

# 전력산업의 경쟁체제에서 유효전력 손실을 부하에 배분하는 방법

(Allocation of Real Power Losses to Individual Loads Under Competition of Deregulated Power Industries)

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## 요 약

본 논문은 전력산업이 경쟁체제로 전환되어 전력시장으로 운영될 경우 보조서비스의 하나인 유효전력손실 배분에 관한 것으로 손실배분계수를 이용하여 유효전력 손실을 부하에 배분하는 방법을 제안하며 아울러 제안하는 방법을 한계손실계수를 이용하는 방법과 비교하고자 한다. 제안하는 방법은 먼저 손실배분계수를 정의하며 이것을 이용하여 유효전력 손실에 대한 각 부하의 부담분을 계산한다. 9-모선 샘플시스템에 대한 사례연구를 통하여 제안하는 알고리즘의 효용성을 입증하고 있으며 한계손실계수를 이용하는 것보다 손실배분계수를 이용하여 유효전력손실을 부하에 배분하는 것이 타당하다는 주장을 담고 있다.

## Abstract

The paper proposes a method to calculate the allocations of real power losses in transmission lines to individual loads based on loss distribution factors and compares them with those using marginal loss factors. The proposed method is implemented by defining loss distribution factors and analysing the individual loads' shares in the transmission line losses. Computer simulations on a 9-bus sample system verify effectiveness of the algorithm proposed and give an assertion that it is desirable to allocate power losses to loads using loss distribution factors rather than based on marginal loss factors.

Key Words : Real power losses, Loss distribution factor, Ancillary service

## 1. Introduction

The onset of deregulation for electric power industries promotes competition by enabling

access to transmission services for all wholesale buyers and sellers of electric power. Transmission utilities must provide non-discriminatory transmission service to third parties at cost-based rates. Under the deregulated circumstance, there is a necessity for separate pricing of the component parts of electricity production and delivery such as power generation, transmission, distribution, and

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ancillary services.

Transmission service providers need to know the precise operating costs of providing ancillary services to their customers since the costs vary as a function of time, location, and system status. Ancillary services are defined, by Federal Energy Regulatory Commission(FERC), as “those services necessary to support the transmission of electric power from seller to purchaser given the obligations of control areas and transmitting utilities within those control areas to maintain reliable operations of the interconnected transmission system[1]”. Cost related to each unbundled ancillary service is to be calculated separately, and then is to be added together to get the total ancillary service cost.

Calculating cost of these ancillary services became, recently, one of the most active areas to research. This paper deals with an ancillary service of real power losses allocation. This service is to use generating equipment and fuel to compensate for the transmission system losses associated with power flow from generators to customers.

Losses are always involved in moving power because of the non-zero resistance of each element in the transmission system. The losses depend on network topology and status, generator locations and outputs, and customer locations and demands. A typical transmission system consumes about 3[%] of the system load as real power losses. However, losses vary greatly as the system conditions change. The losses' nonlinear nature makes it difficult to compute and allocate their costs to the associated loads.

Transmission line losses can generally not be measured directly but are calculated with power-flow computer programs. These programs help to calculate real power losses in transmission lines in near real time and their results can then

be applied to allocate the losses to individual loads. Some loss allocation approaches have been presented in the literature. Approaches based on marginal loss factors(MLF) are proposed for allocating losses to generators or loads. [2,3] This approach needs normalization after calculating MLFs because this allocation does not assure complete recovery of the losses. And this procedure depends on the location of the slack bus since the MLFs vary depending on the slack bus and the MLF of the slack bus is zero. Moreover, the MLFs can be positive or negative which may result in negative charge.

A few approaches to find the shares of individual generators(or loads) for particular line flows have been developed[4,5,6] and Macqueen discussed allocation of energy losses for distribution system by using graph theory[7]. This paper uses the supplement charge allocation method described by Bialek[5], proposes a method to calculate the allocations of real power losses to individual loads based on loss distribution factors(LDF), and compares them with those using MLFs to discuss a better approach. The proposed method is implemented by defining a LDF and analysing the loads' shares in the real power losses in transmission lines. Computer simulations on a 9-bus sample system will verify effectiveness of the algorithm proposed.

## 2. Allocation of real power losses

Two different methods for computing power loss allocations are introduced. The first one is using the concept of marginal loss factors while the second one is based on the loss distribution factors. Numerical applications of Chapter 3 will show the results of comparison between the two approaches.

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## 2.1 Loss allocation using MLF

MLFs are defined by the following equation, which calculates the change in total real power losses with respect to a change in real power at each bus  $i$ [3].

$$MLF_i = \frac{\partial P_{loss}}{\partial P_i} = \eta_i \quad (1)$$

Since losses are deemed to be supplied from the slack bus in power flow calculations, total power losses are insensitive to a change in real power at the slack bus, which means that

$$\eta_s = \frac{\partial P_{loss}}{\partial P_s} = 0 \quad (2)$$

where the subscript  $s$  denotes the slack bus. Thus, the location of slack bus has a considerable impact on the value of MLFs.

Computation of MLFs starts with the results of power flow calculations for a system operation point and then applies the chain rule at that point to derive the following equation when assuming that bus 1 be the slack bus.

$$\begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_2} & \dots & \frac{\partial P_n}{\partial \theta_2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial P_2}{\partial \theta_n} & \frac{\partial P_3}{\partial \theta_n} & \dots & \frac{\partial P_n}{\partial \theta_n} \end{bmatrix} \begin{bmatrix} \frac{\partial P_{loss}}{\partial P_2} \\ \frac{\partial P_{loss}}{\partial P_3} \\ \vdots \\ \frac{\partial P_{loss}}{\partial P_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_{loss}}{\partial \theta_2} \\ \frac{\partial P_{loss}}{\partial \theta_3} \\ \vdots \\ \frac{\partial P_{loss}}{\partial \theta_n} \end{bmatrix} \quad (3)$$

The matrix in equation (3) is the transpose of the jacobian matrix in the Newton-Raphson method for solving power-flow problem. Therefore the MLFs, are computed by multiplying both sides of equation (3) by the inverse of the matrix.

The right-hand side represents sensitivities of total power loss with respect to voltage angles.

Since the power loss in a transmission line  $i$ - $j$  is represented in equation (4), total power loss can be calculated by equation (5).

$$P_{loss,ij} = G_{ij}|E_i|^2 + G_{ij}|E_j|^2 - 2G_{ij}|E_i||E_j|\cos(\theta_i - \theta_j) \quad (4)$$

$$P_{loss} = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n G_{ij} [|E_i|^2 + |E_j|^2 - 2|E_i||E_j|\cos(\theta_i - \theta_j)] \quad (5)$$

where  $G_{ij}$  is the conductance of a line  $i$ - $j$ ,  $|E_i|$  is the voltage magnitude of a bus  $i$ ,  $\theta_i$  is the voltage angle of a bus  $i$ , and  $P_{loss,ij}$  is the real power loss of a line  $i$ - $j$ .

The elements of the right-hand side of equation (3) can be developed like

$$\frac{\partial P_{loss}}{\partial \theta_i} = 2 \sum_{j=1}^n G_{ij} [|E_i||E_j|\sin(\theta_i - \theta_j)] \quad (6)$$

Next, to assure that sum of real power losses allocated to load buses by the MLFs be the total loss, a reconciliation process is needed. Thus the reconciled MLFs are computed by making the following equation satisfied.

$$\sum_{i \in N} [\epsilon \eta_i P_i] = P_{loss} \quad (7)$$

$$\tilde{\eta}_i = \epsilon \eta_i \quad (8)$$

where  $\epsilon$  is a reconciliation factor,  $\tilde{\eta}_i$  is a reconciled MLF, and  $N$  is a set of whole load buses.

Then, the total power loss responsible for  $k$ -th load is computed in the following equation.

$$P_{loss,k} = \tilde{\eta}_k P_k \quad (9)$$

### 2.2 Loss allocation based on LDF

Running a power-flow program can result in computing voltage values at all buses and real-power losses in transmission lines. The sharing of k-th load for the real power flow in a transmission line i-j,  $R_{ij,k}$ , can be calculated as follows[5].

$$R_{ij,k} = \frac{P_{ij}}{P_i} [A_d^{-1}]_{ik} \quad (10)$$

where  $P_i$  is a bus power at bus i, and  $P_{ij}$  is a line flow in line i-j.  $A_d$  is the matrix that satisfies the equation below,

$$A_d P^f = P_R \quad (11)$$

where  $P^f$  is an unknown vector of fictitious bus powers, and  $P_R$  is a vector of load demands. Equation (11) is the matrix notation of the following equation.

$$P_i^f - \sum_{j \in I_i} \frac{|P_{ij}|}{P_j} P_j^f = P_{Ri} \quad (12)$$

where  $P_i^f$  is an unknown fictitious bus power at bus i,  $P_{Ri}$  is a load demand at bus i, and  $I_i$  is a set of buses whose power flows from bus i (The detailed derivation for the above equations is shown in Reference[5]).

Now, we are going to define loss distribution factors using the sharing of loads for transmission line flows,  $R_{ij,k}$ , described in equation (10).

Claim : The responsible shares of loads for the real power losses in transmission lines can be represented as

$$P_{loss,ij,k} = U_{ij,k} P_{loss,ij} \quad (13)$$

$$U_{ij,k} = \frac{R_{ij,k} P_{Rk}}{\sum_{l \in B_{ij}} R_{ij,l} P_{Rl}} \quad (14)$$

where  $P_{loss,ij,k}$  is the responsible share of k-th load for the loss in a transmission line i-j,  $P_{loss,ij}$  is the real power loss in a transmission line i-j, and  $B_{ij}$  is a set of load buses that influence the real power loss in a transmission line i-j.  $U_{ij,k}$  is now called a LDF.

Proof : Since the real power loss in a transmission line i-j is the sum of the responsible shares of all loads for the line loss, the following equation holds.

$$P_{loss,ij} = \sum_{k \in \beta_{ij}} P_{loss,ij,k} \quad (15)$$

Expanding the right-hand side of equation (15) using equations (13) and (14),

$$\begin{aligned} \sum_{k \in \beta_{ij}} P_{loss,ij,k} &= P_{loss,ij,1} + \dots + P_{loss,ij,b} \\ &= \frac{R_{ij,1} P_{R1}}{\sum_l R_{ij,l} P_{Rl}} P_{loss,ij} + \dots + \frac{R_{ij,b} P_{Rb}}{\sum_l R_{ij,l} P_{Rl}} P_{loss,ij} \\ &= P_{loss,ij} \end{aligned} \quad (16)$$

where  $M_{ij}$  is the magnitude of the set  $\beta_{ij}$ .

Next, the total power loss responsible for k-th load is computed in the following equation.

$$P_{loss,k} = \sum_{ij \in A_i} U_{ij,k} P_{loss,ij} \quad (17)$$

where  $A_i$  is a set of all transmission lines. Equation (16) says that summing up all shares of one load for individual line losses becomes the

load's total responsible loss.

### 3. Numerical applications

A 9-bus sample system, shown in Fig. 1, is used to verify the algorithm proposed in the previous section.

#### 3.1 Results using MLF

The results of applying the algorithms for calculating MLFs to the 9-bus sample system are shown in Table 1. The table, in a row order, illustrates loads, MLFs, reconciled MLFs, and allocated losses as to load buses, which parameters are previously discussed in section 2-1.

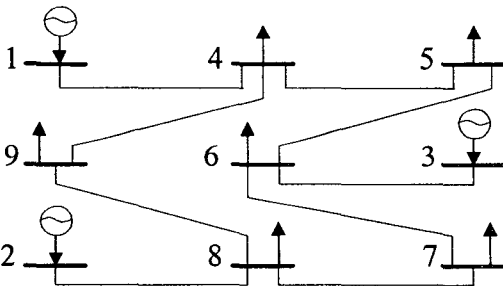


Fig. 1. A 9-bus sample system

Table 1. Results of loss allocation using MLF

	load4	load5	load6	load7	load8	load9
$P_i$ [MW]	8	90	10	100	14	125
$\eta_i$	0.0007	0.0424	-0.0127	0.0163	-0.0288	0.0512
$\tilde{\eta}_i$	0.00076	0.04622	-0.01384	0.01777	-0.03139	0.05581
$P_{loss,i}$ [MW]	0.0061	4.0597	-0.1384	1.7768	-0.4395	6.9764

#### 3.2 Results based on LDF

Table 2 illustrates the result of power-flow calculations for the sample system. Positive values imply that power-flow direction is out of the bus and the opposite direction is for negative values.

Sum of the two power flow values for each line represents the real power loss in the transmission line. Thus, the real power loss in a transmission line 1-4 is 1.082 MW and summing up all line losses results in the total line losses of 12.341[MW].

Table 3 shows sharing factors of loads for the real power flows in individual transmission lines. For an example, since the load magnitude of the bus 5 is 90[MW], real power flow of the line 4-5 is computed as 45.99[MW](90\*0.511), which is almost same as the real power(46.01[MW]) entering to the bus 5 through the line 4-5.

Loss distribution factors for loads are given in Table 4 using the LDF equations of (13). From Table 4, the loads of the buses 4, 5, and 9 are responsible for the real power loss in the line 1-4. Their responsible shares are computed as follows:

$$\begin{aligned} \text{Bus 4 : } & 0.0742 \times 1.082 = 0.0803[\text{MW}] \\ \text{Bus 5 : } & 0.4268 \times 1.082 = 0.4618[\text{MW}] \\ \text{Bus 9 : } & 0.4989 \times 1.082 = 0.5398[\text{MW}] \\ \text{total : } & 1.0820[\text{MW}] \end{aligned}$$

Table 5 represents the amount of power losses allocated to each load using the method based on LDFs together with the result using MLFs. Whereas the values based on LDFs are all positive, the values resulted from the method using MLFs are positive or negative.

Table 2. Result of real power flow calculations [MW]

from	to	P(from)	P(to)	loss
1	4	111.34	-110.26	1.082
4	5	46.88	-46.01	0.871
5	6	-43.99	45.26	1.264
3	6	85.00	-84.13	0.868
6	7	28.88	-28.38	0.499
7	8	-71.62	73.24	1.621
8	2	-160.99	163.00	2.009
8	9	73.75	-71.22	2.529
9	4	-53.78	55.38	1.598

**Table 3. Sharing factors of loads for each line flow**

line	load4	load5	load6	load7	load8	load9
4-1	1.000	0.511	0.0	0.0	0.0	0.430
5-4	0.0	0.511	0.0	0.0	0.0	0.0
5-6	0.0	0.489	0.0	0.0	0.0	0.0
6-3	0.0	0.489	1.000	0.284	0.0	0.0
7-6	0.0	0.0	0.0	0.284	0.0	0.0
7-8	0.0	0.0	0.0	0.716	0.0	0.0
8-2	0.0	0.0	0.0	0.716	1.000	0.570
9-8	0.0	0.0	0.0	0.0	0.0	0.570
9-4	0.0	0.0	0.0	0.0	0.0	0.430

**Table 4. LDFs for loads**

line	load4	load5	load6	load7	load8	load9
4-1	0.0742	0.4268	0.0	0.0	0.0	0.4989
5-4	0.0	1.0000	0.0	0.0	0.0	0.0
5-6	0.0	1.0000	0.0	0.0	0.0	0.0
6-3	0.0	0.5341	0.1214	0.3445	0.0	0.0
7-6	0.0	0.0	0.0	1.0000	0.0	0.0
7-8	0.0	0.0	0.0	1.0000	0.0	0.0
8-2	0.0	0.0	0.0	0.4566	0.0893	0.4541
9-8	0.0	0.0	0.0	0.0	0.0	1.0000
9-4	0.0	0.0	0.0	0.0	0.0	1.0000

**Table 5. Amounts of power losses allocated to loads(MW)**

	load4	load5	load6	load7	load8	load9	total
LDFs	0.060	3.060	0.165	3.336	0.179	5.579	12.381
MLFs	0.006	4.160	-0.138	1.777	-0.440	6.976	12.341

How do we understand the meaning of the negative sign? The loads with positive sign are charged for the amount of allocated loss. Then are the loads with negative sign paid for the amount of loss allocated? The negative sign of MLFs means that when the bus load increases, the loss would decrease. Because of the fact, are the loads with negative sign paid? Reversely, when the bus load with negative sign decreases, the loss would

increase. How do we interpret this fact?

MLFs are just an indicator for the trend of loss change with respect to variation of bus loads. They don't indicate how much losses should be allocated from the present losses existed. So it is not desirable to allocate losses to bus loads based on MLFs, rather desirable using loss distribution factors.

#### 4. Conclusion

As competition is promoted for electric power industries by enabling access to transmission services for all wholesale buyers and sellers of electric power, there is a necessity for separate pricing of the component parts of electricity production, delivery, and ancillary services. This paper deals with one ancillary service of real power losses allocation, and proposes a simple method to calculate the allocations of power losses to individual loads. Two different methods for computing power loss allocations are introduced; one is using MLFs concept and the other is based on LDFs. LDFs identify the loads responsible for the real power loss in a specific transmission line and indicate how much are the responsible shares of these loads for the line loss. According to the results of sample studies, it is desirable to allocate power losses to loads using LDFs rather than based on MLFs.

Moreover, it is anticipated that the loss allocation method can be utilized to compute the price of real power losses under a deregulated environment in electric power industries.

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