An Integrated Design Process for Manufacturing and Multidisciplinary Design Under System Uncertainty

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ABSTRACT

Necessity to address engineering system uncertainties in design processes has long been acknowledged. To obtain quality of product, a safety factor is traditionally used by many design engineers due to its easy of use and comprehension. However, the safety factor approach often yields either conservative or unreliable designs, since it ignores the type of probability distribution and the mechanism of uncertainty propagation from the input to the output. For a consistent reliability-based design, two fundamental issues must be investigated thoroughly. First, the design-decision process that clearly identifies a mechanism of uncertainty propagation under system uncertainties needs to be developed, which must be an efficient and accurate process. To identify the mechanism more effectively, an adaptive probability analysis is proposed by adaptively setting probability levels through a posteriori error estimation. The second is to develop the design process that not only yields a high quality design but also a cost-effective optimum design from manufacturing point of view. As a result, a response surface methodology is specially developed for RBDO thus enhancing numerical challenges of efficiency and complicatedness. Side crashworthiness application is used to demonstrate the integrated design process for product and manufacturing process design.

Key Words : Design Optimization, Reliability, Response Surface, Moving Least Squares, Design of Experiment, Crashworthiness

Nomenclature

- **X** Random parameter; $\mathbf{X} = [X_1, X_2,\ldots, X_n]^T$
- **x** Realization of **X**; $\mathbf{x} = [x_1, x_2,\ldots, x_n]^T$
- **U** Independent, standard normal random parameter
- **u** Realization of **U**; $\mathbf{u} = [u_1, u_2,\ldots, u_n]^T$
- **d** Design parameter; $\mathbf{d} = [d_1, d_2,\ldots, d_n]^T=\boldsymbol{\mu}(\mathbf{X})$
- $P(\bullet)$ Probability function
- $f_{\mathbf{X}}(\mathbf{x})$ Joint probability density function of the random vector
- $\Phi(\bullet)$ Standard normal probability distribution function
- $G(\mathbf{X})$ Design performance function
- $F_G(\bullet)$ Probability distribution function of performance
- β_s Safety reliability index
- β_t Target reliability index
- G_p Probabilistic performance; $G_p = F_G^{-1}(\Phi(-\beta_t))$
- **T** Transformation between X- and U-spaces
- $\hat{g}(\mathbf{d})$ Approximate response using moving least squares
- $\mathbf{a}(\mathbf{d})$ Coefficient vector of response approximation
- $E(\mathbf{d})$ Weighted residual of responses
- $\mathbf{W}(\mathbf{d})$ Weight matrix
- $\mathbf{H}(\mathbf{d})$ Basis matrix of response approximation
- $\mathbf{M}(\mathbf{d})$ Full moment matrix; $\mathbf{M}(\mathbf{d}) = \mathbf{H}^T\mathbf{W}(\mathbf{d})\mathbf{H}$
- $\mathbf{B}(\mathbf{d})$ Half moment matrix; $\mathbf{B}(\mathbf{d}) = \mathbf{H}^T\mathbf{W}(\mathbf{d})$

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1. Introduction

In this paper, a new methodology is described for integrated manufacturing and multidisciplinary design optimization process with a high quality control. To achieve high levels of product quality, the integrated process first identifies critical manufacturing sequences, and then incorporates the design process to the statistical variability of the manufacturing process. This design process is referred as the reliability-based design optimization (RBDO). Furthermore, the envisioned integrated process entails the design-evaluation process to obtain the probability distribution function of the output (or response), which can be performed at the pre- and post-design processes, as shown in Fig. 1. This design-evaluation process is called an output probability analysis, which identifies the uncertainty propagation and probability distribution function.

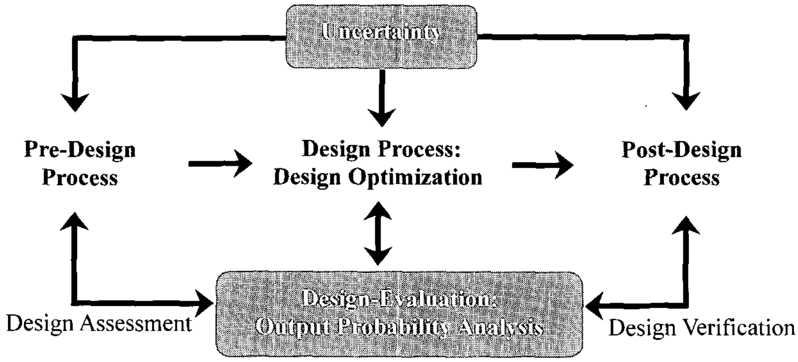


Fig. 1 Integrated Design Process

In the design process, RBDO[1,2] involves the evaluation of probabilistic constraints, which can be executed by several different approaches in the first-order reliability method. Compared to other approaches, the performance measure approach (PMA) is effective in terms of efficiency, stability, and accuracy, since the probabilistic constraints are more naturally defined by converting the probability measure to a performance measure. A stable and efficient hybrid mean value (HMV) method is employed for the numerical solution of the inverse PMA problem[1,2].

The design-evaluation process should provide accurate distribution of the system response at the pre- and post-design processes, which highlights the need of probability analysis methods such as a sampling method, moment matching method, and reliability index method[3,4]. Due to expensiveness of the sampling method and inaccuracy of the moment matching method, the reliability index method, which is developed by defining the most probable point (MPP) and a rotationally invariant reliability index, provides fairly accurate results. However, the existing probability analysis tools of the reliability index method are not only ineffective in reliability analysis (i.e., MPP search), but also unable to properly set the number of probability levels, since a set of probability levels must be prescribed without knowing a degree of the nonlinearity of the probabilistic system. To overcome these limitations, a probability analysis is adaptively carried out by approximating an MPP locus that provides an initial search point for the next probability levels[3]. The number of probability levels is increased and adaptively decided until all initial search points from the MPP locus approximation are close to all MPPs, depending on the nonlinearity of probabilistic system performances. Consequently, this paper proposes an integrated methodology for manufacturing and multidisciplinary design processes under system uncertainty by incorporating the design optimization and design-evaluation processes.

2. Design Process: Reliability-Based Design Optimization

As a parametric design process, the RBDO model[1,2] can be generally defined as

$$\begin{aligned} &\min \quad \text{Cost}(\mathbf{d}) \\ &\text{s.t.} \quad P(G_i(\mathbf{X}) \le 0) - \Phi(-\beta_t) \le 0,\ i = 1, \ldots, NP \quad (1) \\ &\qquad \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U,\quad \mathbf{d} \in R^{NDV} \text{ and } \mathbf{X} \in R^{NRV} \end{aligned}$$

where $\mathbf{d} = \mu(\mathbf{X})$ is the design vector, $\mathbf{X}$ is the random vector, and the probabilistic constraints are described by the performance function $G_i(\mathbf{X})$ with $G_i(\mathbf{X})<0$ as a failure, their probabilistic models, and their prescribed confidence level β_t.

Through inverse transformation, the probabilistic constraint in Eq. (2) can be further expressed in two distinct forms as:

$$\beta_{s_i} = (-\Phi^{-1}(F_{G_i}(0))) \geq \beta_t \tag{2}$$

$$G_{p_i} = F_{G_i}^{-1}(\Phi(-\beta_t)) \geq 0 \tag{3}$$

where β_{s_i} and G_{p_i} are respectively referred to as the safety reliability index and the probabilistic performance measure for the i^{th} probabilistic constraint. Using the reliability index, Eq. (3) is then employed to describe the probabilistic constraint in Eq. (1), i.e., the so-called reliability index approach (RIA). Similarly, Eq. (4) can replace the probabilistic constraint in Eq. (1) with the performance measure, referred to as the performance measure approach (PMA).

There are three major advantages in using PMA as compared to RIA[1,2,5]. First, it is found that PMA is inherently robust and more effective when the probabilistic constraint is either very much feasible or very much infeasible. Second, and more significantly, PMA always yields a solution, whereas RIA may not yield solutions for certain types of distributions, such as Gumbel or uniform distributions. Third, it is also found that PMA is more effective than RIA when RSM is used for RBDO. Therefore, PMA is only presented in this study, rather than employing ineffective RIA.

2.1 First-Order Reliability Analysis in PMA

The first-order reliability analysis in PMA can be formulated as the inverse of the first-order reliability analysis in RIA. The first-order probabilistic performance measure G_P is obtained from an optimization problem with an n-dimensional explicit sphere constraint in U-space, defined as

$$\begin{aligned} &\text{To find } \mathbf{u}^*_{\beta_t}, \quad \text{minimize} \quad G(\mathbf{U}) \\ &\qquad\qquad\quad \text{subject to} \quad \|\mathbf{U}\| = \beta_t, \end{aligned} \tag{4}$$

The optimum point on a target reliability surface is identified as the MPP $\mathbf{u}^*_{\beta_t}$.

2.2 PMA Methods of Reliability Analysis

Three numerical methods[1-3,5,6] were mainly used to solve Eq. (4): the advanced mean value method[6] in Eq. (5), the conjugate mean value method[1-3,5] in Eq. (6), and the hybrid mean value (HMV) method[1-3,5] in Eq. (7).

$$\begin{aligned} &\mathbf{u}^{(1)}_{\text{AMV}} = \beta_t \mathbf{n}(\mathbf{0}), \ \mathbf{u}^{(k+1)}_{\text{AMV}} = \beta_t \mathbf{n}(\mathbf{u}^{(k)}_{\text{AMV}}) \\ &\text{where } \mathbf{n}(\mathbf{u}^{(k)}_{\text{AMV}}) = -\frac{\nabla_U G(\mathbf{u}^{(k)}_{\text{AMV}})}{\|\nabla_U G(\mathbf{u}^{(k)}_{\text{AMV}})\|} \end{aligned} \tag{5}$$

$$\begin{aligned} &\mathbf{u}^{(0)}_{\text{CMV}} = \mathbf{0}, \ \mathbf{u}^{(1)}_{\text{CMV}} = \mathbf{u}^{(1)}_{\text{AMV}}, \ \mathbf{u}^{(2)}_{\text{CMV}} = \mathbf{u}^{(2)}_{\text{AMV}}, \\ &\mathbf{u}^{(k+1)}_{\text{CMV}} = \beta_t \frac{\mathbf{n}(\mathbf{u}^{(k)}_{\text{CMV}}) + \mathbf{n}(\mathbf{u}^{(k-1)}_{\text{CMV}}) + \mathbf{n}(\mathbf{u}^{(k-2)}_{\text{CMV}})}{\|\mathbf{n}(\mathbf{u}^{(k)}_{\text{CMV}}) + \mathbf{n}(\mathbf{u}^{(k-1)}_{\text{CMV}}) + \mathbf{n}(\mathbf{u}^{(k-2)}_{\text{CMV}})\|} \quad \text{for } k \geq 2 \\ &\text{where } \mathbf{n}(\mathbf{u}^{(k)}_{\text{CMV}}) = -\frac{\nabla_U G(\mathbf{u}^{(k)}_{\text{CMV}})}{\|\nabla_U G(\mathbf{u}^{(k)}_{\text{CMV}})\|} \end{aligned} \tag{6}$$

$$\begin{aligned} &\mathbf{u}^{(k+1)}_{\text{HMV}} = \begin{cases} \mathbf{u}^{(k+1)}_{\text{AMV}} \text{ in Eq. (5)}, & \text{if } G(\mathbf{u}^{(k)}_{\text{HMV}}) \text{ is convex} \\ \mathbf{u}^{(k+1)}_{\text{CMV}} \text{ in Eq. (6)}, & \text{if } G(\mathbf{u}^{(k)}_{\text{HMV}}) \text{ is concave} \end{cases} \\ &\text{with } \varsigma^{(k+1)} = (\mathbf{n}^{(k+1)} - \mathbf{n}^{(k)}) \cdot (\mathbf{n}^{(k)} - \mathbf{n}^{(k-1)}) \\ &\text{sign}(\varsigma^{(k+1)}) > 0: \ \text{Convex at } \mathbf{u}^{(k+1)}_{\text{HMV}} \text{ w.r.t. design } \mathbf{d} \\ &\qquad\qquad\quad \leq 0: \ \text{Concave at } \mathbf{u}^{(k+1)}_{\text{HMV}} \text{ w.r.t. design } \mathbf{d} \end{aligned} \tag{7}$$

Although the advanced mean value method shown behaves well for convex performance functions in PMA, it was found to have some numerical shortcomings, such as slow convergence, or even divergence, when applied to concave performance functions[1]. To overcome this difficulty, the conjugate mean value method was proposed[1]. The conjugate steepest descent direction significantly improves the rate of convergence as well as the stability, as compared to the advanced mean value method for the concave performance function. However, as seen in the literature[1], the conjugate mean value method is not as efficient as the advanced mean value method for the convex function. Consequently, the hybrid mean value (HMV) method was proposed to attain both stability and efficiency in the MPP search for PMA[1]. The HMV method employs the criterion for the performance function type, $\varsigma^{(k+1)}$. Once the performance function type is identified, either advanced mean value or conjugate mean value is adaptively selected for the MPP search. The numerical procedure of the HMV method was summarized and followed by some numerical examples in the literature[1].

3. Design Decision Process: Adaptive Probability Analysis[3]

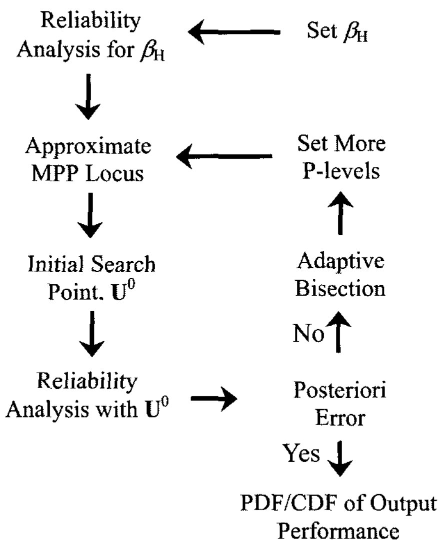


Fig. 2 Flow Chart of Adaptive Probability Analysis

Using the HMV method, a new adaptive probability analysis method is proposed[3] to effectively estimate the probability distribution of the performance function. For this, two key ideas are proposed to be integrated to develop the adaptive probability analysis. The first idea is using an MPP locus interpolation using a moving least squares method. The second idea is an adaptive method employing a posteriori-error estimator when choosing probability levels properly to refine the MPP locus. Thus, the adaptive method provides a close-looped probability analysis, as shown in Fig. 2. Detail discussions on two new ideas are described in next two subsections.

3.1 MPP Locus Approximation Using Moving Least Squares Method[2,7]

The MPP locus approximation[3] using the moving least squares[2,7] method is formulated as

$$\begin{aligned}\hat{U}_j(\beta) &= \sum_{i=1}^{NB} h_i(\beta)a_{ij}(\beta) \\ &= \mathbf{h}^T(\beta)\mathbf{a}_j(\beta) \quad \text{for } j = 1,\cdots,NRV\end{aligned} \tag{8}$$

where $\mathbf{h}$ is the basis vector and $\mathbf{a}_j(\beta) = [a_{1j}(\beta),\ldots,a_{NBj}(\beta)]^T$ is the j^{th} coefficient vector, which is a function of reliability β. Mutually independent monomials are used as basis functions. In this study, a quadratic polynomial basis is used to approximate the MPP locus.

To obtain the coefficient vector the residual functional E_{MLS} can be defined as

$$\begin{aligned}E^j_{MLS}(\beta) &= \sum_{I=1}^{NPL} w(\beta-\beta_I)[\hat{U}_j(\beta_I) - U_j(\beta_I)]^2 \\ &= \sum_{I=1}^{NPL} w(\beta-\beta_I)[\mathbf{h}^T\mathbf{a}_j - U_j(\beta_I)]^2 \\ &= \left[\mathbf{H}\mathbf{a}_j(\beta)-\mathbf{U}_j\right]^T \mathbf{W}(\beta)\left[\mathbf{H}\mathbf{a}_j(\beta)-\mathbf{U}_j\right]\end{aligned} \tag{9}$$

where

$$\begin{aligned}\mathbf{H} &= \begin{bmatrix} h_1(\beta_1) & \cdots & h_{NB}(\beta_1) \\ \vdots & \ddots & \vdots \\ h_1(\beta_{NPL}) & \cdots & h_{NB}(\beta_{NPL}) \end{bmatrix}, \\ \mathbf{U}_j &= \left[U_j(\beta_1) \ \cdots \ U_j(\beta_{NPL})\right], \text{ and} \\ \mathbf{W}(\beta) &= \begin{bmatrix} w(\beta-\beta_1) & 0 & \cdots & 0 \\ 0 & w(\beta-\beta_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w(\beta-\beta_{NPL}) \end{bmatrix}\end{aligned} \tag{10}$$

where NPL is the number of probability levels and the subscript 'MLS' denotes moving least squares. It is found that the approximation $U_j(\beta)$ is drawn closer to $U_j(\beta_i)$ at those points β_i with a relatively large weight w_i by introducing a weight inversely proportional to the distance between sample and interpolation points

Differentiation of E^j_{MLS} with respect to the coefficients $\mathbf{a}_j$ gives the coefficient vector for the j^{th} random parameter as

$$\mathbf{a}_j(\beta) = \mathbf{M}^{-1}(\beta)\mathbf{B}(\beta)\mathbf{U}_j, \quad j = 1,\ldots,NRV \tag{11}$$

where $\mathbf{M}(\beta)$ is referred to as the moment matrix and is given by

$$\mathbf{M}(\beta) = \mathbf{H}^T\mathbf{W}(\beta)\mathbf{H} \quad \text{and} \quad \mathbf{B}(\beta) = \mathbf{H}^T\mathbf{W}(\beta) \tag{12}$$

The MPP locus approximation using the moving least squares method is then expressed as

$$\hat{U}_j(\beta) = \mathbf{h}^T(\beta)\mathbf{M}^{-1}(\beta)\mathbf{B}(\beta)\mathbf{U}_j, \quad j = 1,\ldots,NRV \tag{13}$$

Since the coefficient vector is a function of the reliability index, Eq. (11) must be solved at different probability levels of interest. By setting the weight matrix to an identity matrix, the moving least squares approximation in Eq. (13) simply becomes the least squares approximation. Therefore, the moving least squares method is a more general approximation technique.

3.2 Adaptive Set of Probability Levels

Using moving least squares method, the approximate MPP locus is refined, as more probability levels are included. When adding more probability levels to the current set of probability levels, an adaptive method is applied by using a posteriori error estimator[3], which is defined as

$$\varepsilon_1 = \begin{cases} \left\| \hat{\mathbf{u}}^0 - \mathbf{u}^* \right\| & \text{for } \beta_i > 1 \\ \left\| \hat{\mathbf{u}}^0 - \mathbf{u}^* \right\| / \beta_i & \text{for } \beta_i \le 1 \end{cases} \quad \text{and}$$

$$\varepsilon_2 = \begin{cases} \left| G(\hat{\mathbf{u}}^0) - G(\mathbf{u}^*) \right| & \text{for } \left| G(\mathbf{u}^*) \right| > 1 \\ \left| G(\hat{\mathbf{u}}^0) - G(\mathbf{u}^*) \right| / \left| G(\mathbf{u}^*) \right| & \text{for } \left| G(\mathbf{u}^*) \right| \le 1 \end{cases} \tag{14}$$

where $\hat{\mathbf{u}}^0$ is an initial search point, $\mathbf{u}^*$ is the MPP for β_i, and β_i is the current probability level. In the proposed probability analysis, a posteriori-error estimation on approximated MPP locus enables to perform the adaptive scheme to decide the number of probability levels appropriately. Subsequently, the MPP locus is iteratively refined, by adding more probability levels adaptively. To add probability levels adaptively, the adaptive bisection method is used, as presented[3].

3.3 Design of Experiment (DOE)

Unlike deterministic design optimization, RBDO requires the additional accuracy of RSM, due to the evaluation of probabilistic constraints and their sensitivities. When the first-order reliability method is used for reliability analysis, accurate response and sensitivity information is required at the MPP, which is a certain distance away from the mean value design point. To meet this requirement, the moving least squares method must be integrated with a DOE that is specifically suited for reliability analysis. It is noted that PMA is more effective than RIA when RSM is used for RBDO, since the size of the DOE building block is clearly defined and a performance response can be consistently used in both reliability analysis and design optimization. However, the size of the DOE building block in RIA depends on how far away the MPP is located on a failure surface. In addition, RIA requires two different responses, such as a performance and reliability response[8].

To properly reproduce the main effects and interactions of design parameters, a new DOE framework for RBDO is proposed, composed of axial star and selective interaction (SI) samplings. Axial star sampling is incorporated to represent the main effects of design parameters by selecting a central point and samples along the design axes. The main effect on random parameters play a significant role in deciding the steepest descent direction required in the reliability analysis (see Section 2.2). As shown in Fig. 3(a), the analysis window in axial star sampling must cover the MPP search space defined in PMA so that a probabilistic constraint can be successfully evaluated. In order to capture interaction behaviors between design parameters, sample design points must be located over the interaction regions. In commonly used (fractional) factorial DOEs, a good deal of

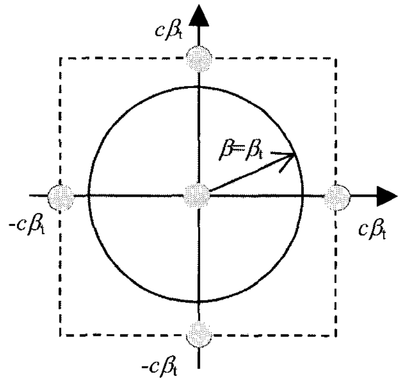


Fig. 3(a) Axial Star Sampling

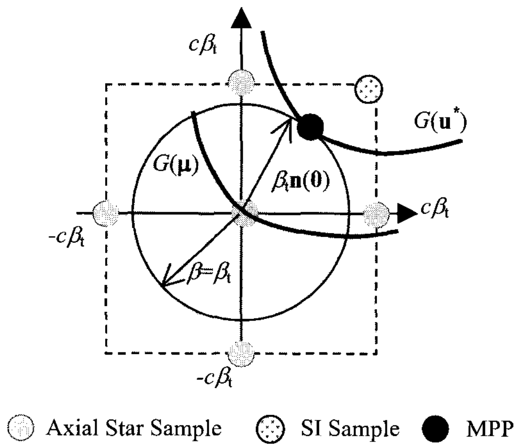


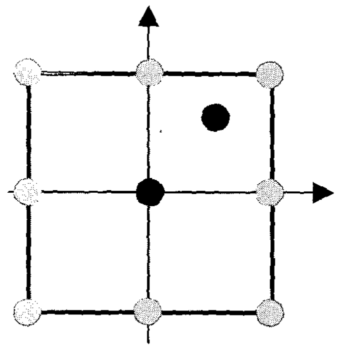
Fig. 3(b) Selective Interaction Sampling effort is spent in gathering interaction information. For example, in a full 2^k factorial DOE with k=10 design parameters, more than 1000 samples are required for the interaction effects of a single DOE. In the RBDO process the challenge is to evaluate probabilistic constraints effectively, which encourages development of the proposed SI sampling in order to capture interaction behavior precisely. The principal concept of SI sampling is the placement of sample points at the corner of the interaction region to include the MPP, where better approximation is required in the RBDO process based on the mean value first-order reliability method. This SI sampling process is illustrated in Fig. 3(b). The size of the DOE building block is defined by $c\beta_t$, where β_t is a target reliability and c is a parameter dependent on the nonlinearity of the response, typically 1.2~1.5.

Combining axial star with SI sampling constitutes a single DOE building block, called ASSI DOE. Next, the computation cost is considered in the proposed ASSI DOE. For example, an ASSI DOE with 10 design parameters and 10 constraints demands less than 31 sample designs, and is far cheaper than any other DOE method.

4. Reliability-Based Design Optimization Using Response Surface Method

The new RSM model addresses our fundamental concern: the effectiveness of RBDO not only for large-scale applications, but also for engineering applications without design sensitivity. The RSM-based RBDO process necessitates a different strategy from RSM-based deterministic optimization, requiring an analysis window to evaluate probabilistic constraints and a design window for design optimization purposes.

An interpolating response surface is generated over the region enclosed by sample points, referred to as an analysis window. Probabilistic constraints can then be computed at any evaluation point over this window. The design window is defined as the space over which any design point allows the probabilistic constraints to be evaluated properly. In a deterministic optimization, a point used to evaluate constraints is identical to a design point. In other words, a design window is identical to an analysis window, as shown in Fig. 4(a). As long as the optimum design exists within the design window, further optimization can be done without any additional design windows.



Design Window=Analysis Window

Fig. 4(a) Windows in Deterministic Optimization

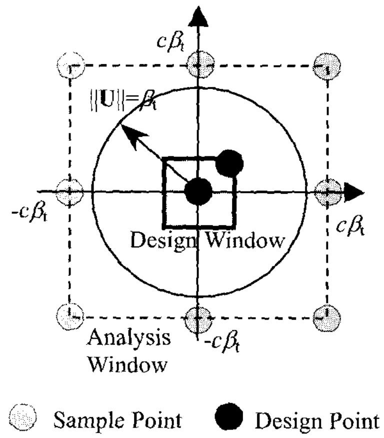


Fig. 4(b) Windows in RBDO

Design and analysis window concepts are then adapted to the RSM-based RBDO process. Unlike deterministic optimization as shown in Fig. 4(a), different mechanisms in the RBDO process distinguish the design window from the analysis window, since probabilistic constraints are not evaluated at the design point but at the evaluation point, such as at the MPP. As illustrated in Fig. 4(b), by definition the design window is a subset of the analysis window in the RBDO process. Therefore, as long as a new design is within the design window, a new analysis window is not necessary. Otherwise, the new analysis window must be created in order to evaluate probabilistic constraints properly. To create a finite design window, the size of the building block (or analysis window) is defined by $c\beta_t$, where β_t is a target reliability and c is a parameter dependent on the nonlinearity of the response, typically 1.2~1.5.

5. Results and Discussions

A side impact crashworthiness[2,3,9] is used to show the manufacturing and multidisciplianry design problem under system uncertainty. In product design process, product manufacturing (X_1 to X_9) and operating (X_{10} and X_{11}) uncertainties are considered, while the design process requires multidisciplinary crash simulation for estimating crash performances.

A large-scale application of vehicle side impact is employed, as illustrated in Fig. 5. The system model includes a full-vehicle FE structural model, an FE side impact dummy model, and an FE deformable side impact barrier model, consisting of 85,941 shell elements and 96,122 nodes. In the FE simulation of the side impact event, the barrier has an initial velocity of 49.89kph and impacts the vehicle structure[2,3,9]. The optimal Latin hypercube sampling with a total of 33 runs was used to generate a sample of design points for construction of the stepwise regression response surface. The explicit 9 responses used in the RBDO are summarized[9]. In this example, the explicit approximations of responses are regarded as exact responses of vehicle side impact to demonstrate the integrated design process. A total of 11 random parameters are employed, as listed in Table 1.

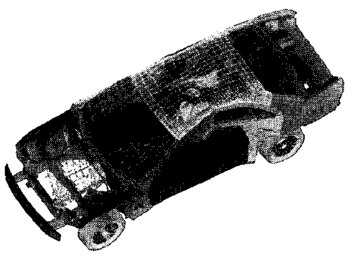


Fig. 5 Vehicle Side Impact Model

Table 1 Properties of Random Parameters

Random Variable	Mean	Std. Dev.
1 (B-pillar inner)	1.0	0.030
2 (B-pillar reinforce)	1.0	0.030
3 (Floor side inner)	1.0	0.030
4 (Cross member)	1.0	0.030
5 (Door beam)	1.0	0.030
6 (Door belt line)	1.0	0.030
7 (Roof rail)	1.0	0.030
8 (Mat. Floor Inside)	0.3	0.006
9 (Mat. Floor side)	0.3	0.006
10 (Barrier height)	0.0	10.0
11 (Barrier hitting)	0.0	10.0

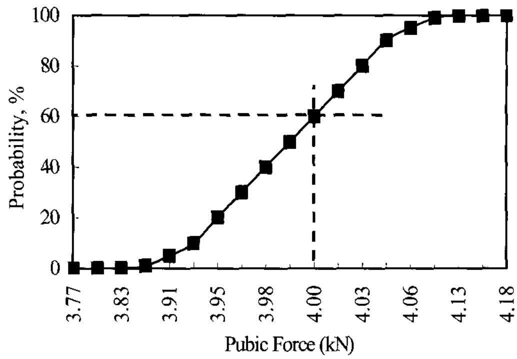


Fig. 6 CDF of Pubic Force for Vehicle Side Impact

This study starts with a pre-design process by assessing a current design through the proposed probability analysis tool. Since many responses are initially less reliable as shown in Fig. 6, they need to be improved through a design process, RBDO. Therefore, under the existence of uncertainties in either the manufacturing process or engineering simulations, a reliability-based design model for crashworthiness as a design process is formulated as

$$
\begin{aligned}
&\min \ \text{Weight}(\mathbf{d}) \\
&\text{s.t.}\ \ P(\text{abdomen load} \le 1.0kN) \ge P_s \\
&\qquad P(\text{upper/middle/lower VC} \le 0.32m/s) \ge P_s \\
&\qquad P(\text{upper/middle/lower rib defl.} \le 32mm) \ge P_s \\
&\qquad P(\text{pubic symphysis F, F} \le 4.0kN) \ge P_s \\
&\qquad P(\text{vel. of B-pillar at mid-point} \le 9.9mm/ms) \ge P_s \\
&\qquad P(\text{vel. of front door at B-pillar} \le 15.7mm/ms) \ge P_s \\
&\qquad \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \quad \mathbf{d} \in R^9 \text{ and } \mathbf{X} \in R^{11}
\end{aligned}
\tag{15}
$$

The reliability-based design obtained through the design process is verified by employing an adaptive probability analysis in the post-design process. In this vehicle side impact problem, the outcome probability analysis result is extremely accurate, with 0.25 % error, as compared to Monte Carlo simulation with one million samples. For this verification, a safety rating score of the vehicle[9] is measured at different four designs and statistically plotted through using the proposed output probability analysis method. Larger safety rating score indicates better design. The four designs: design 1 at the initial stage, design 2 at the deterministic design optimization, design 3 at PMA-RBDO with P_s=90%

target reliability, and design 4 at PMA-RBDO with P_s=99.87% target reliability are compared for the vehicle safety-rating in Fig. 7. As shown in Fig. 7, the CDFs of safety-rating score at four different designs demonstrate distributive levels of human safety from a vehicle side impact. The initial design has the largest deviation of safety-rating score for a vehicle side impact, while the deterministic optimum design has smaller deviation and the reliability-based optimum designs have the smallest safety-rating score over all probability levels and deviations. Accordingly, design 4 yields the highest achieves the highest target reliability at the same time. Consequently, the reliability-based optimum design not only improves the crashworthiness of a vehicle side impact, but also obtains a reliable and robust design by reducing randomness in the safety-rating score.

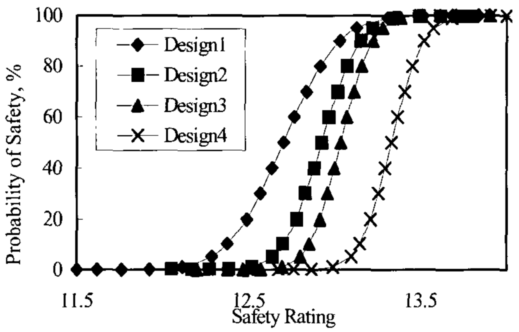


Fig. 7 CDF Plot of Safety Rating for Crashworthiness

6. Conclusion

This paper presents an integration of design process under system uncertainty, which aids a design process for manufacturing and multidisciplinary design under system uncertainty. It has been shown that designs are successfully assessed, optimized, and verified through the integrated design process under system uncertainty. Moreover, the integrated design process can be done very efficiently and accurately by developing an effective RBDO and adaptive probability analysis. Therefore, it provides not only an enhanced design in terms of the crashworthiness of a vehicle side impact, but also a reliable and robust design by managing the uncertainty.

References

1. Youn, B.D., Choi, K.K., Park, Y.H., "Hybrid Analysis Method for Reliability-Based Design Optimization," Journal of Mechanical Design, ASME, Vol. 125, No. 2, pp. 221-232, 2003.
2. Youn, B.D., Choi, K.K., "A New Response Surface Methodology for Reliability-Based Design Optimization," Computers and Structures, Vol. 82, No. 2, pp. 241-256, 2004.
3. Youn, B.D., Choi, K.K., "An Adaptive Probability Analysis Using Performance Measure Approach," AIAA Journal, Vol. 28, No. 3, pp. 219-233, 2004.
4. Du, X., Chen, W., "A Most Probable Point-Based Method for Efficient Uncertainty Analysis," Design Manufacturing, Vol. 4, No. 1, pp. 47-66, 2001.
5. Youn, B.D., Choi, K.K., "An Investigation of Nonlinearity of Reliability-Based Design Optimization Approaches," Journal of Mechanical Design, ASME, Vol. 126, No. 3, pp. 403-411, 2004.
6. Wu, Y.-T., Wirsching, P.H., "New Algorithm for Structural Reliability Estimation," Journal of Engineering Mechanics, ASCE, Vol. 113, No. 9, pp. 1319-1336, Sept. 1987.
7. Lancaster, P., Salkauskas, K Curve and Surface Fitting; An Introduction, Academic Press, London, 1986
8. Qu, X., Venkataraman, S., Haftka, R.T., Johnson, T.F., "Reliability, Weight, and Material Improvement Tradeoffs in the Design of Composite Laminates for Cryogenic Environments," 42nd AIAA/ASME/ASCE/ AHS/ASC Structures, Structural Dynamics, and Materials Conference, AIAA 2001-1327, 2001.
9. Gu, L., Yang, R.J., Cho, C.H., Makowski, M., Faruque, M., Li, Y., 2001, "Optimization and Robustness for Crashworthiness," International Journal of Vehicle Design, Vol. 26., No. 4.