카오스 이동 로봇에서의 카오스 거동 해석

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Chaotic Behaviour Analysis for Chaotic Mobile Robot

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요 약

본 논문에서는 Arnold 방정식, Chua 방정식, 하이퍼카오스 방정식을 이동 로봇에 내장한 카오스 이동 로봇에서의 카오스 거동을 해석하였다. 이동 로봇에서의 카오스 거동을 분석하기 위해서 시계열 데이터, 임베딩 위상공간의 정성적인 분석뿐만 아니라 리아프노프 지수와 같은 정량적인 분석을 수행하였다.

ABSTRACT

In this paper, we propose that the chaotic behavior analysis in the chaotic mobile robot embedding Arnold, equation, Chua's equation and hyper-chaos equation. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time-series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle.

키워드

Chaos robot, Lyapunov exponent, Time series

Ⅰ. Introduction

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into the real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1-2], chaos synchronization and secure/crypto communication [3-7], Chemistry [8], Biology [9], and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot, where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot is represented by Arnold equation. They applied obstacle with chaotic trajectory, but they have not mentioned about the chaotic behavior except Lyapunov exponent.

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In this paper, we propose that the chaotic behavior analysis in the mobile robot of embedding Arnold, Chua's and hyper-chaos equations with obstacle. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time-series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle. In order to avoid obstacles, we assume that all obstacles in the chaos trajectory surface have an unstable limit cycle with Van der Pol equation. When chaos robots meet obstacles among the arbitrary wondering in the chaos trajectory, which is derived using chaos circuit equation such as Chua's equation, obstacles pull out the chaos robots out of chaos trajectory because obstacles have unstable limit cycle with Van der Pol equation.

II. Chaotic Mobile Robot

2.1. Mobile Robot

As the mathematical model of mobile robots, we assume a two- wheeled mobile robot as shown in Fig. 1.

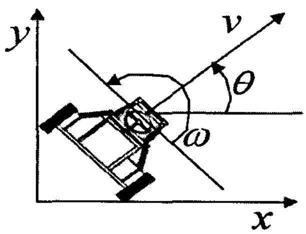


Fig. 1 Two-wheeled mobile robot

Let the linear velocity of the robot v [m/s] and angular velocity [rad/s] be the input to the system. The state equation of the four-wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} \quad (1)$$

where (x,y) is the position of the robot and θ is the angle of the robot..

2.2 Chaos equations

In order to generate chaotic motions for the mobile robot, we apply some chaos equations such as an Arnold equation or Chua's equation.

1) Arnold equation [10]

We define the Arnold equation as follows:

$$\begin{aligned} \dot{x}_1 &= A\ \sin x_3 + C\cos x_2 \\ \dot{x}_2 &= B\ \sin x_1 + A\ \cos x_3 \\ \dot{x}_3 &= C\ \sin x_2 + B\ \cos x_1 \end{aligned} \quad (2)$$

where A, B, C are constants.

2) Chua's equation

Chua's circuit is one of the simplest physical models that has been widely investigated by mathematical, numerical and experimental methods. We can derive the state equation of Chua's circuit.

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \quad (3)$$

where.

$$g(x) = m_{2n-1}x + \frac{1}{2}\sum_{k=1}^{2n-1}(m_{k-1} - m_k)(|x+c_k| - |x-c_k|)$$

3) Hyper-chaos equation

Hyper-chaos equation is one of the simplest physical models that have been widely investigated by mathematical, numerical and experimental methods for complex chaotic dynamic. We can easily make hyper-chaotic equation by using some of connected N-double scroll. We can derive the state equation of N-double scroll equation as followings.

$$\dot{x}=\alpha[y-h(x)] \qquad (4)$$
$$\dot{y}=x-y+z$$
$$\dot{z}=-\beta y$$

Where,

$$h(x)=m_{2n-1}x+\frac{1}{2}\sum_{i=1}^{2n-1}(m_{i-1}-m_i)|x+c_i|-|x-c_i|)$$

In order to make a hyper-chaos, we have compose to 1 dimensional CNN(Cellular Neural Network) which are identical two N-double scroll circuits and then we have to connected each cell by using unidirectional coupling or diffusive coupling. In this paper, we used to diffusive coupling method. We represent the state equation of x-diffusive coupling and y-diffusive coupling as follows.

x-diffusive coupling

$$\dot{x}^{(j)}=\alpha[y^{(j)}-h(x)^{(j)}]+D_x(x^{(j-1)}-2x^{(j)}+x) \qquad (5)$$
$$\dot{y}^{(j)}=x^{(j)}-y^{(j)}+z^{(j)}$$
$$\dot{z}^{(j)}=-\beta y^{(j)}, \qquad j=1,2,....L$$

y-diffusive coupling

$$\dot{x}^{(j)}=\alpha[y^{(j)}-h(x)^{(j)}] \qquad (6)$$
$$\dot{y}^{(j)}=x^{(j)}-y^{(j)}+z^{(j)}+D_y(x^{(j-1)}-2x^{(j)}+x)$$
$$\dot{z}^{(j)}=-\beta y^{(j)}, \qquad j=1,2,....L$$

where, L is number of cell.

2.3 Embedding of Chaos circuit in the Robot

In order to embed the chaos equation into the mobile robot, we define and use the Arnold equation and Chua's circuit equation as follows.

1) Arnold equation

We define and use the following state variables:

$$\dot{x}_1 = D\dot{y} + C\cos x_2$$
$$\dot{x}_2 = D\dot{x} + B\sin x_1$$
$$\dot{x}_3 = \theta \qquad (7)$$

where B, C, and D are constant.

Substituting (1) into (2), we obtain a state equation on $\dot{x}_1$, $\dot{x}_2$, and $\dot{x}_3$ as follows:

$$\dot{x}_1 = Dv + C\cos x_2$$
$$\dot{x}_2 = Dv + B\sin x_1$$
$$\dot{x}_3 = \omega \qquad (8)$$

We now design the inputs as follows [10]:

$$v = A/D$$
$$\omega = C\sin x_2 + B\cos x_1 \qquad (9)$$

Finally, we can get the state equation of the mobile robot as follows:

$$\dot{x}_1 = A\sin x_3 + C\cos x_2$$
$$\dot{x}_2 = B\sin x_1 + A\cos x_3$$
$$\dot{x}_3 = C\sin x_2 + B\cos x_1$$
$$\dot{x} = V\cos x_3$$
$$\dot{y} = V\sin x_3 \qquad (10)$$

Equation (10) includes the Arnold equation.

2) Chua's equation

Using the methods explained in equations (7)-(10), we can obtain equation (11) with Chua's equation embedded in the mobile robot.

$$\dot{x}_1 = \alpha(x_2 - g(x_1))$$
$$\dot{x}_2 = x_1 - x_2 + x_3$$
$$\dot{x}_3 = -\beta x_2$$
$$\dot{x} = V\cos x_3$$
$$\dot{y} = V\sin x_3 \qquad (11)$$

Using equation (11), we obtain the embedding chaos robot trajectories with Chua's equation.

3) Hyper-chaos equation

Combination of equation (1) and (5) or (6), we define and use the following state variables (11) or (12)

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \alpha[y^{(j)} - h(x^{(j)}] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)})) \\ x^{(j)} - y^{(j)} + z^{(j)} \\ -\beta y^{(j)} \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \quad (11)$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \alpha[y^{(j)} - h(x^{(j)}] \\ x^{(j)} - y^{(j)} + z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ -\beta y^{(j)} \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \quad (12)$$

Using equation (8) and (9), we obtain the embedding chaos robot trajectories with Hyper-chaos equation.

III. VDP(Van der Pol) obstacle.

In this section, we will discuss the mobile robot's avoidance of Van der Pol(VDP) equation obstacles. We assume the obstacle has a VDP equation with an unstable limit cycle, because in this condition, the mobile robot can not move close to the obstacle and the obstacle is avoided.

In order to represent an obstacle of the mobile robot, we employ the VDP, which is written as follows:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= (1 - y^2)y - x \end{aligned} \quad (13)$$

From equation (13), we can get the following limit cycle as shown in Fig. 2.

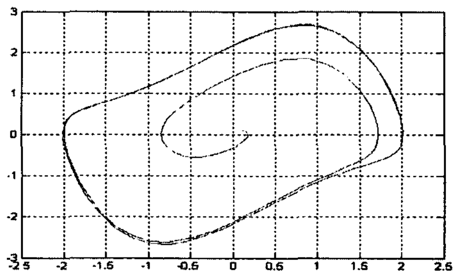


Fig.2 Limit cycle of VDP

IV. Chaotic behavior analysis in the Mobile Robot

To analysis of chaotic behavior in the mobile robot, we investigated the chaotic characteristics from the mobile robot trajectories data. Firstly, we applied embedding method as a qualitative analysis and then we get the reconstruction phase plane from the one dimensional mobile robot trajectories data. Second, we calculated Lyapunov exponent as quantitative analysis.

4.1 Embedding method

In order to reconstruct phase plane from data of robot's single variable, we applied an embedding method proposed by Takens [11]. The embedding method is referring to the process in which a representation of the attractor can be constructed from a set of scalar time-series. The form of such reconstructed state is given as follows:

$$X_t = [x(t), x(t+\tau), \ldots\ldots, x(t+(m-1)\tau] \quad (13)$$

Where $x(t)$ is a robot trajectory data, τ is a delay time, and m is an embedding dimension. It is significant factor to get reasonable embedding phase plane. In chaos mobile case, we choose τ is 400 using an auto-correlation time and m is chosen 5 because nearest false neighbor disappears in that dimension. Fig. 3 Fig. 4 and Fig. 5 shows time series of embedding Arnold equation chaos robot from equation (10), Chua's equation chaos robot from equation (11) and hyper-chaos equation robot from equation (12) receptively.

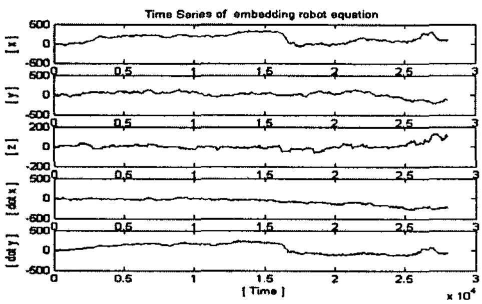


Fig. 3 Arnold chaos robot time-series

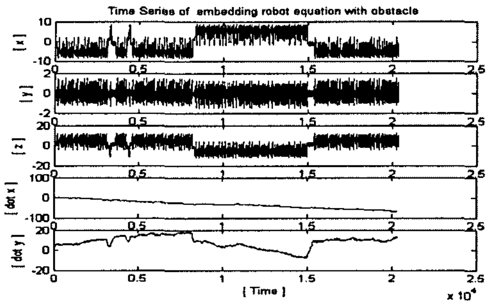


Fig. 4 Chua's chaos robot time-series

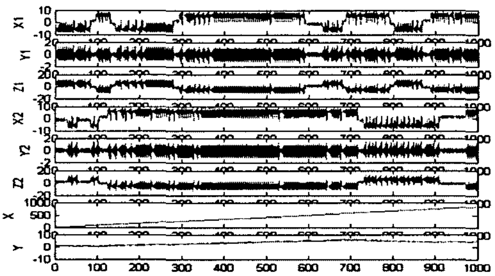


Fig.5 Hyper-chaos robot time-series

4.2 Qualitative Analysis

With reconstructed state, the qualitative chaotic degree of chaotic robot path is analyzed in this section using embedding phase plane. Fig. 6 shows phase plane of these embedding state which are originally robot paths when robot has a (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle from the Arnold embedding chaos robot. Fig. 7 shows phase plane of these embedding state which are originally robot paths when robot has a (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle from the Chua's embedding chaos robot. Fig. 8 shows phase plane of these embedding state which are originally robot paths when robot has a (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle from the hyper-chaos embedding chaos robot.

In Fig. 6, Fig. 7 and Fig. 8 we can recognize that reconstructed robot path from one dimensional mobile trajectories are very complicated signal seems like chaos signal. We can also confirmed that when the robots has a obstacle, reconstructed phase planes are more complicated compare with no obstacle.

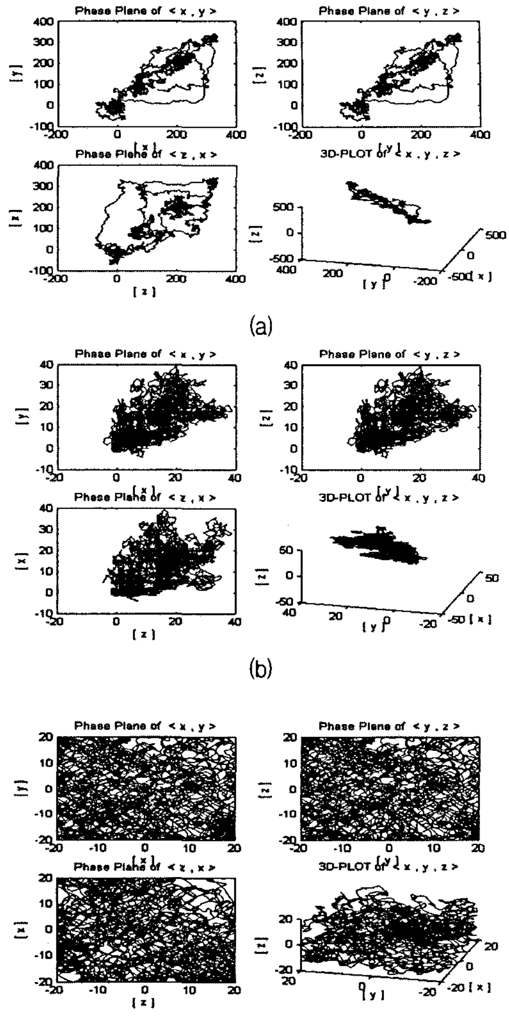
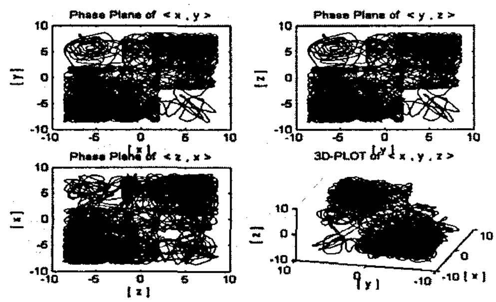
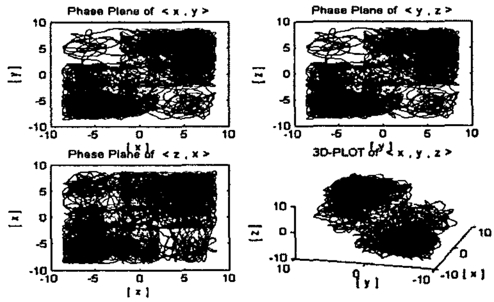


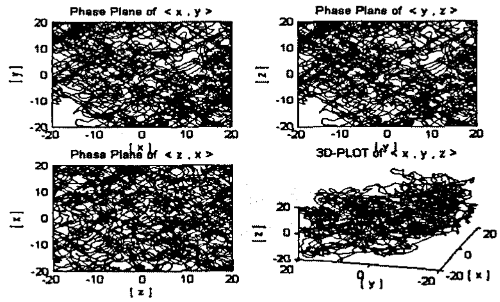
Fig. 6 Reconstructed phase plane (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle.



(a)

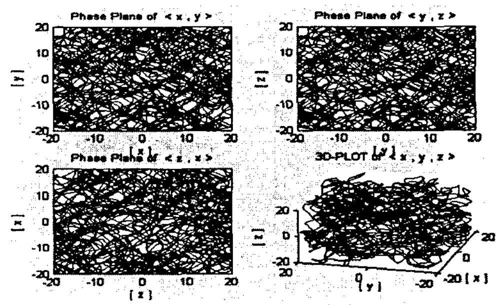


(b)

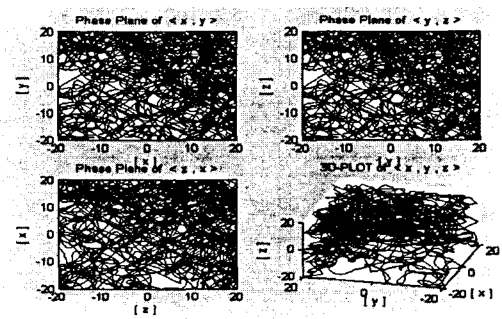


(c)

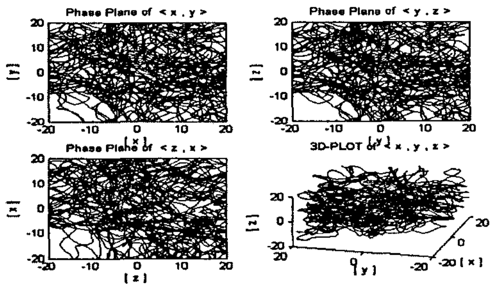
Fig.7 Reconstructed phase plane (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle.



(a)



(b)

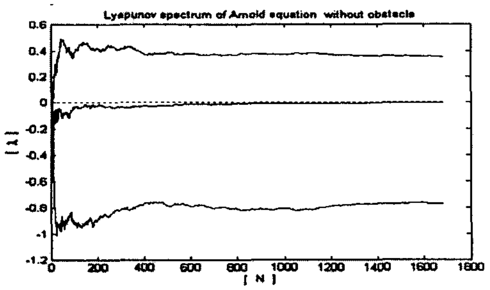


(c)

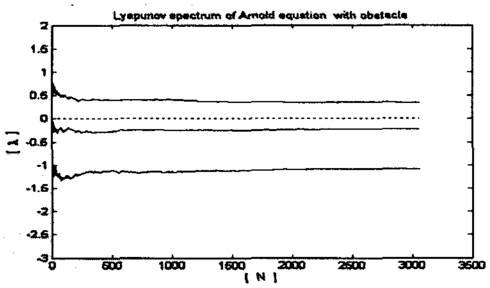
Fig.8 Reconstructed phase plane (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle.

3.3. Quantiative Analysis

In this section, we evaluate Lyapunov spectrum [12] in the mobile robot as a quantitative chaos analysis and shows Arnold chaos robot in Fig. 9 and Chua's chaos robot in Fig. 10 and also shows hyper-chaos robot in Fig. 11. Generally speaking, when the largest Lyapunov exponent more than zero we can say chaotic motion and less than or equal zero, we say periodic motion. In Fig. 9, Fig 10 and Fig 11, we can confirm that reconstructed phase planes are chaotic motion because there are largest Lyapunov exponents more than zero.

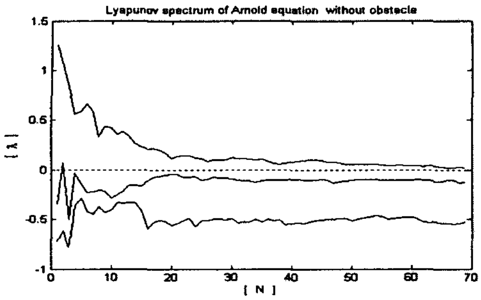


(a)

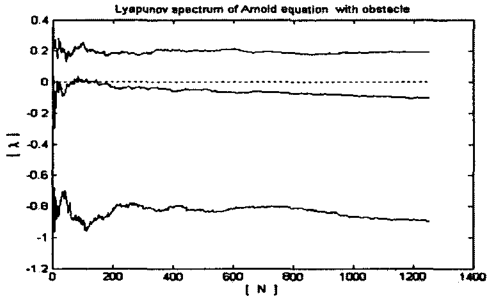


(b)

Fig. 10 Lyapunov spectrum of mobile robot (a) without obstacle, (b) with obstacle of Arnold robot

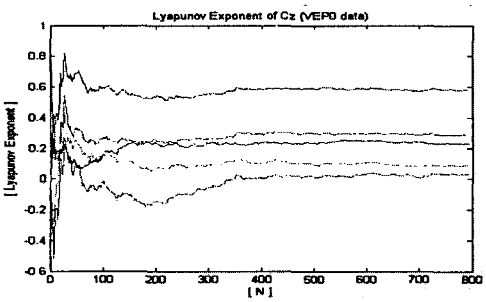


(a)

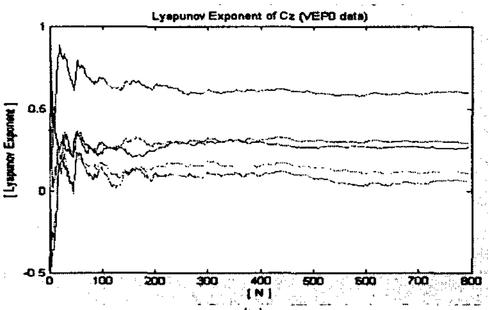


(b)

Fig. 11 Lyapunov spectrum of mobile robot (a) without obstacle, (b) with obstacle of Chua's robot.



(a)



(b)

Fig. 11 Lyapunov spectrum of mobile robot (a) without obstacle, (b) with obstacle of hyper-chaos robot.

V. Conclusion

In this paper, we propose that the chaotic behavior analysis in the mobile robot of embedding Arnold equation, Chua's equation and hyper-chaos equation with obstacle. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time-series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle. In the obstacle, we only assume that all obstacles in the chaos trajectory surface in which robot workspace has an unstable limit cycle with Van der Pol equation.

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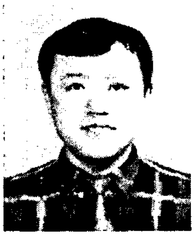

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