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Stability Analysis and Proposal of a Simple Form of a Fuzzy PID Controller

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Abstract : This paper suggests the simple form of a fuzzy PID controller and describes the design principle, tracking performance, stability analysis and changes of parameters of a suggested fuzzy PID controller. A fuzzy PID controller is derived from the design procedure of fuzzy control. It is well known that a fuzzy PID controller has a simple structure of the conventional PID controller but posses its self-tuning control capability and the gains of a fuzzy PID controller become nonlinear functions of the inputs[1]. Nonlinear calculation during fuzzification, defuzzification and the fuzzy inference require more time in computation. To increase the applicability of a fuzzy PID controller to digital computer, a simple form of a fuzzy PID controller is introduced by the backward difference mapping and the analysis of the fuzzy input space.

To guarantee the BIBO stability of a suggested fuzzy PID controller, 'small gain theorem' which proves the BIBO stability of a fuzzy PI[2] and a fuzzy PD[1] controller is used. After a detailed stability analysis using 'small gain theorem', from which a simple and practical method to decide the parameters of a fuzzy PID controller is derived. Through the computer simulations for the linear and nonlinear plants, the performance of a suggested fuzzy PID controller will be assured and the variation of the gains of a fuzzy PID controller will be investigated.

Key words : Fuzzy controller, Fuzzy PID controller Stability analysis, Small gain theorem

1. Introduction

PID controllers have been widely used for industrial processes due to its simplicity and effectiveness. They provide high sensitivity and stability of the overall feedback control system and reduce overshoot and steady-state error. It has been well known that PID controllers can be effectively used for 1st and 2nd-order linear systems, but they can suffer

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from problems on higher-order and nonlinear systems.

On the other hand, fuzzy controllers in general are suitable for many nontraditionally modeled industrial processes such as linguistically controlled devices and systems that cannot be precisely described by mathematical formulation and have significant unmodeled effects and uncertainties. Fuzzy controllers are composed of fuzzifier, defuzzifier and fuzzy inference engine and have an ability of nonlinear compensation by their nonlinear characteristics. There are several types of control systems that adopt a fuzzy logic controller as an essential system component. The majority of applications during the past two decades belong to the class of fuzzy PID controllers.

This paper suggests a simple form of a fuzzy PID controller and describes the design principle, stability analysis and change of parameters of a fuzzy PID controller. To increase the applicability of a fuzzy PID controller to digital computer, a simple form of a fuzzy PID controller is introduced. After a detailed stability analysis using 'small gain theorem', from which a simple and practical decision method of the parameters of a suggested fuzzy PID controller is derived.

Finally several computer simulations are executed to confirm the effectiveness of a suggested fuzzy PID controller. The outputs and changes of gains of the suggested controller are compared with those of the conventional linear digital PID controller and a fuzzy PID controller with various parameter.

2. A simple form of a fuzzy PID controller

In this section, a simple form of a fuzzy PID controller is derived. The structure of a fuzzy PID controller is established, a fuzzy PID controller is formulated through the three steps : the fuzzification, the control-rule base and the defuzzification. A simple form of a fuzzy PID controller is introduced through the analysis of fuzzified spaces and its steady-state behavior is examined.

2.1 The structure of a fuzzy PID controller

The structure of a fuzzy PID controller in this work is shown in Fig. 1[3].

Although most popular fuzzy controllers have a 1-input FLC or a 2-input FLC or a 3-input FLC, the suggested fuzzy PID controller has two independent parallel 2-input FLCs. One has a 2-input FLC for an error('*error*') and a change of error('*rate*'), another has a 2-input FLC for a change of error and an accelerated change of error('*acc*'). Then the incremental output of the controller is formed by algebraically adding the two outputs of FLCs. The notations used in Fig. 1 is such as

$$e(nT) = ref(nT) - y(nT) \tag{1}$$

$$e^* = GE \times e(nT) \tag{2}$$

$$r(nT) = [e(nT) - e(nT-T)]/T \tag{3}$$

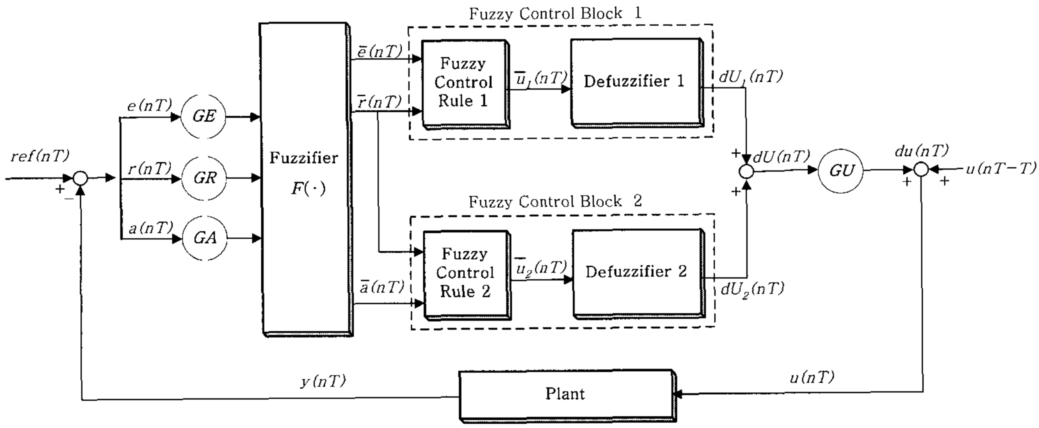


Fig. 1 Structure of a fuzzy PID control system

$$r^* = GR \times r(nT) \quad (4)$$

$$a(nT) = [r(nT) - r(nT-T)]/T$$
$$= [e(nT) - 2e(nT-T) + e(nT-2T)]/T^2 \quad (5)$$

$$a^* = GA \times a(nT) \quad (6)$$

$$dU(nT) = dU_1(nT) + dU_2(nT) \quad (7)$$

$$du(nT) = GU \times dU(nT) \quad (8)$$

$$u(nT) = du(nT) + u(nT-T) \quad (9)$$

where n is a positive integer and T is a sampling period. The $y(nT)$, $e(nT)$, $r(nT)$ and $a(nT)$ denote process output, error, rate, acc at sampling time nT, respectively. The *GE*, *GR*, *GA* and *GU* are the input scaler for *error*, the input scaler for *rate*, the input scaler for *acc* and the output scaler of the FLC and finally they are used as control gain. The e^*, r^* and a^* are the scaled inputs for *error*, *rate*, acc. The dU_1, dU_2, du and u are the incremental output of fuzzy control block 1 and 2, an incremental control input and a control input at sampling time nT, respectively.

2.2 Fuzzification

An input membership function for fuzzification of a fuzzy PID controller is shown in Fig. 2. where the three membership functions for e^*, r^* and a^* have been combined in the same figure, since they are chosen to be the same. The e^* has two members EP and EN, the r^* has two members RP and RN and the a^* has two members AP and AN[3].

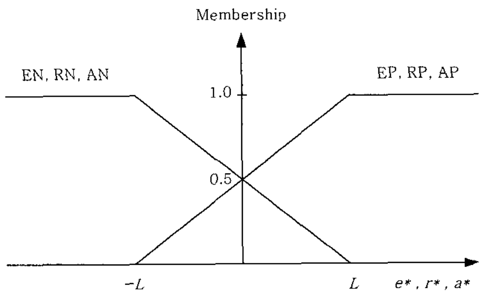
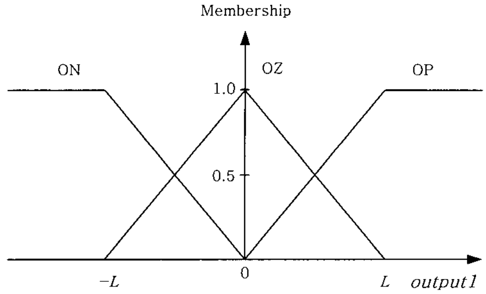
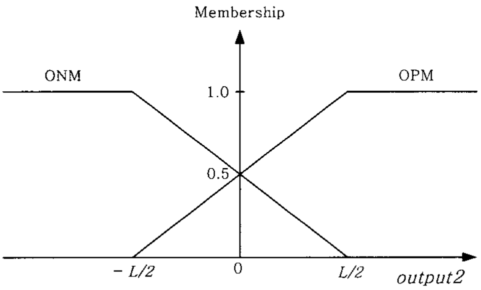


Fig. 2 Membership functions of e*, r* and a*

The output membership function for *output1* and *output2* is shown in Fig. 3. The *output1* has three members OP, OZ and ON shown in Fig. 3(a) and the *output2* has two members OPM and ONM shown in Fig. 3(b).



(a) *output1* (Fuzzy control block 1)



(b) *output2* (Fuzzy control block 2)

Fig. 3 Output membership functions

2.3 Fuzzy control rules

There are four fuzzy control rules in the fuzzy control block 1 and 2, respectively.

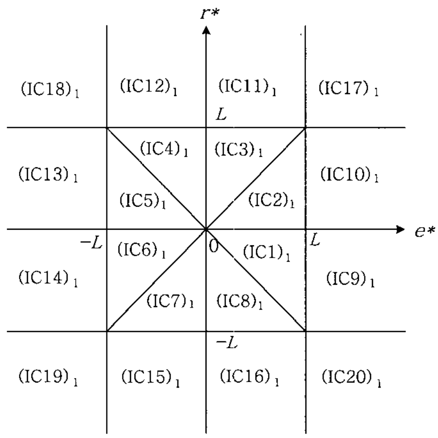
fuzzy control block 1

$(R1)_1$: IF *error*=EP and *rate*=RP THEN *output*=OP
$(R2)_1$: IF *error*=EP and *rate*=RN THEN *output*=OZ
$(R3)_1$: IF *error*=EN and *rate*=RP THEN *output*=OZ
$(R4)_1$: IF *error*=EN and *rate*=RN THEN *output*=ON

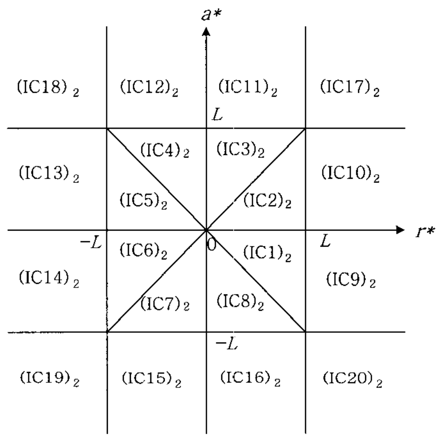
fuzzy control block 2

$(R1)_2$: IF *rate*=RP and *acc*=AP THEN *output*=OPM
$(R2)_2$: IF *rate*=RP and *acc*=AN THEN *output*=ONM
$(R3)_2$: IF *rate*=RN and *acc*=AP THEN *output*=OPM
$(R4)_2$: IF *rate*=RN and *acc*=AN THEN *output*=ONM

These eight rules together yield the control action for the fuzzy PID control law. In the fuzzy PID control block 1, the eight different input combinations of scaled *error* and scaled *rate* to the control rules are shown in Fig. 4(a) and in the fuzzy control block 2, the eight input combinations of scaled *rate* and scaled *acc* to the control rules are shown in Fig. 4(b).



(a) e^* & r^*



(b) r^* & a^*

Fig. 4 Possible input combinations

2.4 Defuzzification

To defuzzify the incremental output of the fuzzy control block 1 and 2, the center average method is employed such as[4],[5]

$$dU = \frac{\sum_{i=0}^{n} \mu_{output}(w_i) \times w_i}{\sum_{i=0}^{n} \mu_{output}(w_i)}. \quad (10)$$

where n is the number of rules, w_i is the member and $\mu_{output}(w_i)$ is the membership of w_i.

The incremental output of the fuzzy control block 1 for the scaled *error* and the scaled *rate* within [$-L$, L] can be described by

$$IF \quad GR \times |r(nT)| \le GE \times |e(nT)| \le L, \quad (11)$$
$$dU_1(nT) = \frac{0.5 \times L}{2L - GE \times |e(nT)|}[GE \times e(nT) + GR \times r(nT)],$$

$$IF \quad GE \times |e(nT)| \le GR \times |r(nT)| \le L, \quad (12)$$
$$dU_1(nT) = \frac{0.5 \times L}{2L - GE \times |r(nT)|}[GE \times e(nT) + GR \times r(nT)]$$

and the incremental output of fuzzy control block 2 for the scaled *rate* and the scaled *acc* within [$-L$, L] is

$$IF \quad GA \times |a(nT)| \le GR \times |r(nT)| \le L,$$
$$dU_2(nT) = \frac{0.25 \times L}{2L - GR \times |r(nT)|}[GA \times a(nT)] \quad (13)$$

$$IF \quad GR \times |r(nT)| \le GA \times |a(nT)| \le L,$$
$$dU_2(nT) = \frac{0.25 \times L}{2L - GA \times |a(nT)|}[GA \times a(nT)]. \quad (14)$$

In cases of either the scaled *error* or the *rate* beyond [$-L$, L] in the fuzzy control block 1 and either the scaled *rate* and the scaled *acc* beyond [$-L$, L] in the fuzzy control block 2, the incremental outputs of the fuzzy control block 1 and 2 are listed in Table 1.

Consequently, the overall incremental output of the FLC, $dU(nT)$, can be obtained by adding incremental output $dU_1(nT)$ of the fuzzy control block 1 and $dU_2(nT)$ of the fuzzy control block 2.

$$dU(nT) = dU_1(nT) + dU_2(nT) \quad (15)$$

Then the incremental control input, $du(nT)$, can be obtained by multiplying $dU(nT)$ by output scaler GU.

$$du(nT) = GU \times dU(nT) \quad (16)$$

Table 1 The incremental output of fuzzy control block 1 and 2 when e* or r* and r* or a* are not within the interval [-L, L]

	Input combinations		Output	
fuzzy control block 1	e^* & r^*	$(IC9)_1$, $(IC10)_1$	$dU_1(nT)$	$[GR \times r(nT) + L]/2$
		$(IC11)_1$, $(IC12)_1$		$[GE \times e(nT) + L]/2$
		$(IC13)_1$, $(IC14)_1$		$[GR \times r(nT) - L]/2$
		$(IC15)_1$, $(IC16)_1$		$[GE \times e(nT) - L]/2$
		$(IC17)_1$		L
		$(IC18)_1$, $(IC20)_1$		0
		$(IC19)_1$		$-L$
fuzzy control block 2	r^* & a^*	$(IC9)_2$, $(IC10)_2$, $(IC13)_2$, $(IC14)_2$	$dU_2(nT)$	$0.5 \times GA \times a(nT)$
		$(IC11)_2$, $(IC12)_2$, $(IC17)_2$, $(IC18)_2$		$0.5 \times L$
		$(IC15)_2$, $(IC16)_2$, $(IC19)_2$, $(IC20)_2$		$-0.5 \times L$

2.5 A simple form of a fuzzy PID controller

A fuzzy PID controller has the excellent performance but also has the structural complexity and the large computational time owing to the 'IF~THEN~' rules and nonlinear computation in fuzzy inference procedure as compared with the conventional PID controller. Investigation of the fuzzy input spaces in fuzzification provide the simple form of a fuzzy PID controller. Fuzzy input spaces generated by fuzzifier are shown in Table 2.

Table 2 Fuzzy spaces generated by fuzzifier

	below $-L$	between $-L$ and L	above L
e^*	$-L$	$GE \cdot e$	L
r^*	$-L$	$GR \cdot r$	L
a^*	$-L$	$GA \cdot a$	L

Applying limited input spaces, a following simple form of a fuzzy PID controller can be derived Table 2 and (11)~(14).

IF $GE \cdot |e| > L$ THEN $GE \cdot |e| = L$
IF $GR \cdot |r| > L$ THEN $GR \cdot |r| = L$
IF $GA \cdot |a| > L$ THEN $GA \cdot |a| = L$

$$\begin{aligned} du(nT) &= K_i e(nT) + K_p r(nT) + K_d a(nT) \\ &= \frac{0.5\times L\times GU\times GE}{2L - max(GE\times|e(nT)|, GR\times|r(nT)|)} e(nT) \\ &+ \frac{0.5\times L\times GU\times GA}{2L - max(GE\times|e(nT)|, GR\times|r(nT)|)} r(nT) \\ &+ \frac{0.25\times L\times GU\times GA}{2L - max(GR\times|r(nT)|, GA\times|a(nT)|)} a(nT) \end{aligned} \quad (17)$$

2.6 Discretization of a conventional continuous-time PID controller

Recall that the conventional continuous-time PID control law is described by

$$U_{\pi d}(t) = K_p^c e(t) + K_d^c \dot{e}(t) + K_i^c \int e(t)dt \quad (18)$$

where K_p^c, K_i^c and K_d^c are the proportional, derivative and integral gains of the controller, respectively, and $e(t)$ is the error signal defined by $e(t) := ref(t) - y(t)$ with $ref(t)$ being the reference signal and $y(t)$ the system output. In the frequency domain the control law of this controller is given by

$$U_{pid}(s) = (K_p^c + sK_d^c + \frac{K_i^c}{s}) E(s) \quad (19)$$

where s is the complex frequency variable of the Laplace transform.

A differential conventional continuous-time PID control law is as follows.

$$sU_{pid}(s) = (sK_p^c + s^2K_d^c + K_i^c) E(s) \quad (20)$$

Let T be the sampling period of the continuous -time signals in the digital control system. By applying the backward difference mapping[6] $s = (1 - z^{-1})/T$ to obtain the simple form, we can convert the continuous-time system to its discrete-time setting in the complex z-frequency domain.

$$\frac{1-z^{-1}}{T} U_{pid}(z) = \left\{\frac{1-z^{-1}}{T} K_p^c + (\frac{1-z^{-1}}{T})^2 K_d^c + K_i^c\right\} E(z) \quad (21)$$

where K_p^c, K_d^c, and K_i^c are the continuous-time and discrete-time proportional, derivative and integral gains. After an inverse z-transform, we obtain

$$\begin{aligned} \frac{u_{pid}(nT) - u_{pid}(nT-T)}{T} &= \\ & K_p^c \frac{e(nT) - e(nT-T)}{T} \\ & + K_d^c \frac{e(nT) - 2e(nT-T) + e(nT-2T)}{T^2} \\ & + K_i^c e(nT) \end{aligned} \quad (22)$$

Define

$$du_{pid}(nT) = \frac{u_{pid}(nT) - u_{pid}(nT-T)}{T} \quad (23)$$

$$r(nT) = \frac{e(nT) - e(nT-T)}{T} \quad (24)$$

$$\begin{aligned} a(nT) &= \frac{r(nT) - r(nT-T)}{T} \\ &= \frac{e(nT) - 2e(nT-T) + e(nT-2T)}{T^2} \end{aligned} \quad (25)$$

where $du_{pid}(nT)$, $r(nT)$ and $a(nT)$ are the incremental control input, the rate of change and accelerated rate of change of the error signal e, respectively. Substituting (23)-(25) into (22) gives

$$du_{pid} = K_i^c e(nT) + K_p^c r(nT) + K_d^c a(nT) \quad (26)$$

where the control action is equal to the sum of the weighted error, the weighted rate of change and the weighted accelerated rate of change of error signal. (23) can be rewritten as

$$u_{pid}(nT) = u_{pid}(nT-T) + T \cdot du_{pid}(nT) \quad (27)$$

3. Stability analysis

In this section, the BIBO stability of the overall control system with a suggested fuzzy PID controller is analyzed. There are a considerable amount of efforts devoted to stability analysis of fuzzy control system in the past[7]~[11]. Especially this paper owes a great deal to [1] and [2] to employ the small gain theorem[12],[13] for stability analysis. The suggested fuzzy PID controller (17) is analyzed as compared with the digital PID controller (26).

Theorem 3.1 Small gain theorem

Consider the system of Fig. 5 and suppose that both a controller S_1 and a nonlinear plant S_2 are causal and L_p-*stable wb*(*wb : with finite gain and zero bias*), and let the gain of a controller S_1 $\gamma_{1p} = \gamma_p(S_1)$ and the gain of a nonlinear plant S_2 $\gamma_{2p} = \gamma_p(S_2)$. Under the conditions, the system Fig. 5 is L_p-*stable wb*[12],[13] if

$$\gamma_{1p}\gamma_{2p} < 1. \quad (28)$$

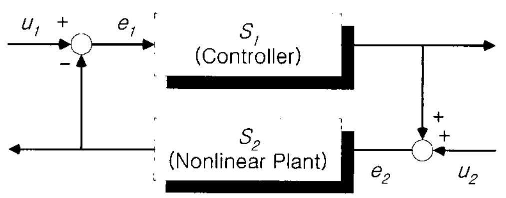


Fig. 5 A nonlinear feedback control system

Theorem 3.2 The BIBO stability conditions of a fuzzy PID control system

If the following conditions are satisfied, the nonlinear fuzzy PID control system is BIBO stable by the small gain theorem.

(1) The nonlinear plant is bounded such that $\|N\| < \infty$.

(2) The gains K_p^c, K_d^c, and K_i^c of the steady-state PID controller are chosen nonlinear plant stable.

(3) The parameters GE, GR, GA, and GU of a fuzzy PID controller satisfy

$$\frac{GU \cdot GE}{4} = K_i^c,\ \frac{GU \cdot GR}{4} = K_p^c,$$
$$\frac{GU \cdot GA}{8} = K_d^c. \quad (29)$$

proof :

We consider the general case that the process under control is a nonlinear plant N, the fuzzy PID control law is (9) and (17), and the reference signal is $ref(nT)$[12,13]. By defining

$$\begin{cases} e_1(nT) = e(nT) \\ e_2(nT) = u_{fpid}(nT) \\ u_1(nT) = ref(nT) \\ u_2(nT) = u_{fpid}(nT-T) \\ S_1(e_1(nT)) = GU\,du_{fpid}(nT) \\ S_2(e_2(nT)) = N(e_2(nT)) \end{cases} \quad (30)$$

In Fig. 5

$$\begin{cases} u_1(nT) = ref(nT) = e_1(nT) + S_2(e_2(nT)) \\ \qquad = e(nT) + N(u(nT)) \\ u_2(nT) = u_{fpid}(nT-T) = e_2(nT) - S_1(e_1(nT)) \\ \qquad = u_{fpid}(nT) - GU\Delta u_{fpid}(nT) \end{cases} \quad (31)$$

We can obtain the norm of the incremental fuzzy PID control law from (17).

$$\|S_1(e_1(nT))\| = \left| \frac{L\cdot GU\cdot (GE\cdot e(nT)+GR\cdot r(nT))}{2(2L-\max(GE|e(nT)|), GR|r(nT)|))} + \frac{0.5L\cdot GU\cdot GA\cdot a(nT)}{2(2L-\max(GR|r(nT)|), GA|a(nT)|))}\right|$$

$$\leq \left| \frac{L\cdot GU\cdot (GE\cdot e(nT)+GR\cdot r(nT))}{2(2L-max(GE|e(nT)|), GR|r(nT)|))}\right| + \left|\frac{0.5L\cdot GU\cdot GA\cdot a(nT)}{2(2L-max(GR|r(nT)|), GA|a(nT)|))}\right|$$

$$\leq \frac{L\cdot GU\cdot (GE\cdot |e(nT)|+GR\cdot |r(nT)|)}{2(2L-\max(GE|e(nT)|), GR|r(nT)|))} + \frac{0.5L\cdot GU\cdot GA\cdot |a(nT)|}{2(2L-\max(GR|r(nT)|), GA|a(nT)|))}$$

$$\leq \frac{L\cdot GU\cdot (GE\cdot |e(nT)|+GR\cdot |r(nT)|)+0.5\,GA\cdot |a(nT)|}{2(2L-\max(GE|e(nT)|), GR|r(nT)|, GA|a(nT)|))}$$

$$\leq \frac{L\cdot GU\cdot (GE+\frac{2GR}{T}+\frac{2GA}{T^2})}{2(2L-\max(GE|e(nT)|), GR|r(nT)|, GA|a(nT)|))}\cdot |e(nT)|$$

$$= \gamma_{1p}\cdot |e(nT)| \qquad (32)$$

where

$$\gamma_{1p}=\frac{L\cdot GU\cdot (GE+\frac{2GR}{T}\frac{+2GA}{T^2})}{2(2L-\max(GE|e(nT)|), GR|r(nT)|, GA|a(nT)|))},$$

$$sup|r(nT)|=sup\left|\frac{e(nT)-e(nT-T)}{T}\right|=sup\frac{2}{T}|e(nT)|,$$

$$sup|a(nT)|=sup\left|\frac{r(nT)-r(nT-T)}{T}\right|=sup\frac{4}{T^2}|e(nT)|.$$

For a general tracking problem, $sup|e(nT)|$ is either the reference value or the maximum overshoot value.

The norm of the nonlinear plant can be obtained such as

$$\|S_2(e_2(nT))\|\leq\|N\||e_2(nT)|=\gamma_{2p}\cdot |e_2(nT)| \qquad (33)$$

where γ_{2p} = $\|N\|$ in which is defined by

$$\|N\|:=sup\frac{|N(v_1(nT))-N(v_2(nT))|}{|v_1(nT)-v_2(nT)|}. \qquad (34)$$

From the stability condition (28) of the small gain theorem

$$\gamma_{1p}\cdot\gamma_{2p}<1$$

$$\frac{L\cdot GU\cdot (GE+\frac{2GR}{T}+\frac{2GA}{T^2})}{2(2L-G_{\max})}\cdot\|N\|<1. \qquad (35)$$

where

$$G_{\max}=\max(GE|e(nT)|), GR|r(nT)|, GA|a(nT)|).$$

According to the design condition of a fuzzy PID controller, we always have $0\leq G_{max}\leq L$. In (35) the minimum of $\frac{L}{2(2L-G_{\max})}$ is 1/4. This minimum is acquired when $e(nT)=0$, $r(nT)=0$, and $a(nT)=0$ i.e. under the steady-state condition of the system output. According to the minimum, we can simplify and rewrite the stability condition (35) as

$$\frac{GU}{4}\cdot(GE+\frac{2GR}{T}+\frac{2GA}{T^2})\cdot\|N\|<1. \qquad (36)$$

Through the procedure from (30) to (35), the corresponding stability condition about the incremental linear PID controller (22) is

$$(K_i^c+\frac{2}{T}K_p^c+\frac{4}{T^2}K_d^c)\cdot\|N\|<1 \qquad (37)$$

Hence if we let

$$\frac{GU\cdot GE}{4}=K_i^c,\ \frac{GU\cdot GR}{4}=K_p^c,\ \frac{GU\cdot GA}{8}=K_d^c \qquad (38)$$

$$GU=T,\ GE=\frac{4K_i^c}{T},\ GR=\frac{4K_p^c}{T},\ GA=\frac{8K_d^c}{T} \qquad (39)$$

(36) will be identical with (37). This result implies that a fuzzy PID controller has the same stability as the incremental PID controller. It we can decide the gains of the PID controller with the gains obtained by (38) or (39) can make the nonlinear plant stable. This is true whether the plant is linear or nonlinear.

4. Simulations

In this section, several computer simulations for the fuzzy PID controller developed in this paper is accomplished and their results are compared with the results of the conventional digital PID controller. The computer simulations are accomplished for the 2nd-order linear plant and nonlinear plant. With the gains K_p^c, K_i^c and K_d^c of the digital PID controller are obtained, the gains of a fuzzy PID controller are determined through (38).

4.1 Linear plant

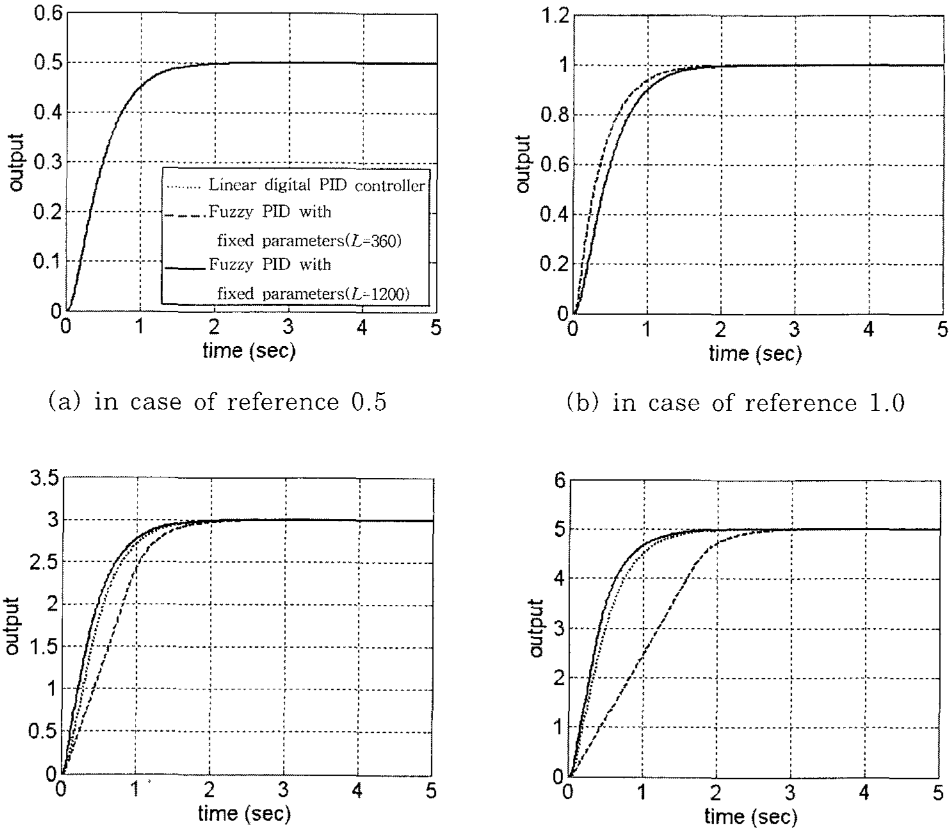
The plant transfer function used in the simulation is

$$G(s)=\frac{10}{s(s+1)}\ . \tag{40}$$

For the references 0.5, 1.0, 3.0 and 5.0, the outputs of the control system with a linear digital PID controller and fuzzy PID controllers with L=360 and L=1200 are shown in Fig. 6.

In case of reference 0.5(Fig. 6(a)), the output of the fuzzy PID controller system is similar to the output of the digital PID control system.

Because a fuzzy PID controller is designed to be identical to the linear digital PID controller in the steady-state, in either the small control input or the



(a) in case of reference 0.5

(b) in case of reference 1.0

(c) in case of reference 3.0

(d) in case of reference 5.0

Fig. 6 Response curves of controllers for linear system simulations

narrow operational range around 0, the gain scheduling of a fuzzy PID controller has reduced and is almost fixed such as the gains of a linear digital PID controller.

In case of reference 1 (Fig. 6(b)), the output of a fuzzy PID controller with $L=1200$ is similar to the output of the linear digital PID controller but the output of a fuzzy PID controller with $L=360$ has the superior performance to the output of linear digital PID controller by gain scheduling.

In case of reference 3, (Fig. 6(c)), a fuzzy PID controller with $L=360$ has the more limited control performance than the linear digital PID controller because the FLC input beyond L is limited to L by a fuzzifier and a limited FLC input generates insufficient control input at the initial. But a fuzzy PID controller with $L=1200$ has the sufficient fuzzy input spaces and has the superior control performance to the linear digital PID controller by gain scheduling. In case of reference 5(Fig. 6(d)), the larger reference input make the rising-time of a fuzzy control system late.

A fuzzy PID controller with small L has the excellent performance in case of the small FLC input, but may cause time-delay and bad transient response in case of the large FLC input beyond L. A fuzzy PID controller with large L has excellent performance in case of the large FLC input within $|L|$, but has the same performance as the linear PID controller in case of the small FLC input around 0. Therefore, according to the reference input and the operation range of the control system, the choice of the appropriate L is required. Parameters of the controllers used in computer simulation is shown in Table 3.

Table 3 Parameters of controllers for linear system simulations

	Parameters							
	K_i^c	K_p^c	K_d^c	GE	GR	GA	GU	L
Linear Digital PID Controller	90	45	5					
Fuzzy PID Controller with Fixed Parameters				360	180	40	1	360
				360	180	40	1	1200

4.2 Nonlinear plant

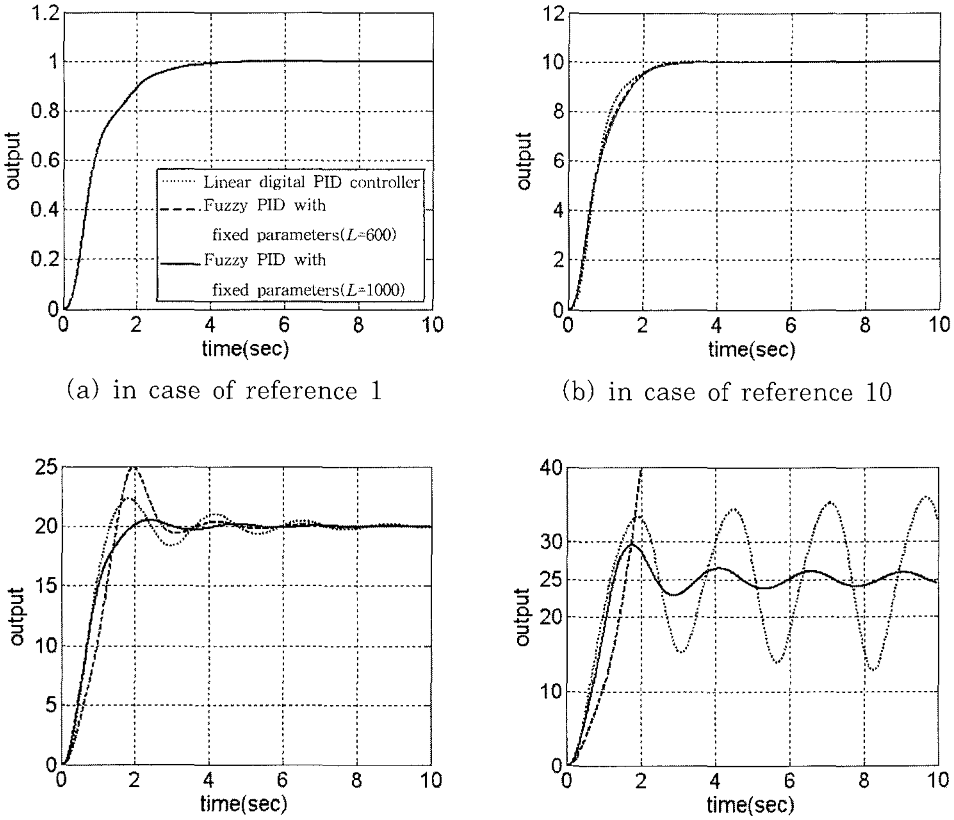
The plant transfer function used in the simulation is

$$\ddot{y}+\dot{y}=0.5y^2+2u. \tag{41}$$

For the references 1, 10, 20 and 25, the outputs of the control system with the linear digital PID controller and fuzzy PID controllers with $L=600$ and $L=1000$ are shown in Fig. 9.

In cases of reference 1 and 10, fuzzy PID controllers have the same control performance as the linear digital PID controller.

In case of reference 20. A fuzzy PID controller with $L=600$ has the large overshoot but a fuzzy PID controller with $L=1000$ has the small overshoot and becomes stable fast. The overshoot of a fuzzy PID controller with $L=600$ is larger than that of a fuzzy PID controller with $L=1000$ because the former has the more limited input space than the latter and generates the insufficient control input in the initial state.



(a) in case of reference 1

(b) in case of reference 10

(c) in case of reference 20

(d) in case of reference 25

Fig. 7 Response curves of controllers for nonlinear system simulations

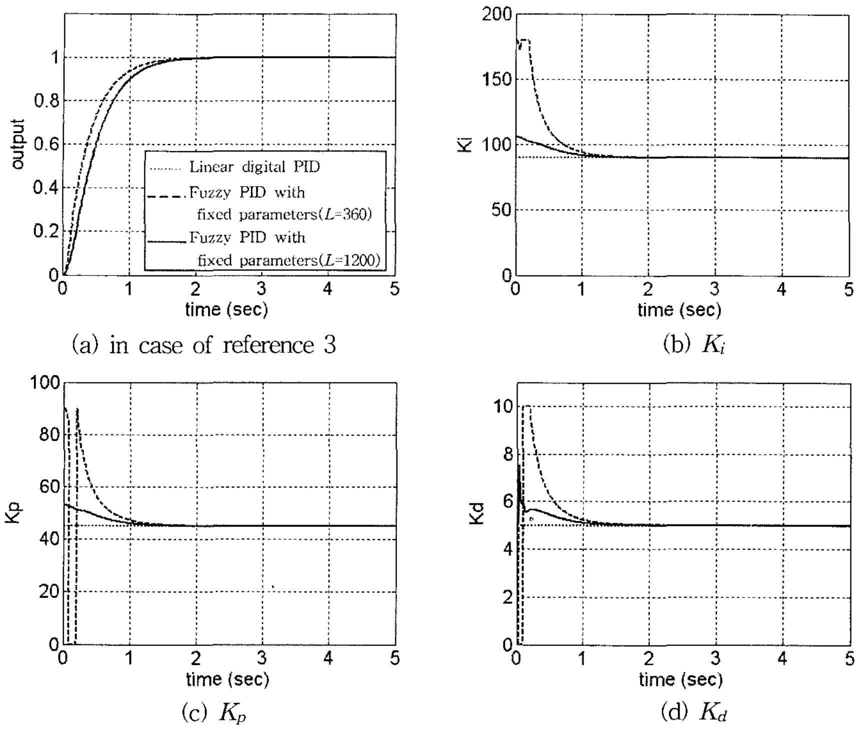
Table 4 Parameters of controllers for nonlinear system simulations

	Parameters							
	K_i^c	K_p^c	K_d^c	GE	GR	GA	GU	L
Linear Digital PID Controller	15	15	2					
Fuzzy PID Controller with Fixed Parameters				60	60	16	1	600
				360	180	40	1	1000

In case of reference 25, the output of the linear digital PID controller begins to diverge. While the fuzzy PID control system with L=600 diverges in initial state because the input fuzzy space limited by L makes the controller generate the insufficient control input, a fuzzy PID control system with L=1000 becomes the stable undamped system. Parameters of conrollers used in computer simulation is shown in Table 4.

4.3 Analysis of effect of gains of controller

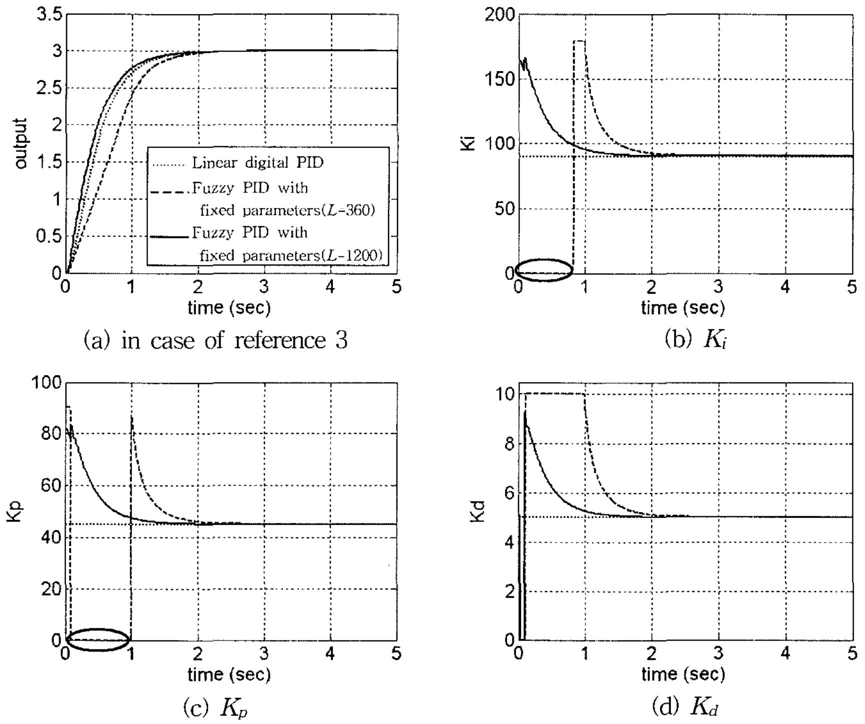
We analyze the gains K_p, K_i and K_d of fuzzy PID controllers with various L as compared with a linear digital PID controller. The simulation result and the changes of controller parameters are shown in Fig. 8 and 9, for the cases of reference 1 and 3, respectively.



(a) in case of reference 3 (b) K_i

(c) K_p (d) K_d

Fig. 8 Response of control system and changes of controller parameters in case of reference 1



(a) in case of reference 3 (b) K_i

(c) K_p (d) K_d

Fig. 9 Response of control system and changes of controller parameters in case of reference 3

In case of reference 1 (Fig. 8), while a fuzzy PID controller with L=360 has the abundant gain scheduling and resultantly has the fast response, a fuzzy PID controller with L=1200 has the poor gain scheduling and is similar to the linear digital PID controller with fixed gains. This result is caused by the fuzzy input space designed by L.

In case of reference 3 (Fig. 9), a fuzzy PID controller with L=360 has the delayed gain scheduling shown in ⬭ of (b) and (c) and the scaled *error*, *rate* and *acc* beyond L are restricted to L until the scaled inputs come inside L. The restricted fuzzy input spaces generate insufficient control inputs at initial state and resultantly cause time-delay and slow response as (c) and (d) and make the nonlinear system have the bad transient response as (c) or the unstable response like (d). A fuzzy PID controller with larger L like 1200 has the proper gain scheduling at initial state and the better system response than a fuzzy PID controller with small L.

5. Conclusion

In this paper, we have designed a simple form of a fuzzy PID controller, and described the design principle, tracking performance, stability analysis and change of parameters of the suggested fuzzy PID controller. For a new simple form of a fuzzy PID controller, we have the conclusions that :

(1) The suggested fuzzy PID controller has the similar structure as a linear digital PID and has the variable gains in comparison with a linear digital PID controller. A simple form of a fuzzy PID controller is introduced to increase the applicability of a fuzzy PID controller to digital computer.

(2) The BIBO stability of a overall fuzzy PID control system has proven by the small gain theorem from a stable linear digital PID control system. The derived stability condition of a fuzzy PID control system makes the parameters of a fuzzy PID controller decided from the stable linear digital PID controller.

(3) Through the computer simulations for the linear and nonlinear plants, It has assured that a suggested fuzzy PID controller has the prior control performance to a linear digital PID controller and the appropriate design of L should be required to improve the stability and the transient response of a fuzzy control system along the reference and the operation range.

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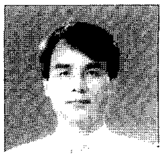

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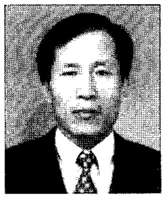

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