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Dispersion Characteristics of Periodically Loaded Conducting Posts in a Rectangular Waveguide as an Interaction Circuit of Broadband Gyro-TWT

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Abstract - A new type of interaction circuitry for a broadband gyro-TWT is presented in this paper. A method for the analysis of dispersion characteristics of a periodic structure, which is composed of conducting posts in a rectangular waveguide, is presented. This method utilizes a mode matching technique and equivalent circuit model. The calculated and measured results demonstrate that this periodic structure could be utilized for the interaction circuit of a wide band Gyro-TWT.

Keywords: Dispersion equation, Gyro-TWT, Mode matching technique, Periodic structure

1. Introduction

Extensive research has been performed on the periodic structure [1-2], which can be utilized in the interaction circuit of the wide band gyrotron traveling wave tube (Gyro-TWT). The Gyro-TWT is one of the most promising millimeter wave amplifiers because of its high power capability, resulting from an efficient fast wave interaction in a metal guide structure. However, the relatively narrow bandwidth of the Gyro-TWT is a significant limitation. Numerous methods to increase its bandwidth have been recommended. Among these suggested methods, one approach for increasing bandwidth is to reduce the dispersion of its interaction circuit. This waveguide interaction circuit can be made less dispersive by loading dielectric or corrugating the waveguide's wall.

All-metal circuits [3], such as the periodic disk-loaded waveguide, have been previously investigated for use as interaction circuits of the Gyro-TWT. However, the structure periodically loading conducting posts in a rectangular waveguide, which has the advantage of tuning, has not yet been investigated in this capacity.

In this paper, a dispersion characteristic of periodically loaded conducting posts in a rectangular waveguide is calculated using mode matching technique and equivalent circuit model to explore the use of the Gyro-TWT interaction circuit. The results indicate that the periodic structure in this paper can be employed in the interaction circuit of the wide band Gyro-TWT. In section II, the scattering parameters of an off-centered post in a waveguide are derived for mode matching techniques. In section III, the dispersion equation of periodically loaded conducting posts in a rectangular waveguide is obtained using a circuit model consisting of a transmission line and T-model. The calculated and measured results, indicating that the structure can be applied to the wide band Gyro-TWT, are presented in section IV. Finally, the conclusion is presented in section V.

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2. Scattering Parameter of an Off-centered Conducting Post

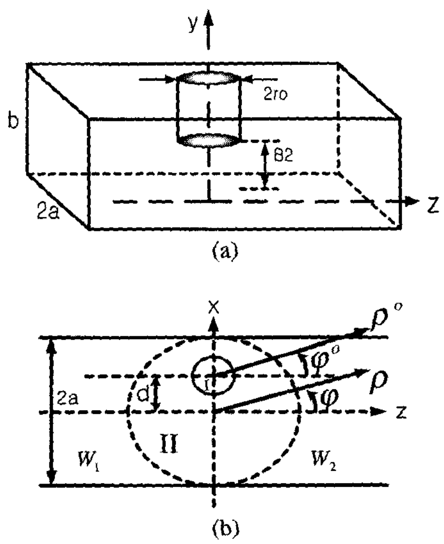

Fig. 1 Geometry of an off-centered post in a rectangular waveguide showing (a) a three-dimension view and (b) a top view.

Consider an off-centered conducting post in a rectangular waveguide as shown in Fig. 1. To analyze this structure, the cross section can be divided into four regions, waveguide region($\rho \geq$ a): W_1 and W_2 , cylindrical region: region I $(\rho^o \leq r_o, 0 \leq y \leq B_2)$ and region II $(r_o \leq \rho^0, \rho \leq a, 0 \leq y \leq b)$ by introducing an artificial boundary at ρ =a. Since the coordinate system (ρ^O, φ^O, y) of cylindrical region I is not concentric with the coordinate system (ρ, φ, y) of cylindrical region II, it is necessary to transform the coordinate system of region I into the coordinate system of region II, which is concentric with a main waveguide.

In the coordinate system (ρ^O, φ^O, y) of regions I and II, the tangential field in y - direction and φ° - direction can be expressed as (1) [4]

$$\bar{E}^p_{ct} = \sum_n \sum_m (C'^{ep}_{nm} J'_n(\beta^{ep}_p \rho^o) + D'^{ep}_{nm} Y'_n(\beta^{ep}_p \rho^o)) \, \bar{e}^{ep}_{ct}(\rho^o, \varphi^o, y) + \sum_n \sum_m (C'^{hp}_{nm} J_n(\beta^{hp}_p \rho^o) + D'^{hp}_{nm} Y_n(\beta^{hp}_p \rho^o)) \bar{e}^{hp}_{ct}(\rho^o, \varphi^o, y) \quad (1a)$$

$$\bar{H}^p_{ct} = \sum_n \sum_m (C'^{ep}_{nm} J_n(\beta^{ep}_p \rho^o) + D'^{ep}_{nm} Y_n(\beta^{ep}_p \rho^o)) \, \bar{h}^{ep}_{ct}(\rho^o, \varphi^o, y) + \sum_n \sum_m (C'^{hp}_{nm} J'_n(\beta^{hp}_p \rho^o) + D'^{hp}_{nm} Y'_n(\beta^{hp}_p \rho^o)) \, \bar{h}^{hp}_{ct}(\rho^o, \varphi^o, y) \quad (1b)$$

where $\bar{e}^{qp}_{ct}$, $\bar{h}^{qp}_{ct}$ (q = e for TE_y mode , h for TM_y mode and p = I, II) represent eigenfield and C'^{qp}_{nm} and D'^{qp}_{nm} are field coefficients in the coordinate system (ρ^O, φ^O, y), respectively. In the coordinate system (ρ, φ, y), the tangential field can also be expressed as in the above form.

By applying boundary conditions on an artificial boundary ($\rho = r_o$), one can obtain a matrix equation with respect to the field coefficient vector in region II and in the coordinate system (ρ^O, φ^O, y) shown in (2)

$$[\, M^{II}_{CD} \,] \begin{bmatrix} C'^{II} \\ D'^{II} \end{bmatrix} = 0 \quad (2)$$

Since the coordinate of cylindrical region I is not concentric with the coordinate of cylindrical region II, it is necessary to transform the coordinate system of region I into the coordinate system of region II [5]. Then, the field coefficient vector can be arranged as in (3)

$$\bar{C}'_m = \begin{bmatrix} C'^C_{0m} \\ C'^S_{1m} \\ C'^S_{2m} \\ \vdots \\ C'^C_{1m} \\ C'^C_{2m} \\ \vdots \end{bmatrix}, \quad \bar{D}'_m = \begin{bmatrix} D'^C_{0m} \\ D'^S_{1m} \\ D'^S_{2m} \\ \vdots \\ D'^C_{1m} \\ D'^C_{2m} \\ \vdots \end{bmatrix}, \quad \bar{C}_m = \begin{bmatrix} C^C_{0m} \\ C^S_{1m} \\ C^S_{2m} \\ \vdots \\ C^C_{1m} \\ C^C_{2m} \\ \vdots \end{bmatrix}, \quad \bar{D}_m = \begin{bmatrix} D^C_{0m} \\ D^S_{1m} \\ D^S_{2m} \\ \vdots \\ D^C_{1m} \\ D^C_{2m} \\ \vdots \end{bmatrix} \quad (3)$$

where C'^r_{nm} (D'^r_{nm}) and C^r_{nm} (D^r_{nm}) are field coefficients (r = s,c) in the coordinate system (ρ^O, φ^O, y) and (ρ, φ, y), respectively. s and c also represent the field coefficient of sinϕ and cosϕ, respectively.

The coordinate transformation yields the following relations between the coefficient vectors [5]:

$$\bar{C}_m = G^J \cdot \bar{C}'_m, \quad \bar{D}_m = G^J \cdot \bar{D}'_m \quad (4a)$$

$$G^J = \begin{bmatrix} g^{J00} & g^{J0S}_1 & g^{J0S}_2 & \cdots & g^{J0C}_1 & g^{J0C}_2 & \cdots \\ g^{JS0}_1 & g^{JSS}_{11} & g^{JSS}_{12} & \cdots & g^{JSC}_{11} & g^{JSC}_{12} & \cdots \\ g^{JS0}_2 & g^{JSS}_{21} & g^{JSS}_{22} & \cdots & g^{JSC}_{21} & g^{JSC}_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ g^{JC0}_1 & g^{JCS}_{11} & g^{JCS}_{12} & \cdots & g^{JCC}_{11} & g^{JCC}_{12} & \cdots \\ g^{JC0}_2 & g^{JCS}_{21} & g^{JCS}_{22} & \cdots & g^{JCC}_{21} & g^{JCC}_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \end{bmatrix} \quad (4b)$$

where

$$\begin{aligned} & g^{J00} = J_0(kd) \, , \quad g^{J0S}_n = (-1)^n J_n(kd) Sin\frac{n\pi}{2} \\ & g^{J0C}_n = (-1)^n J_n(kd) Cos\frac{n\pi}{2} \\ & g^{JS0}_p = 2J_p(kd) Sin\frac{p\pi}{2} \, , \quad g^{JC0}_p = 2J_p(kd) Cos\frac{p\pi}{2} \\ & g^{JSS}_{pn} = J_{p-n}(kd) Cos\frac{(p-n)\pi}{2} - (-1)^n J_{p+n}(kd) Cos\frac{(p+n)\pi}{2} \\ & g^{JSC}_{pn} = J_{p-n}(kd) Sin\frac{(p-n)\pi}{2} + (-1)^n J_{p+n}(kd) Sin\frac{(p+n)\pi}{2} \\ & g^{JCS}_{pn} = -J_{p-n}(kd) Sin\frac{(p-n)\pi}{2} + (-1)^n J_{p+n}(kd) Sin\frac{(p+n)\pi}{2} \\ & g^{JCC}_{pn} = J_{p-n}(kd) Cos\frac{(p-n)\pi}{2} + (-1)^n J_{p+n}(kd) Cos\frac{(p+n)\pi}{2} \end{aligned} \quad (4c)$$

Using the transformation shown in (4), a matrix equation in the coordinate system (ρ^O, φ^O, y) can be transformed into a matrix equation in the coordinate system (ρ, φ, y) shown in (5)

$$[\, M^{II}_{CD} \,][\, T^G \,] \begin{bmatrix} C^{II} \\ D^{II} \end{bmatrix} = 0 \quad (5)$$

where $[T^G]$ is a transformation matrix, which is arranged to fit for a mode combination of 'n' and 'm'.

In the waveguide region (W_1,W_2), the tangential field with respect to artificial boundary (ρ=a) can be represented as (6)

$$\begin{Bmatrix} \bar{E}_{wt}^{(1)}(x,y,z) \\ \bar{E}_{wt}^{(2)}(x,y,z) \end{Bmatrix} = \sum_{q=e,h}\sum_{n}\sum_{m}\left[\begin{Bmatrix} A_{nm}^{(1)q} \\ B_{nm}^{(2)q} \end{Bmatrix}\bar{e}_{wtnm}^{qF}(x,y,z) + \begin{Bmatrix} A_{nm}^{(1)q} \\ B_{nm}^{(2)q} \end{Bmatrix}\bar{e}_{wtnm}^{qF}(x,y,z)\right] \tag{6a}$$

$$\begin{Bmatrix} \bar{H}_{wt}^{(1)}(x,y,z) \\ \bar{H}_{wt}^{(2)}(x,y,z) \end{Bmatrix} = \sum_{q=e,h}\sum_{n}\sum_{m}\left[\begin{Bmatrix} A_{nm}^{(1)q} \\ B_{nm}^{(2)q} \end{Bmatrix}\bar{h}_{wtnm}^{qF}(x,y,z) + \begin{Bmatrix} A_{nm}^{(1)q} \\ B_{nm}^{(2)q} \end{Bmatrix}\bar{h}_{wtnm}^{qF}(x,y,z)\right] \tag{6b}$$

where $\bar{e}_{wtnm}^{qp}$ and $\bar{h}_{wtnm}^{qp}$ (q = e for TE_z , h for TM_z and p =F(forward wave), B(backward wave)) represent the transverse fields of an eigenmode.

By transforming a rectangular coordinate into a cylindrical coordinate and applying the boundary condition at the junction (ρ =a), the following equation can be obtained.

$$[R_{CD}]\begin{bmatrix} C^{II} \\ D^{II} \end{bmatrix} = [M_A]\begin{bmatrix} A^{(1)} \\ A^{(2)} \end{bmatrix} + [T_B]\begin{bmatrix} B^{(1)} \\ B^{(2)} \end{bmatrix} \tag{7}$$

By employing (5) and (7), the generalized scattering matrix $[S^P]$ of a post as shown in (8) can be obtained.

$$[S^P] = -\left\{[M_{CD}^{II}][T^G][R_{CD}]^{-1}[T_B]\right\}^{-1} \times\left\{[M_{CD}^{II}][T^G][R_{CD}]^{-1}[M_A]\right\} . \tag{8}$$

Thus, the scattering parameters of an off-centered conducting post in a waveguide can be calculated.

3. Analysis of Dispersion Characteristics

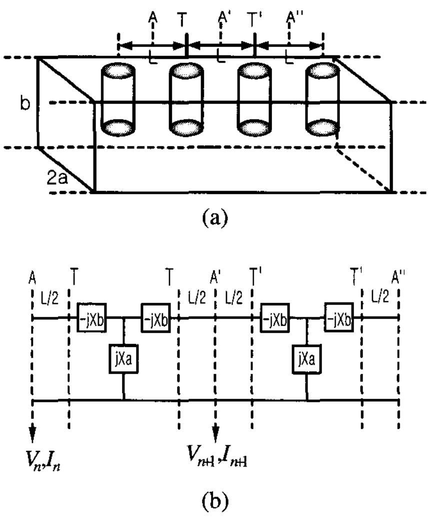

Fig. 2 Geometry of the investigated structure showing (a) a three dimensional view and (b) an equivalent circuit model from A to A.

The periodically loaded conducting posts in a rectangular waveguide and its equivalent circuit model are indicated in Fig. 2. Fig. 2(b) illustrates the voltage-current relationships at the input (V_n,I_n) and output (V_{n+1},I_{n+1}) of the n-th section in the infinity long cascade connection. The dispersion equation of the periodic structure can be obtained with the equivalent circuit model presented in Fig. 2(b). It is possible to express a post as an equivalent T-model circuit. The value of X_a and X_b in the T-model circuit can be obtained as in (9) from the s-parameters of a post

$$X_b = -j\frac{2S_{21} - 1 + S_{11}^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}, \quad X_a = -j\frac{2S_{21}}{(1-S_{11})^2 - S_{21}^2} . \tag{9}$$

The circuit for a unit cell is composed of three circuits, i.e., transmission line to T-model circuit to transmission line. The ABCD matrix for a unit cell is obtained by chain rule as demonstrated in (10)

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \tag{10a}$$

where

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} Cos\theta & jSin\theta \\ jSin\theta & Cos\theta \end{bmatrix}\begin{bmatrix} 1-\frac{X_b}{X_a} & j(\frac{X_b^2}{X_a} - 2X_b) \\ -\frac{j}{X_a} & 1-\frac{X_b}{X_a} \end{bmatrix}\begin{bmatrix} Cos\theta & jSin\theta \\ jSin\theta & Cos\theta \end{bmatrix} \tag{10b}$$

and $\theta = \beta_o L/2$. Note that β_0 is the propagation constant for a rectangular waveguide.

If the periodic structure is capable of supporting a propagating wave, the ABCD matrix for a unit cell can be assumed as (11)

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} e^{-\beta L} & 0 \\ 0 & e^{\pm\beta L} \end{bmatrix}\begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \tag{11}$$

where β is a propagation constant for the periodic structure Using (10) and (11), one can obtain

$$Cos\,(\beta L) = \frac{A+D}{2} . \tag{12}$$

The propagation constant (β) of the periodic structure can be determined with (10.b) and (12) where the scattering parameters are calculated from the mode matching technique presented in Section II.

4. Results

In order to discover the dispersion characteristic of the periodically loaded conducting posts in a rectangular waveguide, scattering parameters of one post should be calculated at the first step. The scattering parameters of one post are calculated using the mode-matching techniques formulated in Section II since the distance (d) of a post from a center axis of a waveguide varies. The results are compared with measurements in Fig. 3, displaying good agreement. Note that the amplitude of the scattering parameter was measured by means of the HP8510C Network Analyzer. As expected, the transmission coefficient (S_{21}) increases the further away a post is from a waveguide center. However, it is observed that $|S_{11}|^2 + |S_{21}|^2$ is less than one in the experiment. This is thought to be due to conduction and leakage loss resulting from imperfection of fabrication. These losses may cause a slight difference in the calculation and measurement in Fig. 3.

The calculation was repeated with a different inserting post length ($b-B_2$) as demonstrated in Fig. 4. At this time, the compared results also display good agreement. However, the transmission coefficient (S_{21}) behaves quite differently from that of Fig. 3. This is due to the resonant characteristic of a post. An inherent inductive component of a post combines with a capacitive component generated between a post and a bottom surface of a waveguide, resulting in a resonance. This resonance causes a total reflection at the distance of 4 mm as illustrated in Fig. 4.

The ideal dispersion relation of an interaction circuit of a broadband Gyro-TWT should have a linear curve for a wave-electron interaction since it matches the Doppler shifted electron cyclotron resonance given by [2]

$$\omega = \Omega_c + \beta V_z \tag{13}$$

where Ω_c is an electron cyclotron frequency and V_z is an axial electron velocity, respectively. To investigate the dispersion characteristic of a periodic structure in Fig. 2, a propagation constant (β) was calculated using (10b) and (12) with a different angular frequency ω where scattering parameters were obtained from the mode matching techniques.

The calculated result from the mode-matching technique and equivalent circuit model is compared with the measurement in Fig. 5. To avoid the complexity of fabrication, the radius (r_0) of a post, the distance (L) between posts, and the distance (d) of a post from a center axis were selected as 1 mm, 12 mm, and 6.43 mm, respectively. Ten conducting posts, whose inserting lengths are adjustable, were inserted in the waveguide. The phase of the reflection coefficient (S_{11}) was measured with the HP8510C Network Analyzer employing a short metal plate at the end of the periodic structure. As the metal plate moves by half wavelength ($\lambda_g/2$), the phase of the reflection coefficient varies by 360°. Thus, the propagation constant ($\beta=2\pi/\lambda_g$) can be determined by monitoring the moving distance of the plate.

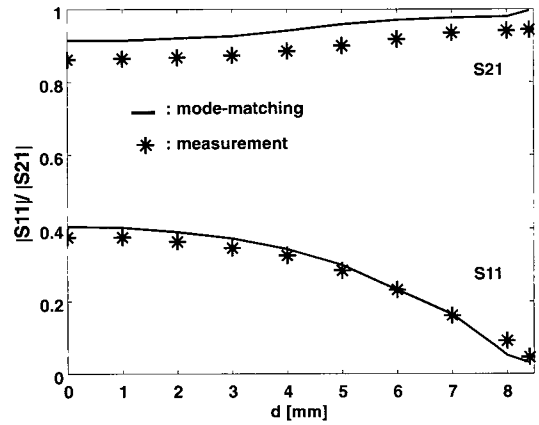

Fig. 3 Comparison of scattering parameters of a post with mode matching and measurement varying d (f=10 GHz, a=11.43mm, b=10.16mm, r_0=3 mm, $b-B_2$=3mm).

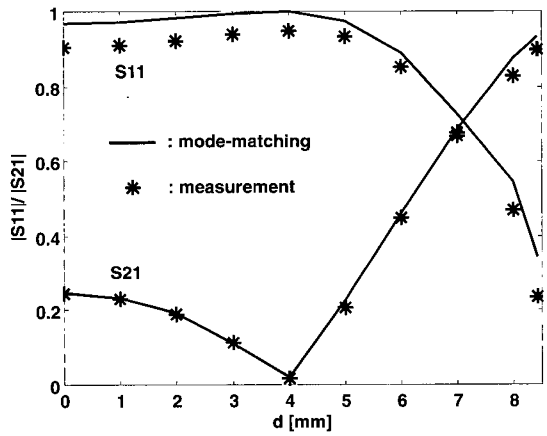

Fig. 4 Comparison of scattering parameters of a post with mode matching and measurement varying d (f=10 GHz, a=11.43mm, b=10.16 mm, r_0=3 mm, $b-B_2$=8mm).

As illustrated in Fig. 5, the compared results indicate excellent agreement. The dispersion curve of the periodic structure is more linear than that of the waveguide over the frequency range of 7 to 9 GHz. The results demonstrate that the periodically loaded conducting posts in a waveguide increase the possibility of interaction with both the electron beam in wide frequency band. Therefore, through control of parameters such as inserting length, radius, and distance between posts, this periodic structure could be utilized in the interaction circuit of the wide band Gyro-TWT.

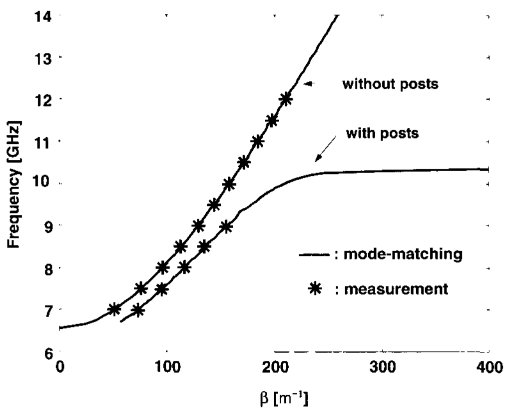

Fig. 5 Comparison of dispersion curve with mode matching and measurement (a=11.43 mm, b=10.16 mm, r_0=1 mm, b-B_2= 5mm, L=12 mm, d=6.43 mm). Note that ten conducting posts are used in the experiment.

5. Conclusion

The dispersion characteristic of periodically loaded conducting posts in a rectangular waveguide was analyzed to investigate the possibility for using it as an interaction circuit of wide band Gyro-TWT. The scattering matrix of an off-centered post in a waveguide was derived and estimated with mode matching techniques. To calculate the propagation constant of the periodic structure, which consists of periodic off-centered posts, an equivalent circuit model was utilized. Both the calculated and the measured results show good agreement. The dispersion curve of the periodic structure is found to be linear over wide frequency range, suggesting that this structure could be utilized for the interaction circuit of wide band Gyro-TWT.

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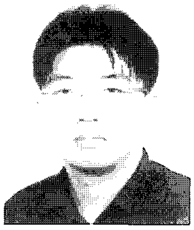

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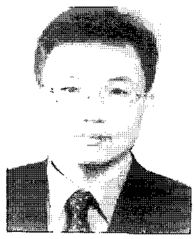

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