Energy-based Approach to Power Transfer System Analysis

Young-Hyun Moon*, Jong-Gi Lee† and Yong-Jun Kwon*

Abstract - This paper presents a new theoretical approach to energy-based power system analysis for multibus power transmission systems. On the basis of mechanical analogy, an exact energy integral expression is derived for lossy multi-bus systems through rigorous energy analysis. A simple rigid rod model of mechanical power transfer system is introduced to address the physical meanings of potential energy terms associated with transfer conductances as well as transfer susceptances. Finally, energy-based analysis has been proposed to show that the energy function has all information of the power system characteristics.

Keywords: Direct method, Energy function, EMM (Equivalent Mechanical Model), Lagrange's Equation, Energy Integral

1. Introduction

This paper presents a new theory of energy-based analysis by using an energy integral based on an Equivalent Mechanical Model (EMM) for stability analysis of multimachine power systems. The energy-based approach has been preferred for long by the system engineers since it provides systematic procedures to set up the system dynamic equations based on the energy function [1, 2].

Many approaches have been presented to develop energy functions for power systems by using mechanical analogy and mathematical approaches. Most of earlier energy functions have some limitations in view of the fact that a lossless power system has been considered with the classical generation model[3, 4]. Introduction of the structure-preserving energy concept[3-6] has recently made considerable progress in the development of energy functions to take into account the effects of reactive powers and transmission-line resistances. The previous works[7, 8] also show that a well-defined energy function can be derived to reflect the transfer conductances under the assumption of uniform R/X ratios for all transmission-lines.

It is a well-known fact that the energy function has all information on the system dynamics, which leads to energy-based system analysis including the Lagrange's equations and the Hamiltonian equations.

The power system is one of the most complicate systems including various kinds of system models and device models. Moreover, the power system is a special system to transfer electric power with some specified angular velocity with invisibility. This makes it more difficult to grasp the physical meanings of each term of the energy functions. For example, the structure preserved energy function includes potential energy terms associated with a power dissipation element of a transfer conductance, which is an uncommon result to bring about a lot of questions. Due to these facts, there has been no attempt to apply the energy-based system analysis to power systems, which keeps power system analysis remaining an area difficult of access for general system engineers.

This paper adopts an exact EMM to represent the multimachine systems with lossy transmission-lines[9, 10] to develop a new approach to energy-based analysis. On the basis of the EMM, an exact integral expression is derived for lossy multimachine systems through rigorous mathematical analysis. This provides the theoretical background of the energy-based power system analysis since the proposed EMM is a mechanical system which exactly represents the power system dynamics. In order to make out the physical meanings of the potential energy terms related to transfer conductances, this study investigates a simple mechanical power transfer system with a rigid rod for mechanical analogy of the electric power transmission system. As a result, it has been shown that the energy terms associated with line susceptances represent the energy store which is inevitably required for power transfer. Similarly, it has been discussed that the conductance-related energy terms can be interpreted as an inevitable power loss which is required to transfer powers in electric power transmission systems. The Rayleigh dissipation functions [15] have been commonly used to model resistive elements in power circuits. However, they are applicable for instantaneous energy analysis of the system, but no longer valid for steady state power transfer analysis. These provide a proper environment for us to

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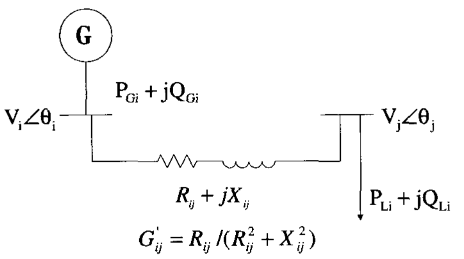
attempt to apply an energy-based approach to power systems.

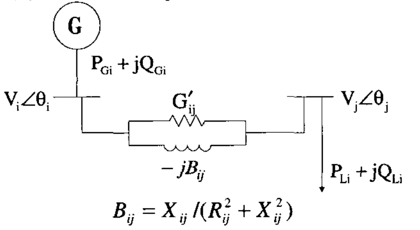
This study shows that Lagrange's equations can be well applied to power system analysis with the proposed energy integral, providing systematic procedures to set up the power system dynamic equations and additionally the load flow equations for the static solutions. This means that the energy integral has all information of power system dynamics in both static and dynamic states. The proposed approach enables us to utilize well-developed theories and mathematical tools for the analysis of power system dynamics.

2. Energy Integral Based on the Exact EMM

To begin with, consider a two-bus system with a resistive and reactive transmission line in Fig.1 (a).



(a) Two-Bus System with a resistive



(b) Equivalent System and reactive line

Fig. 1 Sample of Resistive and Reactive System

This system can be changed to an equivalent system with two parallel lines: one with pure susceptance B_{ij} and the other with pure conductance G'_{ij} as shown in Fig.1 (b). For the system in Fig.1 (b), the following EMM has been successfully developed with introduction of the real and imaginary springs in the previous works[9, 10].

$$\begin{aligned}\mathbf{F}_{ij} &= \mathbf{F}_{ij}^G + \mathbf{F}_{ij}^B \\ &= G_{ij}(V_i - V_j\cos\theta_{ij})\hat{\theta} - G_{ij}V_i\sin\theta_{ij}\hat{\mathbf{r}} \\ &\quad - B_{ij}V_j\sin\theta_{ij}\hat{\theta} + B_{ij}(V_i - V_j\cos\theta_{ij})\hat{\mathbf{r}} \\ &= [B_{ij}(V_i - V_j\cos\theta_{ij}) - G_{ij}V_i\sin\theta_{ij}]\hat{\mathbf{r}} \\ &\quad - [B_{ij}V_j\sin\theta_{ij} - G_{ij}(V_i - V_j\cos\theta_{ij})]\hat{\theta}\end{aligned} \tag{1}$$

where $G_{ij} = -G'_{ij} = \mathrm{Re}[Y_{ij}^{Bus}]$

$G'_{ij} = R_{ij}/(R_{ij}^2 + X_{ij}^2)$

$B_{ij} = X_{ij}/(R_{ij}^2 + X_{ij}^2)$

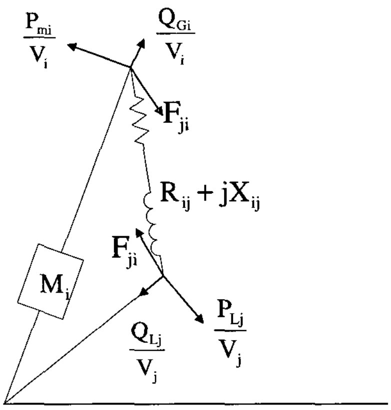


Fig. 2 EMM using Line Impedances

The EMM developed for a general two-bus system can be easily generalized for multibus systems. For example, we will consider the following 3-bus system, which is the smallest system including all types of buses. The internal reactances of the generators can be included in the transmission impedances by eliminating the generator terminal buses. However, the generator internal impedance can be directly taken into account by considering an element of transmission line with the internal impedance of pure reactance.

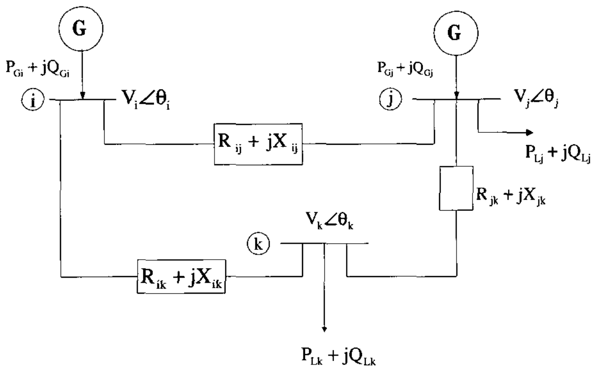


Fig. 3 3-Bus System

By assuming $\theta_i > \theta_j > \theta_k$, we can obtain the following EMM for the above system with the impedance model.

By observing the above force diagram, it can be shown that the following force balance equations hold for arbitrary bus i:

$$\frac{M_i\ddot{\theta}_i}{V_i}\hat{\theta} + \frac{D_i\dot{\theta}_i}{V_i}\hat{\theta} = \frac{P_{mi} - P_{Li}}{V_i}\hat{\theta} + \frac{Q_{Gi} + Q_{Li} - Q_{Li}}{V_i}\hat{\mathbf{r}} + \sum_{j\neq i}\mathbf{F}_{ij} \tag{2}$$

where $i,j \in \{(i),(j),(k)\}$

One can easily check that a substitution of (1) into the above equation provides two directional force balance equations which exactly agree with the power swing equations for a generator bus, and with real and reactive power balance equations for a load bus [10]. The proposed EMM can be applied to any multimachine system if the classical model is adopted for all generators.

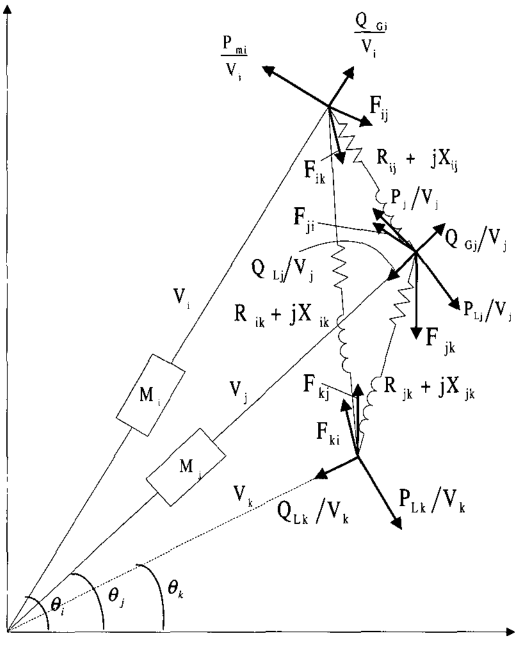


Fig. 4 EMM for 3-bus System

An energy integral expression now be derived with the use of the exact EMM. The classical generator model is adopted in the energy integral derivation. The energy integral of a system can be obtained by integrating the differential energy dW due to its differential displacements. In this section, an energy integral expression for power systems is derived based on rigorous analysis of differential energy changes in the EMM.

Considering the differential energy dW_i due to the differential changes in voltage magnitudes and phase angles, i.e., dV_i and $d\theta_i$ for an EMM of the 3-bus system in Fig. 4. The energy integral of the whole system can be calculated by integrating the total sum of the differential energy dW_i for each bus i as follows:

$$W =_c \int (\sum_i dW_i) \tag{3}$$

where c is an integral path which should be a solution path

In the above equation, differential energy dW_i is given by the scalar product of the total force acting on bus i and the differential distance vector.

That is

$$dW_i = -\mathbf{F}_{i,ttl} \cdot d\mathbf{x}_i = -\mathbf{F}_{i,ttl} \cdot (V_i d\theta_i \hat{\theta} + dV_i \hat{\mathbf{r}}) \tag{4}$$

where

$$\mathbf{F}_{i,ttl} = \left[-\frac{M_i \ddot{\theta}_i}{V_i} - \frac{D_i \dot{\theta}_i}{V_i} + \frac{P_{mi} - P_{Li}}{V_i} \right] \hat{\theta} + \frac{Q_{Gi} + Q_{Ci} - Q_{Li}}{V_i} \hat{\mathbf{r}} + \sum_{j \neq i} \mathbf{F}_{ij} \equiv 0 \tag{5}$$

In the above equations, $\mathbf{F}_{ij}$ is given by (1). It is noted that $\mathbf{F}_{i,ttl}$ is always zero by (2). Substitution of (5) into (4) yields:

$$\begin{aligned} dW_i = &\sum_{j \neq i} \{ [G_{ij}(V_i - V_j \cos\theta_{ij}) + B_{ij} V_j \sin\theta_{ij}] V_i d\theta_i \\ &+ [G_{ij} V_i \sin\theta_{ij} + B_{ij}(V_i - V_j \cos\theta_{ij})] dV_i \} \\ &- (P_{mi} - P_{Li}) d\theta_i - \left[\frac{Q_{Gi} + Q_{Ci} - Q_{Li}}{V_i} \right] dV_i \\ &+ M_i \ddot{\theta}_i d\theta_i + D_i \dot{\theta}_i d\theta_i \equiv 0 \end{aligned} \tag{4'}$$

The total differential energy due to all the differential displacements of $d\theta_i$ and dV_i ($i=1,2,\cdots,N$) can be calculated as follows :

$$dW = \sum dW_i \tag{6}$$

Now, we let $\omega_i = \dot{\delta}_i = \dot{\theta}_i$ and $\delta_i = \theta_i$ for generator bus i. By using the properties of Y_{BUS} and the differential rules, the following equation can be obtained with skillful manipulations [7, 11].

$$\begin{aligned} dW = &-\sum d\left(\frac{1}{2} B_{ii} V_i^2\right) + \frac{1}{2} \sum_i \sum_{j \neq i} d\left(-B_{ij} V_i V_j \cos\theta_{ij}\right) \\ &+ \sum_i G_{ii} V_i^2 d\theta_i + \sum_i \sum_{j \neq i} G_{ij} \begin{pmatrix} V_i V_j \cos\theta_{ij} d\theta_i \\ + V_j \sin\theta_{ij} dV_i \end{pmatrix} \\ &- \sum_i P_{mi} d\theta_i + \sum_i P_{Li} d\theta_i - \sum_i \left[\frac{Q_{Gi} + Q_{Ci} - Q_{Li}}{V_i} \right] dV_i \\ &+ \sum_i M_i \omega_i d\omega_i + \sum_i D_i \omega_i d\delta_i \equiv 0 \end{aligned} \tag{7}$$

The energy integral can be obtained by integrating (7) from the initial state to a certain state.

$$\begin{aligned} W^{(t)} - W^{(t_0)} &=_c \int_{(\theta_0, V_0)}^{(\theta, V)} dW \\ &= \sum_i^N \Big\{ -\frac{1}{2} B_{ii} V_v^2 - \frac{1}{2} \sum_{j \neq i} B_{ij} V_i V_j \cos\theta_{ij} +_c \int_{\theta_0}^{\theta} G_{ii} V_i^2 d\theta_i \\ &+ \sum_{j \neq i}^N G_{ij\,c} \int_{(\theta_0, V_0)}^{(\theta, V)} \left(V_i V_j \cos\theta_{ij} d\theta_i + V_j \sin\theta_{ij} dV_i \right) \\ &-_c \int_{\theta_0}^{\theta} P_{mi} d\theta_i +_c \int_{\theta_0}^{\theta} P_{Li} d\theta_i -_c \int_{V_0}^{V} \left[\frac{Q_{Gi} + Q_{Ci} - Q_{Li}}{V_i} \right] dV_i \\ &+ \frac{1}{2} M_i \omega_i^2 +_c \int_{t_0}^{t} D_i \omega_i^2 dt \Big\} - C(\omega_0, \theta_0, V_0) \equiv 0 \end{aligned} \tag{8}$$

with $\theta_i = \delta_i, V_i = E_{Gi} = constant$ for Gen. Bus i

$P_{mi} = 0, M_i = 0, D_i = 0, Q_{Gi} = 0$ for load bus i

$\theta = [\theta_1, \theta_2, \cdots, \theta_N]^T \quad V = [V_1, V_2, \cdots, V_N]^T$

$C(\omega_0, \theta_0, \mathbf{V}_0)$: *constant* determined by the initial conditions

In the above equation, it is noted that all of the integral terms are path dependent, which are difficult to deal without appropriate assumptions. For practical applications, one can eliminate the path dependency by assuming constant mechanical inputs, constant loads, and lossless transmission system. It is noted that a useful and practical energy function can be derived by equivalent system technique under assumption of constant R/X ratio [10]. However, we will keep path dependency in the energy integral for rigorous energy analysis from the theoretical point of view.

As mentioned earlier, the fact that the above energy integral should always be zero implies the energy conservation law in the EMM for a power system. By taking account $dV_i = 0$ for the voltage-controlled buses, a structure-preserved energy function can be easily obtained by removing the time integral term in (8).

$$
\begin{aligned}
E = \sum_{i}^{m} \frac{1}{2} M_i \omega_i^2 + \sum_{i}^{N} \Big\{ & -\frac{1}{2} B_{ii} V_i^2 - \frac{1}{2} \sum_{j \neq i}^{N} B_{ij} V_i V_j \cos\theta_{ij} \\
& + {}_c\int_{\theta_{i0}}^{\theta_i} G_{ii} V_i^2 d\theta_i + \sum_{j \neq i}^{N} G_{ij\,c}\int_{(\theta_{i0}, V_{i0})}^{(\theta_i, V_i)} \left(V_i V_j \cos\theta_{ij} d\theta_i + V_j \sin\theta_{ij} dV_i \right) \\
& - {}_c\int_{\theta_{i0}}^{\theta_i} P_{mi} d\theta_i + {}_c\int_{\theta_{i0}}^{\theta_i} P_{Li} d\theta_i + {}_c\int_{V_{i0}}^{V_i} \frac{Q_{Li}}{V_i} dV_i \Big\}
\end{aligned}
\tag{9}
$$

The above energy function is the same given in a text book by Fouad and Vittal [16].

From (8), it can be shown that the energy integral has an alternative expression as follows

$$E(t) = E(t_0) - \sum_i \left(\int D_i \omega_i^2 dt \right) \tag{10}$$

By using the above equation, the semi-negativeness of the time derivative of the energy function(9) can be easily proven.

3. Physical Interpretations of Potential Energy Terms in Power Transfer Systems

The potential energy in (9) can be interpreted as the energy stored in the line susceptances and the line losses due to the line conductances which are inevitably required for power transfer. This can be explained by the mechanical analogy of the electric power transfer system. Consider the following mechanical power transfer system composed of a simple rigid rod. It is assumed that the rigid rod is rotating with constant velocity ω_0 to transfer the mechanical torque T_{mi} from node i to node j.

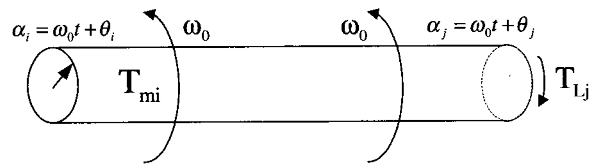


Fig. 5 Rigid Rod Model

When input torque T_{mi} is applied to the rod, the angle difference between α_i and α_j should be increased proportional to the power transfer to the load. This makes some potential energy charged at the rod. Let K_{ij} be a twisting constant like as a spring constant of a spring, then we can obtain the following energy balance equation from the energy conservation law.

$$\int_{\theta_{i0}}^{\theta_i} T_{mi} d\alpha_i - \int_{\theta_{j0}}^{\theta_j} T_{Lj} d\alpha_j = \int_{\theta_{ij0}}^{\theta_{ij}} K_{ij} (\alpha_i - \alpha_j) d(\alpha_i - \alpha_j) \tag{11}$$

$$
\begin{aligned}
\text{where } & d\alpha_i = \omega_0 dt + d\theta_i, \quad d\alpha_{ij} = d\theta_{ij} \\
& d\alpha_j = \omega_0 dt + d\theta_j, \quad \alpha_{ij} = \alpha_i - \alpha_j = \theta_{ij}
\end{aligned}
\tag{12}
$$

With the relationship in (12), (11) can be rewritten as

$$
\begin{aligned}
& \int_{t_0}^{t} (T_{mi} - T_{Li}) \omega_0 dt + \int_{\delta_{i0}}^{\delta_i} T_{mi} d\theta_i - \int_{\delta_{j0}}^{\delta_j} T_{Lj} d\theta_j \\
& = \int_{\delta_{ij0}}^{\delta_{ij}} K_{ij} \theta_{ij} d\theta_{ij}
\end{aligned}
\tag{13}
$$

In the case where many rods are connected, the total energy should satisfy the following equation:

$$
\begin{aligned}
& \sum \int_{t_0}^{t} (T_{mi} - T_{Li}) \omega_0 dt + \sum \int_{\delta_{i0}}^{\delta_i} T_{mi} d\theta_i - \sum \int_{\delta_{j0}}^{\delta_j} T_{Lj} d\theta_j \\
& = \sum \int_{\delta_{ij0}}^{\delta_{ij}} K_{ij} \theta_{ij} d\theta_{ij}
\end{aligned}
\tag{14}
$$

Without loss of generality, it can be assumed that the system be in the steady state in the macro-scopic sense with negligible rod inertia, i.e. the transients related to the rod inertia be negligible. This assumption corresponds to the assumption of stator/network transients being negligible in the electric power systems [15]. Under this assumption, it must hold

$T_{mi} = T_{Lj}$

As a result, (14) reduces to

$$\sum\int_{\delta_{i0}}^{\delta_i} T_{mi}\,d\theta_i - \sum\int_{\delta_{j0}}^{\delta_j} T_{Lj}\,d\theta_j = \sum\frac{1}{2}K_{ij}\theta_{ij}^2 \tag{15}$$

In the EMM of the power system, the transmission line is considered as an ideal rod with mass zero. This guarantees that $T_{mi} = T_{Lj}$, equivalently $I_{ij} = -I_{ji}$ in the steady state of power system. Since the rod rotates with angular velocity ω_0, we have the following power equation by multiplying ω_0 to the both sides of (15).

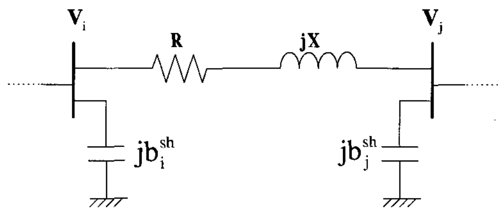
$$\sum\int_{\delta_{i0}}^{\delta_i} T_{mi}\omega_0\,d\theta_i - \sum\int_{\delta_{j0}}^{\delta_j} T_{Lj}\omega_0\,d\theta_j = \sum\frac{1}{2}K_{ij}\omega_0 T_{Lj}^2 \tag{16}$$

By using the mechanical analogy [11], the above equation corresponds to the following equation for the electric power transfer system.

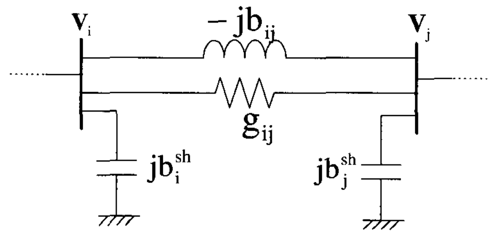
$$\sum\int_{\delta_{i0}}^{\delta_i} P_{mi}\,d\theta_i - \sum\int_{\delta_{j0}}^{\delta_j} P_{Lj}\,d\theta_j = \sum\frac{1}{2}\omega_0 L_{ij} I_{Lj}^2 \tag{17}$$

where $L_{ij} = K_{ij}$

It should be noted that the right side of (17) is the potential energy stored at the line inductance in the electric power system, which indicates that it is necessary to charge some potential energy in the line inductance in order to transfer electric power through the transmission line. The electric transmission line can be represented by the π-equivalent circuit as shown in Fig. 6. Each transmission line in Fig. 6(b) is correspondent to the rigid rod in Fig. 5 by the mechanical analogy.



(a) Transmission Line Model



(b) Equivalent Admittance Model

Fig. 6 The π-equivalent Circuit of Power Transmission Line

For the multibus power systems, the conventional energy function gives the following potential energy associated with the line susceptances and shunt capacitances.

$$E_{p1} = \frac{1}{2}\sum_{i=1}^{N}\left(b_i^{sh} - \sum_{j=1,j\neq i}^{N} b_{ij}\right)V_i^2 - \sum_{i=1}^{N}\sum_{j=i+1}^{N} V_i V_j b_{ij}\cos\theta_{ij} \tag{18}$$

$$\text{where} \quad B_{ii} = b_i^{sh} - \sum_{j=1,j\neq i}^{N} b_{ij} \tag{19}$$

$$B_{ij} = b_{ij} \tag{20}$$

With the relationships (19) and (20), the potential energy in (18) can be transformed into the following form.

$$\begin{aligned} E_{p1} &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=i+1}^{N} b_{ij}(V_i^2 - 2V_iV_j\cos\theta_{ij} + V_j^2) + \frac{1}{2}\sum_{i=1}^{N} b_i^{sh}V_i^2 \\ &= \sum_{i=1}^{N}\sum_{j=i+1}^{N}\frac{1}{2}\frac{1}{b_{ij}}\left|b_{ij}(\mathbf{V}_i - \mathbf{V}_j)\right|^2 + \sum_{i=1}^{N}\frac{1}{2}b_i^{sh}\left|\mathbf{V}_i\right|^2 \\ &= \sum_{i=1}^{N}\sum_{j=i+1}^{N}\frac{1}{2}\frac{1}{b_{ij}}\left|\mathbf{I}_{ij}^{B}\right|^2 + \sum_{i=1}^{N}\frac{1}{2}b_i^{sh}\left|\mathbf{V}_i\right|^2 \end{aligned} \tag{21}$$

where $\mathbf{I}_{ij}^B = jb_{ij}(\mathbf{V}_i - \mathbf{V}_j)$

The above equation shows that the energy terms associated with the line susceptance can be represented by ω_0 times of the energy stored in the corresponding elements, which is inevitably required for the power transfer as mentioned with (17) before. Therefore, the physical meaning of the potential energy terms in (18) can be interpreted as the energy stored in the susceptances or capacitances.

On the other hand, the energy function (9) includes the conductance-related potential energy, which is rather too complicate to make out some physical meaning of the energy terms. However, it can be roughly understood in the same context that the conductance-related potential energy represents the integration of the loss power dissipated by the conductances with respect to the mean rotating angle. This loss power can be interpreted as a potential energy which is inevitably required to support steady state electric power transfer through the parallel conductances as shown in Fig. 6. Therefore, the conductance-related energy in (9) has the properties of the potential energy. Here, it should be noted that the Rayleigh dissipation function has no relation with the conductance-associated energy terms in (9), but it has some relations with generator damping. The generator damping does not concern about the steady state power transfer. For example, any damping loss is not necessarily required to support the steady state power transfer going out from a generator. Therefore, the energy due to the damping loss has no relation with the potential energy. It is mere energy dissipation, which means that a Rayleigh dissipation function should be introduced to consider it. There have been a lot of questions arising about the transfer conductance-related potential energy even though a well-defined energy function can be developed to reflect the transfer conductances under some assumptions [9,10,14].

Example

It is well known that there exists an energy function for a two-bus power systems with lossy transmission lines [9,10]. A well-defined energy function can be easily constructed as given in (22) with the consideration of transfer conductances for a two-machine system for the uniform damping case[10]. This example shows that the transfer conductance produces potential energy terms in steady state power transfer analysis.

$$E=\frac{1}{2}M\omega^2+B\left(1+\frac{G^2}{B^2}\right)\left(\frac{1}{2}V^2-EV\cos(\delta-\theta)\right)-P_m(\delta-\delta_0) \quad (22)$$
$$+\left(P_L-\frac{G}{B}Q_L\right)(\theta-\theta_0)+\frac{G}{B}{}_c\int Q_G d\delta+\left(Q_L+\frac{G}{B}P_L\right)\log\frac{V}{V_0}$$

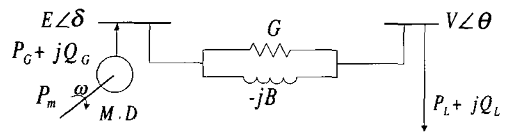


Fig. 7 Two-Bus System with Transmission Loss

The above discussions may give some answer about 'why the energy dissipation element has a potential energy in power transfer systems'. Consequently, the transfer-conductance-related potential energy can be considered to be just normal potential energy in power transfer systems.

4. Energy-Based Power System Analysis

Energy-based system analysis has been preferred so long by the system engineers since it provides a systematic procedure to set up the system dynamic equations with the use of energy functions. The energy function is a scalar function given by the total sum of the energy stored in system elements, most of which are decoupled of others. This provides great advantage in handling the energy functions. In the previous section, the energy integral has been successfully derived for the electric power system. The energy integral is just the same as the mechanical energy concerned to its mechanical analogy system. Consequently, the electric power system can be analyzed in the same manner as a mechanical system. Lagrange's equation is a well known method to analyze the mechanical system based on the system energy [15]. This study attempts to apply Lagrange's equations to power system analysis.

Define the kinetic energy T and the potential energy V for the electric power system as follows:

$$T=\sum\frac{1}{2}M_i\omega_i^2 \quad (23)$$

$$V=\sum_i\left\{-\frac{1}{2}B_{ii}V_i^2-\sum_{j\neq i}^{N}\frac{1}{2}B_{ij}V_iV_j\cos\theta_{ij}\right. \quad (24)$$
$$+{}_c\int_{\theta_{i0}}^{\theta_i}G_{ii}V_i^2 d\theta_i+\sum_{j\neq i}G_{ij}\,{}_c\int_{(\theta_{i0},V_{i0})}^{(\theta_i,V_i)}\left(V_iV_j\cos\theta_{ij}d\theta_i+V_j\sin\theta_{ij}dV_i\right)$$
$$\left.-{}_c\int_{\theta_{i0}}^{\theta_i}P_{mi}d\theta_i+{}_c\int_{\theta_{i0}}^{\theta_i}P_{Li}d\theta_i+{}_c\int_{V_{i0}}^{V_i}\frac{Q_{Li}}{V_i}dV_i\right\}$$

Then, the Lagrangean is defined by

$$L=T-V \quad (25)$$

Define a Rayleigh dissipation function associated with the generator damping as follows [17]:

$$F=\frac{1}{2}\sum_{i=1}^{m}D_i\omega_i^2 \quad (26)$$

The power system dynamics can be described by Lagrange's equation as follows:

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{q}_i}-\frac{\partial L}{\partial q_i}+\frac{\partial F}{\partial\dot{q}_i}=0 \quad (i=1,\cdots N) \quad (27)$$

For the EMM of the power system, (27) gives the power system dynamic equations.

$$M_i\dot{\omega}_i+D_i\omega_i-\frac{\partial L}{\partial\theta_i}=0 \quad i\in\{generator\ buses\} \quad (28)$$

$$-\frac{\partial L}{\partial\theta_i}=0 \quad i\in\{load\ buses\} \quad (29)$$

$$-\frac{\partial L}{\partial V_i}=0 \quad i\in\{voltage\ uncontrolled\ buses\} \quad (30)$$

In the above equations, it is required to pay special attention to evaluation of the partial differential. A comment on the partial differential rules is given in Appendix. By applying the proposed method, one can easily check the above equations provide just the power system swing equations including the real and reactive power balance equations. The power flow equations can be easily derived from the energy function in (9). The power flow solutions are given by the equilibrium points of the system, and the equilibrium condition of the system can be shortly described by using the potential energy V as follows:

$$\frac{\partial V}{\partial \mathbf{q}} = 0 \tag{31}$$

The above equation can be rewritten as

$$\frac{\partial V}{\partial \theta_i} = 0 \qquad i \in \{1,2,\cdots n\} \tag{32}$$

$$\frac{\partial V}{\partial V_i} = 0 \qquad i \in \{\textit{voltage uncontrolled bus}\} \tag{33}$$

It can be also easily checked that (32) and (33) are the same as the real and reactive power balance equations respectively. This study shows that Lagrange's equations can be well applied to power system analysis, providing systematic procedures to set up the power system dynamic equations and additionally the load flow equations for the static solutions with the proposed energy integral of power systems. This means that the energy integral has all information of power system dynamics in both static and dynamic states. The proposed approach enables us to utilize well-developed theories and mathematical tools for the analysis of power system dynamics, providing an easy access to power system analysis for general system engineers.

Example

Consider the following 3-bus system, in which the network is represented by $\mathbf{Y_{BUS}}$ in the form of $\mathbf{Y_{BUS}} = \mathbf{G_{BUS}} + \mathbf{jB_{BUS}}$.

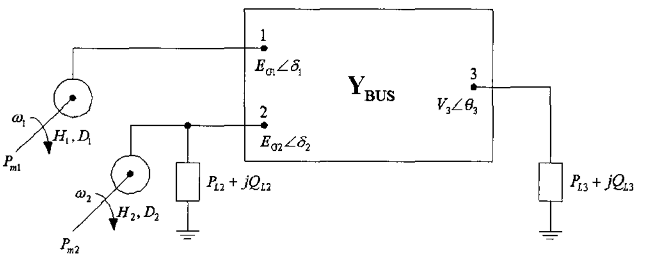


Fig. 8 3-bus Sample System

We will assume that Generator internal voltages E_{G1} and E_{G2} are kept to be constant during the time period in concern, and that $\mathbf{Y_{BUS}}$ is calculated after removing all the generator terminal buses for the post fault period. Then, the kinetic and the potential energies are given by

$$T = \frac{1}{2}M_1\omega_1^2 + \frac{1}{2}M_2\omega_2^2 \tag{34}$$

$$\begin{aligned} V = &-\frac{1}{2}B_{33}V_3^2 - E_{G1}E_{G2}B_{12}\cos(\delta_1-\delta_2) \\ &-\sum_{i=1}^{2}E_{Gi}V_3B_{i3}\cos(\delta_i-\theta_3) + \int G_{33}V_3^2 d\theta_3 \\ &+G_{12}\int E_{G1}E_{G2}\cos(\delta_1-\delta_2)(d\delta_1+d\delta_2) \\ &+\sum_{i=1}^{2}G_{i3}\int\begin{bmatrix} E_{Gi}V_3\cos(\delta_i-\theta_3)(d\theta_3+d\delta_i) \\ +E_{Gi}\sin(\delta_i-\theta_3)dV_3 \end{bmatrix} \\ &-P_{m1}(\delta_1-\delta_{10}) - P_{m2}(\delta_2-\delta_{20}) + P_{L2}(\delta_2-\delta_0) \\ &+P_{L3}(\theta_3-\theta_{30}) + \int\frac{Q_{L3}}{V_3}dV_3 \end{aligned} \tag{35}$$

The Lagrangian function can be organized by (25), having only 4 variables δ_1, δ_2, θ_3 and V_3. With use of the above energy function and Rayleigh function (26), Lagrange's equation yields 4 equations as follows:

$$\begin{aligned} &M_1\dot{\omega}_1 + E_{G1}E_{G2}B_{12}\sin(\delta_1-\delta_2) + E_{G1}V_3B_{13}\sin(\delta_1-\theta_3) \\ &+G_{12}E_{G1}E_{G2}\cos(\delta_1-\delta_2) + G_{13}E_{G1}V_3\cos(\delta_1-\theta_3) \\ &-P_{m1} + D_1\omega_1 = 0 \qquad \text{for } q_i = \delta_1 \end{aligned} \tag{36.a}$$

$$\begin{aligned} &M_2\dot{\omega}_2 + E_{G1}E_{G2}B_{12}\sin(\delta_2-\delta_1) + E_{G2}V_3B_{23}\sin(\delta_2-\theta_3) \\ &+G_{21}E_{G1}E_{G2}\cos(\delta_2-\delta_1) + G_{23}E_{G2}V_3\cos(\delta_2-\theta_3) \\ &-P_{m2} + P_{L2} + D_2\omega_2 = 0 \qquad \text{for } q_i = \delta_2 \end{aligned} \tag{36.b}$$

$$\begin{aligned} &-\sum_{i=1}^{2}E_{Gi}V_3B_{i3}\sin(\delta_i-\theta_3) + G_{33}V_3^2 + \sum_{i=1}^{2}G_{i3}E_{Gi}V_3\cos(\delta_i-\theta_3) + P_{L3} = 0 \\ &\text{for } q_i = \theta_3 \end{aligned} \tag{36.c}$$

$$\begin{aligned} &-B_{33}V_3 - \sum_{i=1}^{2}E_{Gi}\left(B_{i3}\cos(\delta_i-\theta_3) - G_{i3}\sin(\delta_i-\theta_3)\right) + Q_{L3}/V_3 = 0 \\ &\text{for } q_i = V_3 \end{aligned} \tag{36.d}$$

The first two of the equations are just the same as the swing equations for generators G_1 and G_2, and the last two of them are just the same as the power flow equations at bus 3. Moreover it should be noted that the above equations constitute a complete set of load flow equations when $\omega_i = 0$ and $\dot{\omega}_i = 0$ for all generators.

This example illustrates that the energy function has all information about the power system and that the energy-based method provides a systematic approach to analysis of complicate power systems with use of the energy function which can be easily obtained.

5. Conclusion

A new approach to energy-based power system analysis has been proposed by using an energy integral for multimachine power systems. On the basis of mechanical analogy, an exact energy integral expression is derived for lossy multi-bus systems through rigorous energy-based analysis. A simple mechanical rod model is introduced to make out the physical meanings of potential energy terms associated with transfer conductances as well as transfer susceptances. Finally, an approach to energy-based power system analysis has been proposed with the application of Lagrange's equations. This provides systematic procedures to set up the power system dynamic equations and additionally the load flow equations for the static solutions with the proposed energy integral of power systems. The proposed approach enables us to utilize well-developed theories and mathematical tools for the analysis of power system dynamics, guaranteeing an easy access to power system analysis for general system engineers.

Appendix: Partial Derivative with the Consideration Integral Dummy Variables

Consider the partial differentiation of $F(x, y)$ which is given by the path dependent integral as follows:

$$F(x,y) =_c \int_{(x_0,y_0)}^{(x,y)} [f_1(x,y)dx + f_2(x,y)dy] \quad \text{(A.1)}$$

First, we will examine the partial differential properties the integral function by distinguishing the dummy variables. The exact expression of (A.1) should be given by

$$F(x,y) =_c \int_{(x_0,y_0)}^{(x,y)} [f_1(\xi,\eta)d\xi + f_2(\xi,\eta)d\eta] \quad \text{(A.2)}$$

By using (A.2), we can easily calculate the partial derivatives as follows

$$F_x(x,y) = \frac{\partial}{\partial x}F(x,y) = f_1(x,y) \quad \text{(A.3)}$$

$$F_y(x,y) = \frac{\partial}{\partial y}F(x,y) = f_2(x,y) \quad \text{(A.4)}$$

The above result can easily be checked by comparing the form with the following line integral formula.

$$F(x,y) =_c \int_{(x_0,y_0)}^{(x,y)} [F_x(x,y)dx + F_y(x,y)dy]$$

We can calculate the time derivative of $F(x,y)$ in two ways: one is the method by changing the line integral into an integral with respect to time, and the other is using the differential chain rule. We can check here if both of the methods produce the same results. The integral in (A.1) can be changed into a time integral as follows:

$$F(x,y) = \int_{t_0}^{t} [f_1(x,y)\frac{dx}{dt} + f_2(x,y)\frac{dy}{dt}]dt \quad \text{(A.5)}$$

This directly gives the following time derivative

$$\frac{d}{dt}F(x,y) = f_1(x,y)\frac{dx}{dt} + f_2(x,y)\frac{dy}{dt} \quad \text{(A.6)}$$

By using the differential chain rule with (A.3) and (A.4), we can alternatively obtain the time derivative of $F(x,y)$ which just same as given in (A.6). This makes us confirm the exactness of mathematics regarding the differential rules.

For a special case, we will consider the following integral function

$$G(x,y) =_c \int_{(x_0,y_0)}^{(x,y)} g(x,y)dx \quad \text{(A.7)}$$

where y varies with x along the path c

The differential formulae in (A.3) and (A.4) provide the partial derivatives of $G(x,y)$.

$$\frac{\partial}{\partial x}G(x,y) = g(x,y)$$
$$\frac{\partial}{\partial y}G(x,y) = 0 \quad \text{(A.8)}$$

Here, it should be noted that the following calculation is erroneous unless y is independent of integral path c.

$$\frac{\partial}{\partial y}G(x,y) \neq_c \int_{x_0}^{x} \frac{\partial}{\partial y} g(x,y)dx$$

In the above equation, the equality holds only for the case where y is a constant parameter independent of integral path c. The above discussions can be easily extended to general integral functions with multivariables.

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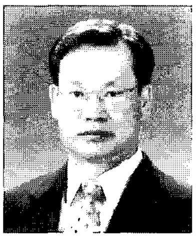

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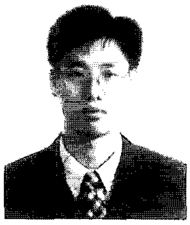

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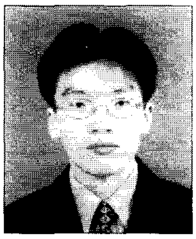

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