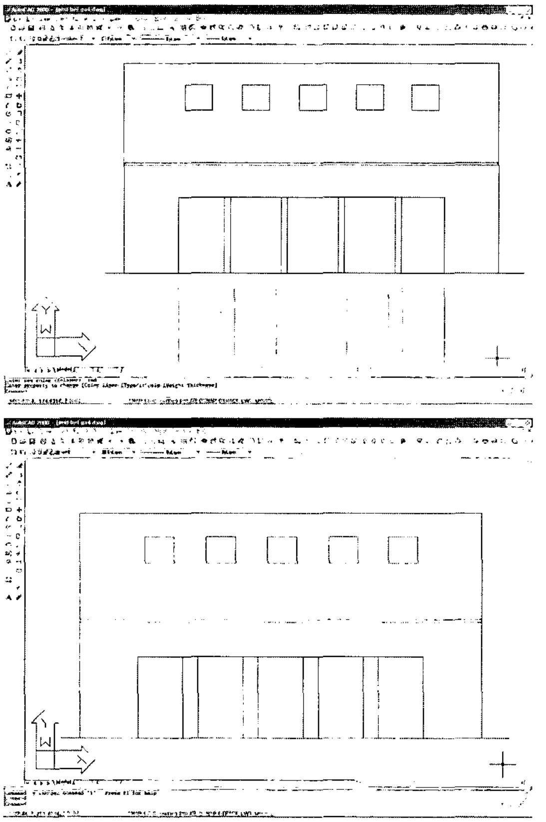


Fig. 9. Alternative thickness of the columns (above) and result after using the system (bottom)

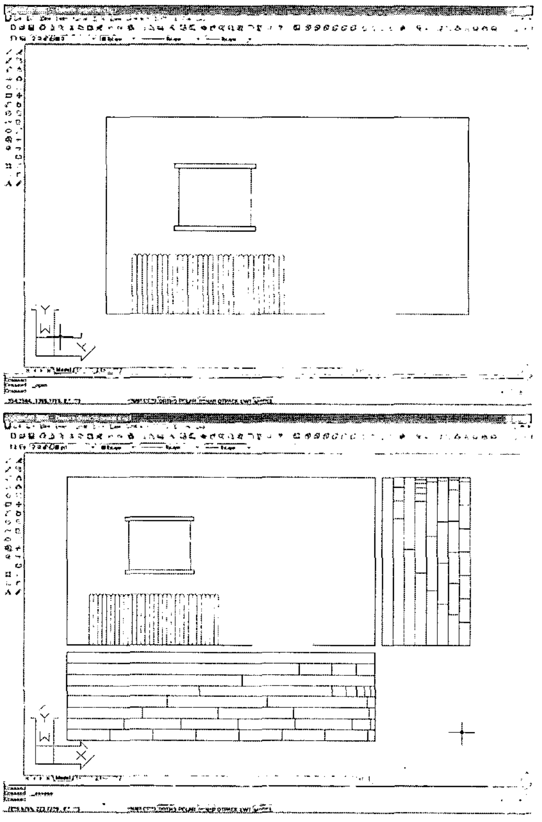


Fig. 10. Interior elevation of a room (above) and recommended series of the Plastic Number (bottom)

The above figure of Fig. 10 shows an interior elevation of a room. Computing Van der Laan's series of numbers, all elements of the interior, such as the height and the length of the room, of the door, of the window and of the radiator, can be compared to the different diagrams that are set in the right- and bottom-side of the interior elevation in Fig. 10. These diagrams are composed of various parts harmoniously, which are derived from the height and the length of the elevation as the major whole in Van der Laan's system, as shown in the bottom figure of Fig. 10. The numerical measurements can be applied to each measurement of the door, the window and the radiator.

V. Conclusion

Le Corbusier's Modulor and Van der Laan's Plastic Number are based theoretically on irrational numbers. In Modulor, the key measurement is fixed by the height of a man, so that each measurement of the Modulor is always given with specific numbers. However, the Plastic Number consists of a scale of proportion without any fixed measurements, as shown the previous examples. Hence, the application range of it is very flexible and the potentiality of it is higher than Le Corbusier's.

When applying this reasonable proportion system to an architectural design, we will meet a problem in practice; it has too complex measurements or ratios to apply it to the design with our head. This proposed proportion-generating system enables to apply Van der Laan's proportional system to our design easily. So far, the world of architecture has not offered a full answer to the above-posed question in Chapter 1.

It is hoped that we will be able to design buildings using the harmonious ratios easily, test the quality of buildings, and then seek the answer to this question, by using this system. Finally, the use of this system based on Van der Laan's Plastic Number is a way to link a proportional theory of architecture into a CAAD system.

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