Rotation-Free Transformation of the Coupling Matrix with Genetic Algorithm-Error Minimizing Pertaining Transfer Functions

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Abstract

A novel Genetic Algorithm(GA)-based method is suggested to transform a coupling matrix to another, without the procedure of Matrix Rotation. This can remove tedious work like pivoting and deciding rotation angles needed for each of the iterations. The error function for the GA is simply formed and used as part of error minimization for obtaining the solution. An 8th order dual-mode elliptic integral function response filter is taken as an example to validate the present method.

Key words : Genetic Algorithm, Transfer Function, Matrix Rotation, Dual-Mode Filter, Elliptic Integral Function Response Filter.

I. Introduction

In designing a filter of a desired frequency response, the first step is to find its transfer function. Transmission and reflection zeros, rejection, ripple, e.t.c. determine the transfer function. The coupling matrix $\bar{M}$ can be obtained as in [1]~[2], which describes the entire relation of the resonance modes of the filter network. Though $\bar{M}$ can be directly obtained by optimization routines available, designers should check out the poles and zeros of the corresponding frequency response. From this point of view, the canonical coupling matrix as the initial $\bar{M}$ provides the required poles and zeros and plays an important role in being the starting or reference coupling matrix. If necessary, the canonical coupling matrix is transformed to an alternative one for a wanted coupling structure via Matrix Rotation[1]~[5], according to constraints like feasibility.

The Matrix Rotation iterates the removal and creation of the elements of $\bar{M}$ using pivots and rotation angles [1]~[5]. Each iteration, these values should be determined on the basis of trial and error, investigating every occurrence or cancellation of unwanted elements in $\bar{M}$. Some guidelines are suggested, but can not be generalized[3] in that the solution is case-dependent.

In this paper, the Genetic-Algorithm(GA) is proposed to avoid the aforementioned cumbersome steps of checking and deciding pivots and rotation angles[6],[7]. That is, the focal point will be that optimization method is employed to transform the reference coupling matrix to another. Minimizing the error function that is efficiently formed with not indefinite frequency points (as the conventional choice), but definite coefficients of the transfer function, the present method transforms the initial coupling matrix to the matrix of the target topology in an 8th order dual-mode elliptic integral function response filter. Besides, the result presents validity and usefulness when an already provided coupling matrix needs to be transformed.

II. Theory

For the frequency response of a filter to be designed, as the first step, the transfer function can be found, represented as

$$Trans(S) = u_0 \frac{S^m + B_{m-1}S^{m-1} + \cdots + B_2S^2 + B_1S^1 + B_0}{S^n + A_{n-1}S^{n-1} + \cdots + A_2S^2 + A_1S^1 + A_0} \quad (1)$$

where $S=\sigma+j\tau$, $j=\sqrt{-1}$ and $n>m$. Coefficients A_i $(i=0, 1, \cdots, n-1)$ and $B_h(h=0, 1, \cdots, m-1)$ of the denominator and numerator of *Trans*(*S*), respectively, are determined by the reflection zeros and transmission zeros for the desired frequency response. As the next step, the coupling coefficients need to be obtained to relate the resonance modes of the 2-port filter network seen in Fig. 1.

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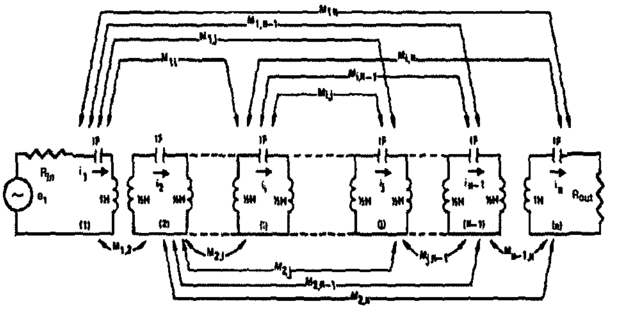

Fig. 1. A two-port network where all resonance modes are assumed coupled.

The coupling coefficient matrix $\bar{\bar{M}}$ as the unknowns can be solved via the generalized impedance and its scattering parameters as follows.

$$\bar{\bar{Z}} \cdot \bar{I} = \bar{e}, \tag{2}$$

where $\bar{e}^T = (1, 0, 0, \cdots, 0, 0)$ is the voltage excitation vector and $\bar{I}^T = (i_1, i_2, i_3, \cdots, i_{n-1}, i_n)$ is the current vector including all the resonators. Also, the generalized impedance $\bar{\bar{Z}}$

$$\bar{\bar{Z}} = j(\tau\bar{\bar{U}} + \bar{\bar{M}}) \tag{3}$$

has self- and mutual coupling values of the network.

$$\tau = \frac{f_0}{\Delta f}\left(\frac{f}{f_0} - \frac{f_0}{f}\right). \tag{4}$$

$\bar{\bar{U}}$ is the identity matrix. Using the above relation of the current and voltage, major S-parameters can be represented as

$$S_{21} = -2\sqrt{R_{in}R_{out}}\, i_n \tag{5}$$

and

$$S_{11} = 1 - 2R_{in} i_1. \tag{6}$$

where R_{in} and R_{out} mean input and output port resistance.

More specifically, when S_{21} is equated to *Trans*(*S*), mathematical manipulation of a well-known method in [1]~[2] results in the canonical coupling(folded coupling structure) matrix as the initial coupling matrix $\bar{\bar{M}}^{(0)}$. This canonical coupling matrix is important to know, since its eigen values are the unique means to verifying the required zeros and poles of *Trans*(*S*). Besides, if it is convenient to realize the couplings, the canonical coupling matrix can be directly used, or it can be a starting point for further transforms to create alternative coupling structures optimally adopted for mechanical and electrical constraints of technology.

Conventionally, the coupling matrix is rotated or transformed through 'similarity transform'. A similarity transform on an $n \times n$ matrix $\bar{\bar{M}}^{(r-1)}$ is pre- and post-multiplication of $\bar{\bar{M}}^{(r-1)}$ by an $n \times n$ rotation matrix $\bar{\bar{R}}^{(r)}$ and its transpose $\bar{\bar{R}}^{(r)t}$. The $(r-1)$-th $\bar{\bar{M}}$ is transformed to the rth matrix as follows.

$$\bar{\bar{M}}^{(r)} = \bar{\bar{R}}^{(r)} \cdot \bar{\bar{M}}^{(r-1)} \cdot \bar{\bar{R}}^{(r)t} \quad \text{and} \quad r \ge 1 \tag{7}$$

The rth rotation matrix is given as

$$\bar{\bar{R}}^{(r)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\theta_r & 0 & 0 & 0 & -\sin\theta_r & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \sin\theta_r & 0 & 0 & 0 & \cos\theta_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

where n and the pivot are assumed as 8 and (3, 7), respectively. The pivot of $\bar{\bar{R}}^{(r)}$ means $R^r_{ii} = R^r_{jj} = \cos\theta_r$, $R^r_{ij} = -R^r_{ji} = -\sin\theta_r$ with $i, j \ne 1$ or n. And θ_r is the rotation angle. Neither of an element of $\bar{\bar{M}}^{(r-1)}$ lies on the ith row or jth column, its value is maintained. However, if either of the row or column of an element coincides with i or j of the pivot, the element is changed this way

$$\begin{aligned} M^{(r)}_{ik} &= \cos\theta_r \cdot M^{(r-1)}_{ik} - \sin\theta_r \cdot M^{(r-1)}_{jk} \\ M^{(r)}_{jk} &= \sin\theta_r \cdot M^{(r-1)}_{ik} + \cos\theta_r \cdot M^{(r-1)}_{jk} \\ M^{(r)}_{ki} &= \cos\theta_r \cdot M^{(r-1)}_{ki} - \sin\theta_r \cdot M^{(r-1)}_{kj} \\ M^{(r)}_{kj} &= \sin\theta_r \cdot M^{(r-1)}_{ki} + \cos\theta_r \cdot M^{(r-1)}_{kj} \end{aligned} \tag{9}$$

where $k(\ne i, j)$=1, 2, 3, ··· , n. If in a specific coupling structure, a nonzero element, say, $M^{(0)}_{27}$ has to be made zero($M^{(1)}_{27}$), the pivot should be first decided as (i=3, j=7) and the 4th of equations (9) is chosen with k=2.

$$M^{(1)}_{27} = 0, \quad \sin\theta_1 \cdot M^{(0)}_{23} = -\cos\theta_1 \cdot M^{(0)}_{27} \tag{10}$$

Then, the angle of rotation is formulated as

$$\theta_1 = \tan^{-1}(-M^{(0)}_{27} / M^{(0)}_{23}) \tag{11}$$

At all times, the decision of the pivot and rotation angle follows designers' investigation of the coupling

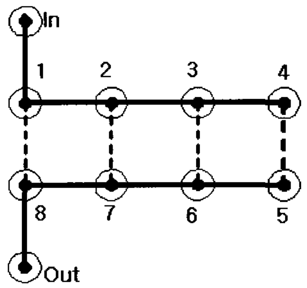

Fig. 2. Coupling structure of the 8th order canonical symmetric filter topology.

matrix and judgement of which element should be cancelled. And it is notable that when one element(e.g. $M_{27}^{(1)}$) is eliminated, unwanted values(e.g. $M_{32}^{(1)}$ and $M_{23}^{(1)}$) are created. For removing the unwanted elements, additional rotation must be adopted and cumbersome routines will go on and on.

This paper suggests a novel method that escapes the tedious steps of determining pivots and rotation angles, via GA-based optimization scheme that minimizes the error between polynomial coefficients due to the original and tried coupling matrices. From this part on, the coupling matrix transform will be explained with maintaining the characteristics of eigen values of the canonical coupling matrix. As an example, the canonical symmetric structure of an 8th order filter is shown as the topology in Fig. 2.

Each node corresponds to a resonance mode. The adjacently placed numbers of nodes are sequential couplings, and the connections of non-adjacent nodes are cross-couplings. And its coupling matrix $\bar{\bar{M}}^{(0)}$ is given as

$$\bar{\bar{M}}^{(0)} = \begin{bmatrix} 0 & m_{12} & 0 & 0 & 0 & 0 & 0 & m_{18} \\ m_{12} & 0 & m_{23} & 0 & 0 & 0 & m_{27} & 0 \\ 0 & m_{23} & 0 & m_{34} & 0 & m_{36} & 0 & 0 \\ 0 & 0 & m_{34} & 0 & m_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{45} & 0 & m_{56} & 0 & 0 \\ 0 & 0 & m_{36} & 0 & m_{56} & 0 & m_{67} & 0 \\ 0 & m_{27} & 0 & 0 & 0 & m_{67} & 0 & m_{78} \\ m_{18} & 0 & 0 & 0 & 0 & 0 & m_{78} & 0 \end{bmatrix} \tag{12}$$

where m_{pq} menas coupling coefficient between nodes p and q. If a different coupling structure with $\bar{\bar{M}}^{(goal)}$ is needed for a practical use, $\bar{\bar{M}}^{(0)}$ is rotated iteratively a possible number of times. For special cases of matrix transformation, pivots and rotation angles are empirically found and suggested [3]~[5]. However, necessary information on matrix rotation is hardly formulated to discover $\bar{\bar{M}}^{(goal)}$ in general cases.

Though $\bar{\bar{M}}^{(goal)}$ is transformed by Matrix Rotation from $\bar{\bar{M}}^{(0)}$, they have the same frequency response and transfer function. The elements of the two coupling matrices are different, but they have the same coefficients A_i and B_h, because the coefficients reflect the eigen values of the original coupling matrix, that is, canonical coupling matrix. This point makes it possible to efficiently form the error function in a solution-search algorithm by using the definite number of errors of the coefficients, compared to the conventional choice of an indefinite number of frequency points in the S-parameters of transmission and reflection. The closed-form expressions of the transfer functions $Trans^{(0)}(S)$ and $Trans^{(goal)}(S)$ for $\bar{\bar{M}}^{(0)}$ and $\bar{\bar{M}}^{(goal)}$ can be calculated with the help of the method in [2]. $A^{(0)}{}_i$ and $B^{(0)}{}_h$ are constant $A_i^{(goal)}$ and $B_h^{(goal)}$ become nonlinear functions of the unknown elements of $\bar{\bar{M}}^{(goal)}$. Thus, the error function *Error* can be formed as

$$Error = \sum_{i=0}^{n-1} |A_i^{(goal)} - A_i^{(0)}| + \sum_{h=0}^{m-1} |B_h^{(goal)} - B_h^{(0)}| \tag{13}$$

and minimized by the GA[6],[7]. This removes the use of pivots and rotation angles.

III. Numerical Results

$\bar{\bar{M}}^{(0)}$ of the canonical coupling structure can be computed for fractional bandwidth of 0.504 %, Insertion Loss(I.L.) of 0.1 dB, Rejection of 55 dB at 1.23 in the normalized frequency and Transmission zeros at $\tau = \pm 1.2,\ \pm 1.7$.

$$\bar{\bar{M}}^{(0)} = \begin{bmatrix} 0 & 0.910 & 0 & 0 & 0 & 0 & 0 & -0.049 \\ 0.910 & 0 & 0.588 & 0 & 0 & 0 & 0.198 & 0 \\ 0 & 0.588 & 0 & 0.375 & 0 & -0.562 & 0 & 0 \\ 0 & 0 & 0.375 & 0 & 0.921 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.921 & 0 & 0.375 & 0 & 0 \\ 0 & 0 & -0.562 & 0 & 0.375 & 0 & 0.588 & 0 \\ 0 & 0.198 & 0 & 0 & 0 & 0.588 & 0 & 0.910 \\ -0.049 & 0 & 0 & 0 & 0 & 0 & 0.910 & 0 \end{bmatrix} \tag{14}$$

Also, the turns ratio of in/output port is given as 1.07. Then, we assume that physical and electrical constraints of a certain system require developers to have a filter of $\bar{\bar{M}}^{(goal)}$ as seen in Fig. 3.

In Fig. 3, the very special cross-coupling like modes 3 and 8 or modes 4 and 7 are presented. It will be

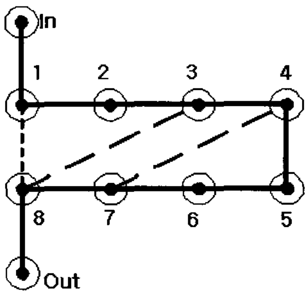

Fig. 3. Desired coupling structure of the 8th order asymmetric filter symmetric filter topology.

considered a good example that any feasible couplings for a specific order of filtration can be obtained from the reference structure through the proposed scheme.

Before mentioning the efficient proposed scheme, the conventional matrix rotation method needs to be recalled. Through inevitable repetition of choosing the pivot and deciding the rotation angle, it is found out that if $M_{27}^{(0)}$ and $M_{36}^{(1)}$ can be eliminated in the first and second similarity transform, respectively, it can prevent unnecessary waste of time. As for the first transform, the pivot is (3, 7) and the rotation angle is $-18.6°$. Also, (4, 6) and 44.2° are the pivot and rotation angle for the second transform, each.

Next, in following the GA-based procedures, the present method adopts 8 bits, 10 parameters, 100 individuals, 50 generations and crossover rate of 0.75. These have been found by numerical experiment. All the parameters vary from -1 to $+1$. Fig. 4 shows the error function with three mutation rates and convergence over 35 of the generation number. 0.095 is chosen as the mutation rate.

Although the choice between the three mutation rates does not seem to make big difference in Fig. 4, if far more genes or individuals or generations had to be used, the mutation rate that accounts for faster convergence would prevent costly situations.

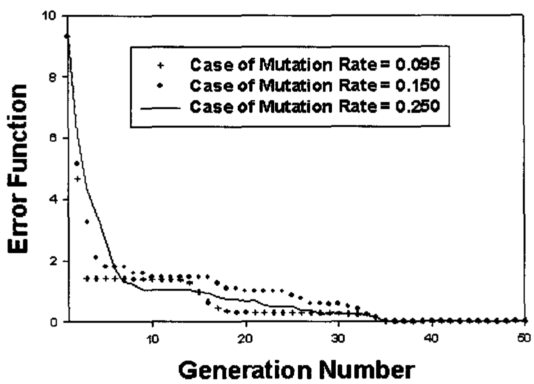

Fig. 4. Error function behavior with different mutation rates.

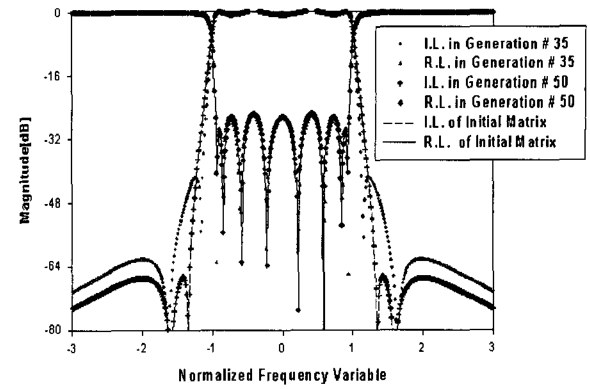

Fig. 5. Frequency responses of the initial and transformed coupling matrices.

Table 1. Inputs for GA application.

Condition	Assignment
1 Gene	8 bits
1 Chromosome	10 Genes
1 Population	100 Chromosomes
Crossover rate	0.750
Generation	50

In Fig. 5, the frequency response of $\bar{\bar{M}}^{(goal)}$ varies from generation to generation. With reference to frequency response of $\bar{\bar{M}}^{(0)}$, some discrepancy, small though, is seen in the earlier generations of tries. However, the frequency response gets to agree with the frequency that of $\bar{\bar{M}}^{(0)}$ in the final generation.

Eventually, with the inputs summarized in Table 1, the GA application provides $\bar{\bar{M}}^{(goal)}$ as follows.

$$\bar{\bar{M}}^{(goal)} = \begin{bmatrix} 0 & 0.910 & 0 & 0 & 0 & 0 & 0 & -0.049 \\ 0.910 & 0 & 0.620 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.620 & 0 & 0.496 & 0 & 0 & 0 & 0.290 \\ 0 & 0 & 0.496 & 0 & 0.399 & 0 & -0.598 & 0 \\ 0 & 0 & 0 & 0.399 & 0 & 0.912 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.912 & 0 & 0.445 & 0 \\ 0 & 0 & 0 & -0.598 & 0 & 0.445 & 0 & 0.862 \\ -0.049 & 0 & 0.290 & 0 & 0 & 0 & 0.862 & 0 \end{bmatrix} \tag{15}$$

The insertion loss of 3.06 dB at f_0 is less than 4 dB. Besides, as to the near-band rejection, S_{21} is less than -60 dB and complies with the design target.

Ⅳ. Conclusion

With the help of a new GA-based method, the initial coupling matrix of a transfer function can be efficiently transformed from the canonical coupling matrix to another, avoiding Matrix Rotation. The proposed method employs a simple error function and has benefits of usefulness of transforming an already obtained coupling matrix to the wanted topology, still maintaining the characteristics of the original coupling matrix.

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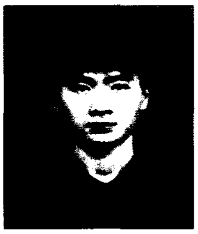
Sungtek Kahng



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