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Development of High-Performance FEM Modeling System Based on Fuzzy Knowledge Processing

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Abstract

This paper describes an automatic finite element (FE) mesh generation for three-dimensional structures consisting of free-form surfaces. This mesh generation process consists of three subprocesses: (a) definition of geometric model, (b) generation of nodes, and (c) generation of elements. One of commercial solid modelers is employed for three-dimensional solid structures. Node is generated if its distance from existing node points is similar to the node spacing function at the point. The node spacing function is well controlled by the fuzzy knowledge processing. The Voronoi diagram method is introduced as a basic tool for element generation. Automatic generation of FE meshes for three-dimensional solid structures holds great benefits for analyses. Practical performances of the present system are demonstrated through several mesh generations for three-dimensional complex geometry.

Key Words : Automatic Mesh Generation, Fuzzy Theory, Advancing Front Method, Voronoi Diagram Method, Computational, Geometry, Finite Element Method, Solid Geometry, Shell Geometry

I. Introduction

The finite element method (FEM) has been widely utilized in simulating various engineering problems such as structural deformation, thermal conduction, fluid dynamics, electromagnetics and so on. The main reason for this is its high capability of dealing with boundary-value problems in arbitrarily shaped domains. On the other hand, a mesh used influences computational accuracy as well as time so signifi- cantly that the mesh generation process is as much important as the FEM analysis itself. Especially, in such large scale nonlinear FEM analyses that approach the limitation of computational capability of so-called supercomputers, it is highly demanded to optimize the distribution of mesh size under the condition of limited total degrees of freedom. Thus, the mesh generation process becomes more and more time-consuming and heavier tasks.

Loads for pre-processing and post-processing are increasing rapidly in accordance with an increase of scale and complexity of analysis models to be solved. Particularly, the mesh generation process, which influences computational accuracy as efficiency and whose fully automation is very difficult in three-dimensional (3D) cases, has become the most critical issue in a whole process of the FE analyses. In this respect, various researches [1-5] have been performed on the development of automatic mesh generation techniques. Among mesh generation methods, the tree model method [6] can generate graded meshes and it uses a reasonably small amount of computer time and storage. However, it is, by nature, not possible to arbitrarily control the changing rate of mesh size with respect to location, so that some smaller projection and notch etc. are sometimes omitted. Also, domain decomposition method [7] does not always succeed, and a designation of such subdomains is very tedious for uses in 3D cases.

In recent years, much attention has been paid to fuzzy knowledge processing techniques [8], which allow computers to treat "ambiguous" matters and processes. In this paper, a novel FE mesh generation system are explained based on fuzzy knowledge processing and computational geometry techniques. Here, the node density distribution, which is a kind of a node spacing function, was well controlled by means of the fuzzy knowledge processing technique, so that even beginners of the FE analyses are able to produce nearly optimum meshes through very simple operations as if they were experts.

The individual techniques in the present study are as follows :

(a) Adoption of practical geometric modelers such as Designbase [9] which are capable of dealing with Bezier-type free-form surfaces.
(b) Adoption of the grid method [10] for fast node generation, which is one of computational geometry techniques.
(c) Adoption of the Voronoi diagram method [2] for fast element generation.

In the following sections, first described are the general requirements for automatic mesh generators, the fundamental principle of the present algorithm. The practical performances of the system are demonstrated through the mesh generation of several 3D structures.

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2. General Requirement for Automatic Mesh Generation System

The phase of pre-processing is very important in the sense that the generation of a valid mesh in a domain with a complex geometry is not a trivial operation and can be very expensive in terms of the time required. On the other hand, it is crucial to create a mesh which is well adapted to the physical properties of the problem under consideration, as the quality of the computed solution is strongly related to the quality of the mesh.

Various automatic and semi-automatic mesh generation methods have been investigated so far. The requirements for ideal fully automatic mesh generation systems may be summarized as follows [11] :

(a) Arbitrarily shaped domain can be subdivided into elements.
(b) Mesh size and its changing rate with respect to location can be easily controlled.
(c) Distortion of element shape can be avoided as much as possible.
(d) Total number of nodes can be controlled.
(e) Number of input data is smaller.

The requirement (a) is fundamental, while (b) and (c) are strongly related to mesh quality. The requirement (d) corresponds to the controllability of computational time and storage. If any system satisfies the items (a) through (d), optimum meshes can be generated with the balance of computational accuracy as well as efficiency. The requirement (e) is also indispensable for any systems dealing with 3D complex geometries.

3. Outline of the System

3.1 Definition of Geometric Model

Geometric modelers are utilized to define geometries of analysis domains. One of commercial geometric modelers, Designbase [9] is employed for 3D solid structures. The advantage of Designbase is that a wide range of solid shapes from polyhedra to free-form surfaces can be designed in a unified manner. In these modelers, 3D geometric data are stored as a tree structure of domain - surfaces (free-form surfaces such as Bezier or Gregory type surfaces) - edge (B-spline or Bezier type curves) - vertices.

Designbase allows the user to start with a hierarchical compositional Constructive Solid Geometry (CSG)-view of a part and then to refine it with local but consistent operations on the boundary represen- tation of the object. By basing all operations available to the user on well-defined, invertible Euler-operations, it is possible to keep a compact represen- tation of the complete design history of a part, and thus to "undo" and "redo" any sequence of operations. This encourages the designer to try out ideas without fear of destroying a model in which several hours of design time have already been invested. It also makes it possible to store several alternative versions of a design in a natural and efficient manner. As an example, Fig. 1 shows a geometry model of 3D solid structures using Designbase.

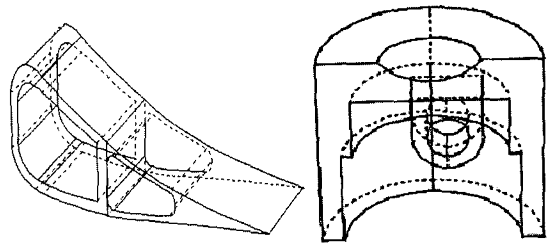

Fig. 1. Examples of geometry model

3.2 Designation of node density distributions

In this section, the connecting process of locally-optimum mesh images is dealt with using the fuzzy knowledge processing technique [12,13].

To begin with, let us consider a mesh generation process performed by the experts on FEM stress analyses, taking an example of an upper half portion of a cracked plate with a circular hole as shown in Fig. 2(a). Fig. 2(b) and 2(c) show the schematic views of the locally-optimum mesh images around a hole and a crack, respectively. It is anticipated that the experts have attained such mesh images through theoretical studies of numerical analyses. If both mesh images are connected smoothly, one could generate a quasi globally-optimum mesh in the whole analysis domain. However, it will be soon noticed that none of the conventional mesh generation techniques, which are strictly based on mathematical principles, enables us to do so since the connecting process is ambiguous and non-algorithmic.

In this paper, such a connection process of locally optimum mesh images is performed using the fuzzy knowledge processing technique.

3.3 Superposition of Locally Optimum Mesh Pattern

Performances of automatic mesh generation methods based on node generation algorithms depend on how to control node spacing functions or node density distributions and how to generate nodes. The basic concept of the present mesh generation algorithm is originated from the imitation of mesh generation processes by human experts on FE analyses. One of the aims of this algorithm is to transfer such experts' techniques to beginners.

In the present system, nodes are first generated, and then a finite element mesh is built. In general, it is not so easy to well control element size for a complex geometry. A node density distribution over a whole geometry model is constructed as follows. The present system stores several local nodal patterns such as the pattern suitable to well capture stress concentration, the pattern to subdivide a finite domain uniformly, and the pattern to subdivide a whole domain uniformly. A user selects some of those local nodal patterns, depending on their analysis purposes, and designates where to locate them.

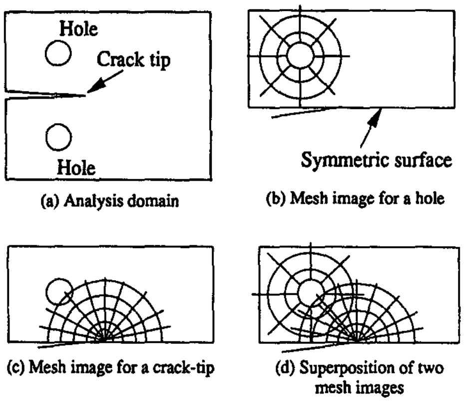

Fig. 2. Mesh images of experts on FEM stress analysis

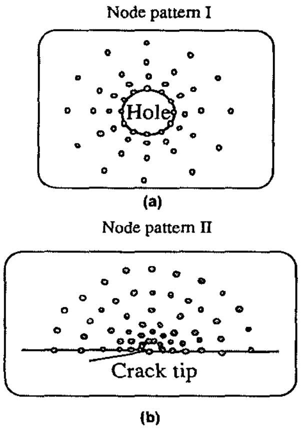

Fig. 3. Examples of locally optimum node patterns
(a) For a hole
(b) For a crack tip

For example, when either the crack of the hole exists solely in an infinite domain, the local node patterns as shown in Figs. 3(a) and 3(b) may be regarded locally-optimum around the crack tip or the hole, respectively. When these stress concentration fields exist closely to each other in the same analysis domain, a simple superposition of both local node patterns gives the result as shown in Fig. 4(a). Namely, extra nodes have to be removed from the superposed region of both patterns.

In the present method, the field A close to the crack-tip and the field B close to the hole are defined in terms of the membership functions used in the fuzzy set theory as shown in Fig. 4(c).

For the purpose of simplicity, each membership function is given a function of one-dimension in the figure. In practice the membership function can be expressed as $\mu(x, y)$ in this particular example, and in 3D cases it is a function of 3D coordinates, i.e. μ (x, y, z). In Fig. 4(c), the horizontal axis denotes the location, while the vertical axis does the value of membership function, which indicates the magnitude of "loseness"of the location to each stress concentration field. That is, a nodal location closer to the stress concentration field takes a larger value of the membership function. As for

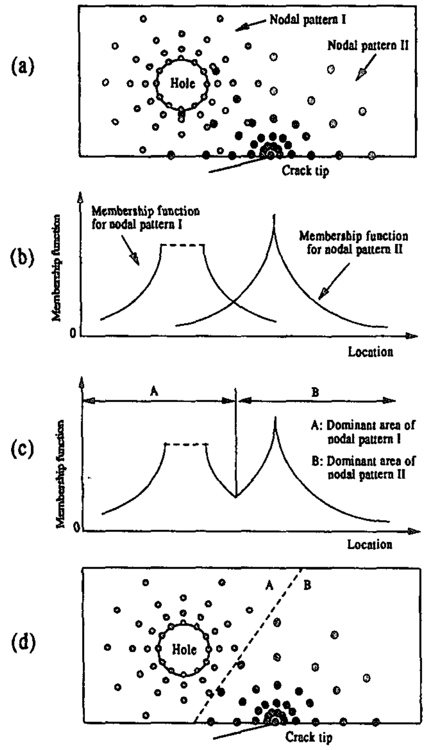

Fig. 4. Superposition of node patterns based on fuzzy knowledge processing

Fig. 4(b), choosing the mesh pattern with a larger value of the membership function in each location, one can obtain an overlapped curve of both membership functions, and the domain can be automatically divided into the following two sub-domains A and B as shown in Fig. 4(c) : the sub-domain close to the crack-tip and that of the hole. Finally, both node patterns are smoothly connected as shown in Fig. 4(d). This procedure of node generation, i.e. the connection procedure of both node patterns, is summarized as follows :

- If $\mu_A(x_p, y_p) \geq \mu_B(x_p, y_p)$ for a node p (x_p, y_p) belonging to the pattern A, then the node p is generated, and otherwise p is not generated.
- If $\mu_A(x_q, y_q) \geq \mu_B(x_q, y_q)$ for a node q (x_q, y_q) belonging to the pattern B, then the node q is generated, and otherwise q is not generated.

It is apparent that the above algorithm can be easily extended to 3D problems and any number of node patterns. In addition, since finer node patterns are generally required to place near stress concentration sources, it is convenient to let the membership function correspond to node density as well. According to this definition, Fig. 5 also indicates the distribution of node density over the whole analysis domain including the two stress concentration fields. When designers do not want any special meshing, they can adopt uniformly subdivided mesh. It is possible to combine the present techniques with an adaptive meshing technique.

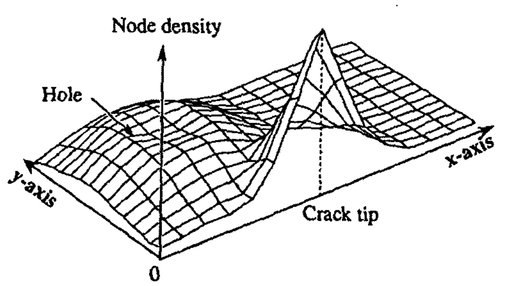

Fig. 5. Distribution of node density of whole domain

3.4 Node Generation

Node generation is one of time consuming processes in automatic mesh generation. The base node pattern and several special ones are placed in the domain, and all the node patterns are smoothly connected based on the principle described in section 3.3.

The input data required are only the node density of the base node pattern such as unit distance of nodes, the kinds of special node patterns, and the location and node densities at the representative points of the special node patterns.

The procedure of two dimensional node generation of the base node pattern is illustrated in Fig. 6. First, either a circumscribed rectangle or box (in the 3D) to the domain is determined, in which nodes are generated regularly. A distance of neighboring nodes of the pattern, which is called "base grid size" here, is inputted by a user. Second, each node is examined whether to be inside the domain by the IN-OUT check criterion, and any nodes outside the domain are removed. Any nodes located very closely to the domain boundary are removed as well to avoid undesirable distortion of mesh shape near the domain boundary. Among algorithms for uniform node generation, the present method may not be the best one

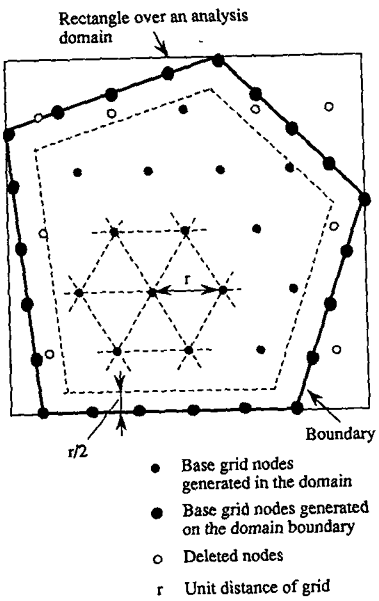

Fig. 6. Generation of base node pattern

as for the uniformity of node locations near the domain boundary, but it is very simple operation. On plane boundary of 3D structures, nodes are generated by the identical procedure to the above 2D case. On line boundary, nodes are generated regularly with the identical space to the base grid size.

Next, several node patterns are generated through an interactive operation between a user and the system. The input data required here are only the node density and the location of each stress concentration point.

The node generation procedure of the special node pattern inside the domain is essentially identical to that of the base node pattern described previously. On the other hand, nodes on the domain boundary are generated one by one by calculating the value of the membership function on the boundary. For example, nodes are generated on straight line as follows:

$$X_{i+1} = X_i + \left(\frac{1}{f^{1/3}}\right) u \qquad (1)$$

where function f is the value of membership function, u the unit base vector of the line, X_{i+1} the vector of the new point, and X_i that of the old point, respectively.

3.5 Creation of Finite Elements

After all the nodes are generated in the analysis domain and on its boundary, the system creates triangular (in the 2D) or tetrahedral (in the 3D) elements. As an example, the algorithm of triangulation used here is described in Fig. 7.

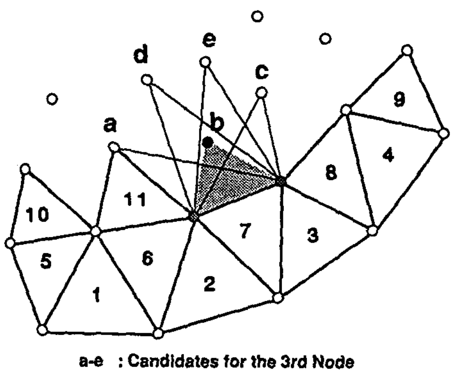

Fig. 7. Schematic view of triangulation mesh

First, created is the table which involves the coordinate data of segmented lines and the numbers of both initial and terminal nodes. In the beginning stages, only the segmented lines on the domain boundary are registered in the table. Following the order of registration, one segmented line is taken. Then, the node such that an inner angle produced by the node and the segmented line is the largest, is chosen among other nodes, and a new triangle and two new segmented lines are created. If the new lines have not been registered in the table yet, they are added at the end of the table. Such operation is repeated until all the segmented lines are processed. Finally the whole domain is subdivided into a number of triangles. In a 3D domain, tetrahedral elements can be generated by the similar algorithm.

3.6 Smoothing Process of Element

The algorithm of element generation mentioned above works well in most cases. However, element shapes obtained are sometimes distorted in a superposed region of several node patterns or near domain boundary. The smoothing method called "Laplacian operation"[19] is here applied to remedy such distorted elements. In this operation, the location of each node is replaced with a mean value of locations of its neighboring nodes. This operation is iterated several times.

4. Examples and Discussions

The performance of the system is demonstrated through the mesh generation of several 3D structures. Fig. 8 to 10 show the examples of the application of this mesh generator for 3D geometry. As shown in figures, a uniform mesh and a nonuniform mesh were connected very smoothly. In case of a half of piston head as shown in Fig. 8, it took about 60 minutes to define this geometry model by using Designbase. The mesh consists of 14,250 tetrahedral elements and 27,458 nodes. Nodes and elements are generated in about 15 minutes and in about 3 minutes, respectively. To complete this mesh, the following two node patterns are utilized ; (a) the base node pattern in which nodes are generated with uniform spacing over a whole analysis domain, (b) a special node pattern for stress concentration of four corners. In case of complex nozzle corner as shown in Fig. 10, nodes and elements were generated in about 30 minutes and in about 20 minutes, respectively. The mesh consists of 38,863 tetrahedral elements and 8,426 nodes.

5. Conclusions

A novel automatic FEM mesh generation system for large scale complex structures. Here several locally optimum node patterns are chosen from a database and are superposed automatically based on the fuzzy knowledge processing technique.

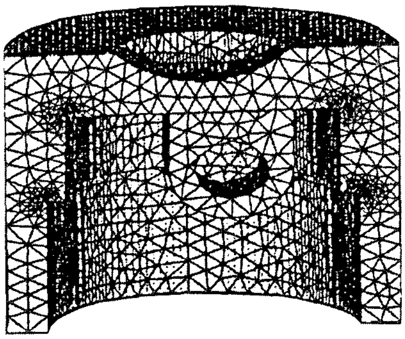

Fig. 8 Mesh for a half of piston head

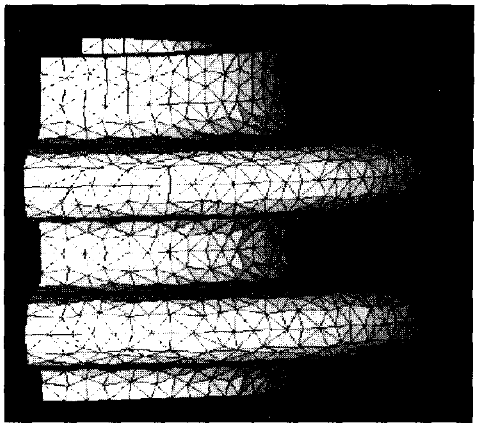

Fig. 9. Mesh for a symmetric 2 convolution of bellows

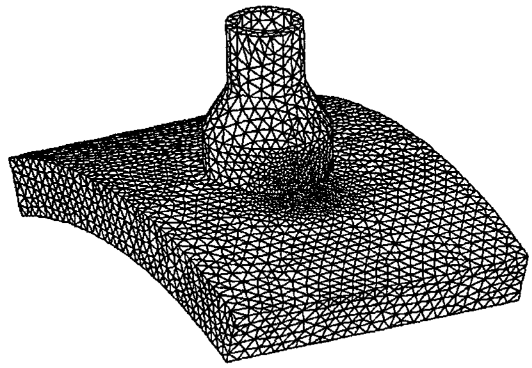

Fig. 10. Mesh for a nozzle part
(No. of nodes = 8,426, No. of elements = 38,863)

Also, several computational geometry techniques were successfully applied to node and element generation, whose processing speed is proportional to the total number of nodes. The key features of the present algorithm are an easy control of complex 3D node density distribution with a fewer input data by means of the fuzzy knowledge processing technique, and fast node and element generation owing to some computational geometry techniques. The effective of the present system is demonstrated through several mesh generations for 3D complex structures.

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