

# A New Approach of BK products of Fuzzy Relations for Obstacle Avoidance of Autonomous Underwater Vehicles

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## Abstract

This paper proposes a new heuristic search technique for obstacle avoidance of autonomous underwater vehicles equipped with a looking ahead obstacle avoidance sonar. We suggest the fuzzy relation between the sonar sections and the properties of real world environment. Bandler and Kohout's fuzzy relational method are used as the mathematical implementation for the analysis and synthesis of relations between the partitioned sections of sonar over the real-world environmental properties. The direction of the section with optimal characteristics would be selected as the successive heading of AUVs for obstacle avoidance. For the technique using in this paper, sonar range must be partitioned into multi equal sections; membership functions of the properties and the corresponding fuzzy rule bases are estimated heuristically. With the two properties Safety, Remoteness and sonar range partitioned in seven sections, this study gives the good result that enables AUVs to navigate through obstacles in the optimal way to goal.

**Key words** : Autonomous underwater vehicles (AUVs), obstacle avoidance, fuzzy relation, BK-products

## 1. Introduction

In recent years, autonomous underwater vehicles (AUVs) have become an intense tendency of research about ocean robotics because of the commercial as well as military potential and the technological challenge in developing them. Navigation with an obstacle execution process is one of the many important capabilities and behaviors of AUVs. The real time operation of this capability is the main requirement needed within the autonomous vehicle's control management system. Many approaches to the problem have been proposed in recent years [1][2][7][17][18][19][21]. Additionally, the heuristic search technique for AUVs obstacle avoidance using BK-products has already presented in [14][15].

In this paper, a new heuristic search technique is suggested. It is required that the range of obstacle avoidance sonar required being divided into many sections, which probably detect forward obstacles distinctly. The fuzzy relation between the partitioned sections of sonar range and the properties toward the real world environment in which AUVs navigate plays an important role in this study. Because the a-priori knowledge and information about underwater environment in which the AUVs operate is often incomplete, uncertain and approximated, fuzzy logic is necessarily used to generate the

possibility of the real time environmental effects to AUVs navigation [20]. Domain experts who are boarding a submarine recognize characteristics and events that can occur in the navigation environment and then analyze to figure out the rule bases and membership functions so used for reasoning. Thus, Bandler and Kohout's fuzzy relational method (BK-products) is applied to the above fuzzy relation and its transposed to select the optimal successive heading direction for the AUVs path planning in case of obstacle presenting in the planned route.

Actually, the proposed algorithm has essentially focused on horizontal movement since the translation cost for vertical movement of AUVs proved in [18] is 1.2 times greater than in horizontal movement. The turning angle of AUVs heading is equal to the angle formed by the current heading and the selected section of sonar. In the exception case obstacle is completely filled up the sonar's coverage, AUVs must go to up one layer at a time and then apply the algorithm to find out the turning. Until obstacle clearance, AUVs are constrained to go back to the standard depth of the planned route. The first part of the paper will introduce the theory of BK-products. Second, the design of the AUV obstacle avoidance technique using BK-products is explained briefly. At last, the case study is elaborated to denote the feasible application of BK-products using for this study and the experiment results are also shown in both 2D and 3D cases.

## 2. BK-products of Fuzzy Relation

Relational representation of knowledge makes it possible to perform all the computations and decision making in a uniform relational way, by mean of special relational compositions called triangle and square products. These were first introduced by Bandler and Kohout and are referred to as the BK-products in the literature. Their theory and applications have made substantial progress since then [3-4][6][8-16].

There are different ways to define the composition of two fuzzy relations. The most popular extension of the classical circular composition to the fuzzy case is so called *max-min* composition [10]. Bandler and Kohout extended the classical circular products to BK-products as sub-triangle ( $\triangleleft$ , “included in”), super-triangle ( $\triangleright$ , “includes”), and square ( $\square$ , “are exactly the same”). Assume the relations  $R$  and  $S$  are fuzzy relations, then the  $R$ -afterset of  $x$ ,  $xR$  and the  $S$ -foreset of  $z$ ,  $Sz$ , obviously are fuzzy sets in  $Y$ . The common definition of inclusion of the fuzzy set  $xR$  in  $Y$  in the fuzzy set  $Sz$  in  $Y$  is given by (1)

$$xR \subseteq Sz \Leftrightarrow (\forall y \in Y)(xR(y) \leq Sz(y)) \quad (1)$$

A fuzzy implication is modeled by mean of a fuzzy implication operator. A wide variety of fuzzy implication operators have been proposed, and their properties have been analyzed in detail [4][10][16]. For this study, we make use only of Lukasiewicz fuzzy implication operator as shown in (2) and the triangle sub product. Using (2), with  $n$  the cardinality of  $Y$ , we easily obtain the mathematical definitions for the BK-sub triangle product as in (3)

$$a \rightarrow_5 b = \min(1, 1 - a + b) \quad (2)$$

$$x_i(R \triangleleft S)z_j = \frac{1}{n} \sum_{y \in Y} \min(1, 1 - x_i R(y) + S z_j(y)) \quad (3)$$

Along with the above definitions, “ $\alpha$ -cut” and “Hasse diagram” are also the two important features in this method. The  $\alpha$ -cut transforms a fuzzy relation into a crisp relation, which is represented as a matrix [4][9][15][22]. Let  $R$  denotes a fuzzy relation on the  $X \times Y$ . The  $\alpha$ -cut relation for  $R$  is defined as the equation (4).

$$R_\alpha = \{(x, y) \mid R(x, y) \geq \alpha \text{ and } 0 \leq \alpha \leq 1\} \quad (4)$$

The Hasse diagram is a useful tool, which completely describes the partial order among the elements of the crisp relational matrix by a Hasse diagram structure. To determine the Hasse diagram on the relation, the following 3 steps should be adopted [5][12][15].

- [Step 1] Delete all edges that have reflexive property.
- [Step 2] Eliminate all edges that are implied by the transitive property.
- [Step 3] Draw the diagraph of a partial order with all edges pointing upward, and then omit arrows from the edges.

## 3. Using BK-Products To AUV Obstacle Avoidance

In this study it is required that obstacle avoidance sonar range can be partitioned into several sub-ranges. One of these represents for the successive heading candidate of AUVs navigation. Whenever obstacle is detected, the sonar return is clustered and the sections in which obstacles are presenting can be identified distinctly. The sonar model is illustrated as in the Fig.1. Domain experts who have wide knowledge about ocean science could give the properties about the environmental effects to the of AUVs navigation.

The looking-forward obstacle avoidance sonar whose coverage range can be divided into multi-sections is used to determine a heading candidate set  $S$ . Otherwise; a property set  $P$  denotes the possibility to which the sonar sections can be characterized by the real world environment. The fuzzy rule bases and membership functions for the corresponding property can be estimated heuristically. The set of the candidates  $S$  and the set of environmental properties  $P$  are shown in (5). The relation  $R$  is built as (6). The elements  $r_{ij}$  of this relation means the possibility the section  $s_i$  can be characterized by the property  $p_j$ . The value of  $r_{ij}$  is calculated by means of the rule bases with the membership functions.

$$S = \{s_1, s_2, \dots, s_i\} \quad P = \{p_1, p_2, \dots, p_j\} \quad (5)$$

$$R = S \times P = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1j} \\ r_{21} & r_{22} & \dots & r_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ r_{i1} & r_{i2} & \dots & r_{ij} \end{bmatrix} \begin{matrix} s_1 \\ s_2 \\ \vdots \\ s_i \end{matrix}$$

$$\begin{matrix} p_1 & p_2 & \dots & p_j \end{matrix}$$

$$T = R \triangleleft R^T = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1i} \\ t_{21} & t_{22} & \dots & t_{2i} \\ \vdots & \vdots & \ddots & \vdots \\ t_{i1} & t_{i2} & \dots & t_{ii} \end{bmatrix} \begin{matrix} s_1 \\ s_2 \\ \vdots \\ s_i \end{matrix} \quad (7)$$

$$R_\alpha = \alpha\_cut(T, \alpha) = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1i} \\ a_{21} & a_{22} & \dots & a_{2i} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ii} \end{bmatrix} \begin{matrix} s_1 \\ s_2 \\ \vdots \\ s_i \end{matrix} \quad (8)$$

In the next step, a new fuzzy relation  $T$  is computed by using sub-product products  $\triangleleft$  to fuzzy relation  $R$  and  $R^T$ , the transposed of  $R$ . The fuzzy relation  $T$  as shown in (7) is the product relation between candidate set  $S$  and candidate set  $S$  that means the degree of implication among elements of candidate set.

Then, the  $\alpha$ -cut is applied to fuzzy relation  $T$  in order to transform into crisp relation as shown in (8). It is important to

select a reasonable  $\alpha$ -cut value because the hierarchical structure of candidate set depends on an applied  $\alpha$ -cut.

Finally, we draw the Hasse diagram, which completely describes a partial order among elements of candidate set. That is to say a hierarchical structure among the elements of candidate set with respect to the optimality and efficiency. Select then the top node of the Hasse diagram as the successive heading of AUVs.

Because the energy consumption in vertical movement of AUVs is much greater than in the horizontal movement (1.2 times)[18], this technique is focus strongly on the horizontal movement. In the case of obstacle occurrence, AUVs just turn left or turn right with the turning angle determined by degree from the current heading to the selected section. But in the exception case a very wide obstacle has completely filled up the sonar's coverage, AUVs must go to upper layer at a time and then apply the algorithm to find out the turning. Until obstacle clearance, AUVs are constrained to go back to the standard depth of the planned route.

The algorithm of the proposed technique can summarize into five below steps and is imitated briefly in control flow as shown in Fig. 2.

**[Step 1]** If AUVs detects obstacle then go to next step, else go to step 5

**[Step 2]** Determine  $P$  and configure  $S$

**[Step 3]** If very wide obstacle is detected in all of  $S$  then go up and return step 1, else go to next step.

**[Step 4]** Apply BK-products to  $S$  and  $P$  to select the successive heading for obstacle avoidance

**[Step 5]** Go on in the planned route

## 4. Case Study

### 4.1 Determination of $S$ and $P$

In the case study, assuming that sonar range can be divided into seven sections as shown in the figure 1. The set  $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$  is set of the heading candidate. Actually, domain expert could determine the two fuzzy properties *safety* and *remoteness*. The set  $P = \{p_1, p_2\}$  is set of these two properties where  $p_1, p_2$  is the safety and remoteness property respectively. The safety property means the degree to which each section of sonar can be characterized in case obstacle appears within the sonar range. The remoteness property means how far of the distance to goal if AUVs follow the direction of the section  $s_i$ .

### 4.2. Calculation of BK-products of fuzzy relations

The a-priori knowledge and information about underwater environment in which the AUVs have to operate is often incomplete, uncertain. Therefore the establishment of

membership functions is very important in order to obtain more reasonable solution using the BK's method. The membership functions of the safety and remoteness properties are heuristically formulated in Eq.9 and Eq.10. The configuration of safety and remoteness characteristics are shown in Fig.3 and Fig.4, respectively.

$$\mu_s(x) = 1 - \frac{1}{1 + e^{\frac{1}{s-x}}} \quad (9)$$

$$\mu_r(y) = \frac{1}{1 + e^{\frac{1}{s-y}}} \quad (10)$$

The two variables  $x$  and  $y$  in Eq.9 and Eq.10 above are two fuzzy variables determined by the fuzzy rule base. In this context, it was assumed that there are four levels of ambiguity as very safe, safe, risky and very risky for safety property. It was also assumed that there are four levels of ambiguity as very far, far, near and very near according to remoteness property. The rule bases for safety and remoteness properties are shown in the table 1 and table 2 respectively.

It is assumed that obstacles are detected by the sections  $s_2, s_4,$  and  $s_5$  as in the figure 1. Table 3 and table 4 show the fuzzy membership value of the heading candidate set with respect to the safety and remoteness degree gain by the fuzzy rule bases and the membership functions.

The fuzzy relation  $R$  in (11) between the heading selection set  $S$  and the property set  $P$  can be done easily from the values in table 3 and table 4.

$$R = \begin{pmatrix} 0.9526 & 0.0180 \\ 0.5000 & 0.0474 \\ 0.8808 & 0.1192 \\ 0.5000 & 0.2689 \\ 0.5000 & 0.1192 \\ 0.9526 & 0.0474 \\ 0.9820 & 0.0180 \end{pmatrix} \quad (11)$$

Then, the new fuzzy relation  $T$  in (12) is calculated using triangle subproducts  $\triangleleft$  to the fuzzy relation  $R$  and  $R^T$ , the transposed of  $R$  as mentioned in section 3.

$$T = \begin{pmatrix} 1 & 0.774 & 0.964 & 0.774 & 0.774 & 1 & 1 \\ 0.985 & 1 & 1 & 1 & 1 & 1 & 0.985 \\ 0.949 & 0.774 & 1 & 0.810 & 0.810 & 0.964 & 0.949 \\ 0.875 & 0.889 & 0.925 & 1 & 0.925 & 0.889 & 0.875 \\ 0.949 & 0.964 & 1 & 1 & 1 & 0.964 & 0.949 \\ 0.985 & 0.774 & 0.964 & 0.774 & 0.774 & 1 & 0.985 \\ 0.985 & 0.759 & 0.949 & 0.759 & 0.759 & 0.985 & 1 \end{pmatrix} \quad (12)$$

### 4.3. Selection of the optimal results\_

This example selected  $\alpha$ -cut = 0.95 and gain  $R_\alpha$  as in (13)

$$R_{\alpha} = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \quad (13)$$

The Hasse diagram in Fig.5 was drawn from crisp relation  $R_{\alpha}$  in (13). The section  $s_3$  is the top node of Hasse Diagram. Obviously,  $s_3$  contains the optimal characteristics about the safe possibility and the remoteness degree to goal. Therefore, AUV should choose this section as the next direction to move.

#### 4.4. Experimental results

Supposing that the velocity of AUV is 2 knots; sonar distance is 50m, sonar coverage is 70 degrees in horizontal, and the computational time of AUV is 10s. There are two scenarios suggested as follows. In the 2D case, AUV met four obstacles in the way to goal as shown in Fig.6. At the point A(40,0), obstacle A can be detected in section  $s_1, s_2, s_3, s_4$ . The section  $s_3$  is the selected of successive heading direction. At B (150,0), obstacles B and C can be detected in  $s_1, s_4, s_5, s_6, s_7$ , so  $s_3$  is chosen for the next turning. Similarly, at C(250,0), obstacle D is detected in  $s_2, s_3, s_4, s_5, s_6, s_7$ , and then  $s_1$  is selected.

In the 3D case, AUV could meet a very wide obstacle as illustrated in Fig.7. The vehicle is suggested to go up with a certain inclination angle. In the upper layer, at the position AUV has just reached, the algorithm is used to select the optimal turning angle to pass if some sections of sonar are detected free obstacles. If not, AUV can go to one more layer, the loop is so done until the obstacle is clearance and AUV follows the route planning for the mission.

### 5. Conclusions

In this paper, the obstacle avoidance capability for autonomous underwater vehicle has been developed and verified by the new heuristic search technique. It has been shown that by using BK-products, AUVs equipped with an obstacle avoidance sonar and the property set about the real time navigation environment determined heuristically can safely navigate in short way to the target. The primary requirement is that the sonar can be partitioned into many sections each of which represents for a successive heading candidate. Otherwise, the property set is also very important to collect as many as characteristics of AUV toward the real time environment in order to control AUVs more effectively. The more properties can be used, the more optimally and effectively AUVs can navigate. The fuzzy rule bases and membership

functions play a very important role for feasibility of this study. In the scope of this paper, in case of 3D scenario, we presented only how to apply the proposed technique but did not mention how to lead AUVs to go up and down.

### References

- [1] Antonelli, G., Chiaverini, S., Finotello, R. and Schiavon, R., "Real-time path planning and obstacle avoidance for RAIS: an autonomous underwater vehicle", Oceanic Engineering, IEEE Journal of, Volume: 26, Issue: 2, April 2001, pp. 216-227.
- [2] Anvar, A. M., "Intelligent navigation process for autonomous underwater vehicles (AUVs) using time-based fuzzy temporal reasoning", Temporal Representation and Reasoning, 2003 and Fourth International Conference on Temporal Logic, Proceedings, 10th International Symposium on, 8-10 July 2003, pp. 56-61.
- [3] Bandler, W. and Kohout, L. J., "Fuzzy Relational Products as a Tool for Analysis and Synthesis of the Behavior of Complex natural and Artificial System", in: Wang, S. K, and Chang, P. P. eds., *Fuzzy Sets: Theory and Application to Analysis and Information Systems*, Plenum Press, New York, 1980, pp. 341-367.
- [4] Bandler, W. and Kohout, L. J., "Semantics of Implication Operators and Fuzzy Relational Products", *Intl. Journal of Man-Machine Studies*, 1980.
- [5] Bandler, W. and Kohout, L. J., "Special properties, closures and interiors of crisp and fuzzy relations", *Fuzzy sets and Systems* 26(3), June 1988, pp. 317-332.
- [6] Hyeokki Kwon, Il Im and Van de Walle, B., "Are you thinking what I am thinking? A comparison of decision makers' cognitive maps by means of a new similarity measure", *System Sciences*, 2002, HICSS, Proceedings of the 35th Annual Hawaii International Conference on, 7-10 Jan. 2002
- [7] Hyland, J. C. and Taylor, F. J., "Mine avoidance techniques for underwater vehicles", Oceanic Engineering, IEEE Journal of, Volume: 18, Issue: 3, July 1993, pp. 340-350.
- [8] Kohout L. J and Kim E., "Semiotic descriptors in fuzzy relational computations", In: Albus JH, Meystel A (eds) *Proc IEEE Int Symp Intelligent Control, IEEE Int Symp Computational Intelligence in Robotics and Autonomous and Intelligent Systems and Semiotic (A Joint Conf Science and Technology of Intelligent Systems)*, Piscataway, 1998, pp. 828-833.
- [9] Kohout L. J and Kim E., "The role of BK-products of Relations in Soft Computing", *Soft Computing* 6, Springer-Verlas, 2002, pp.92-115.
- [10] Kohout L. J., Keravnou, E., and Bandler, W., "Automatic

*Documentary Information Retrieval by Means of Fuzzy Relational Products*”, In Gaines, B. R., Zadeh, L. A. and Zimmermann, H. J., editors, *Fuzzy Sets in Decision Analysis*, North-Holland, Amsterdam, 1984, pp. 308-404.

[11] Kohout, L. J. and Bandler, W., “*Fuzzy relational products in knowledge engineering*”, In V. Novak et al., editor, *Fuzzy Approach to Reasoning and Decision Making*, pp. 51-66, Academia and Kluwer, Prague and Dordrecht, Zchec 1992.

[12] Kohout, L. J. and Harris, M., “*Computer Representation of Fuzzy and crisp relations by means of threaded trees using foresets and aftersets*”, *Journal of fuzzy logic and intelligent System*, Vol.3, no. 1, 1993.

[13] Kohout, L. J., Granville, B., and Kim, E., “*Granular Relational Computing with Semiotic Descriptors Using BK-Products of Fuzzy Relations*”, *Computing with Words*. John Wiley & Sons, Inc., New York, NY, USA 2001.

[14] Lee, Young-Il and Kim, Yong-Gi, “*An intelligent navigation system for AUVs using fuzzy relational products*”, *IFSA World Congress and 20th NAFIPS International Conference, 2001, Joint 9th Volume: 2*, 25-28 July 2001, pp. 709--714 vol.2

[15] Lee, Young-Il, Kim, Yong-Gi and Kohout, L. J, “*An Intelligent Collision Avoidance System for AUVs using Fuzzy Relational Products*”, *Information Sciences*, Elsevier, Vol. 158 (2004) pp. 209-232

[16] Lee, Young-il, Noe, Chan-Sook and Kim, Yong-Gi, “*Implication operators in fuzzy relational products for a local path-planning of AUVs*”, *Fuzzy Information Processing Society, 2002, Proceedings, NAFIPS, 2002 Annual Meeting of the North American, 27-29 June 2002*, pp. 221-226.

[17] Liu, Xuemin, Pengm, Liang, Li, Jiawei, and Xu, Yuru, “*Obstacle Avoidance using fuzzy neural networks*”, *Underwater Technology, 1998. Proceedings of the 1998 International Symposium on, 15-17 April 1998*, pp. 282-286.

[18] Ong, S. M., *A Mission Planning knowledge-based system with Three-Dimensional Path Optimization for the NPS Model 2 Autonomous Underwater Vehicle*, Master’s Thesis, Naval Postgraduate School, 1990.

[19] Petillot, Y., Tena Ruiz, I, and Lane, D. M, “*Underwater Vehicle Obstacle Avoidance and Path Planning Using a Multi-Beam Forward Looking Sonar*”, *IEEE journal of, Volume: 26, No: 2, April 2001*, pp.240-251.

[20] Saffiotti, A. “*The Uses of Fuzzy Logic in Autonomous Robot Navigation*”. *Soft Computing 1*, Springer-Verlas, 1997, pp.180-197.

[21] Sayyaadi, H., Ura, T. and Fujii, T., “*Collision avoidance controller for AUV systems using stochastic real value reinforcement learning method*”, *SICE 2000, Proceedings*

of the 39th SICE Annual Conference, International Session Papers, 26-28 July 2000, pp. 165-170

[22] Zadeh, Lotfi: “*Fuzzy Sets*” *Information and Control*. 8(3), pp.338-353, 1965 (Cited by [Klir 1995, 1997], [Bonissone 1997])

Table 1. The rule base for Safety Property

Degree	Situation	Value of x
Very Safe	IF: $s_i$ is not adjacent to any obstacle	THEN: $x=1$
Safe	IF: $s_i$ is adjacent to obstacle	THEN: $x=2$
Risky	IF: $s_i$ is between the obstacle	THEN: $x=3$
Very Risky	IF: $s_i$ contains obstacle	THEN: $x=5$

Table 2. The rule base for Remoteness Property

Degree	Situation	Value of y
Very Far	IF: the distance to goal by $s_i$ 's direction is very far	THEN: $y=1$
Far	IF: the distance to goal in $s_i$ 's direction is far	THEN: $y=2$
Near	IF: the distance to goal by $s_i$ 's direction is near	THEN: $y=3$
Very Near	IF: the distance to goal by $s_i$ 's direction is very near	THEN: $y=4$

Table 3. The membership value for Safety Property

Candidate	Fuzzy Degree	Fuzzy Value (x)	Membership Degree
$s_1$	Safe	2	0.9526
$s_2$	Very Risky	5	0.5000
$s_3$	Risky	3	0.8808
$s_4$	Very Risky	5	0.5000
$s_5$	Very Risky	5	0.5000
$s_6$	Safe	2	0.9526
$s_7$	Very Safe	1	0.9820

Table 4. The membership value for Remoteness Property

Candidate	Fuzzy Degree	Fuzzy Value (y)	Membership Degree
$s_1$	Very Far	1	0.0180
$s_2$	Far	2	0.0474
$s_3$	Near	3	0.1192
$s_4$	Very Near	4	0.2689
$s_5$	Near	3	0.1192
$s_6$	Far	2	0.0474
$s_7$	Very Far	1	0.0180

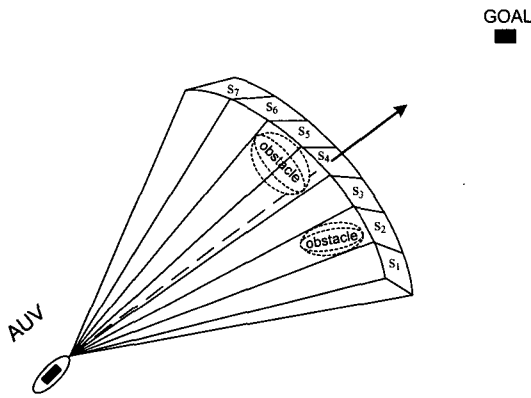


Fig. 1. The model of sonar

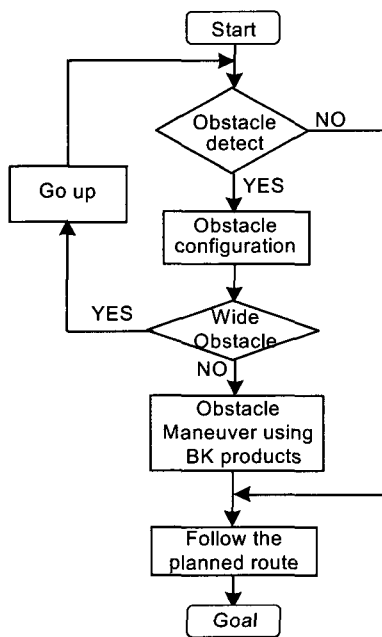


Fig. 2. Control flow of AUVs obstacle avoidance

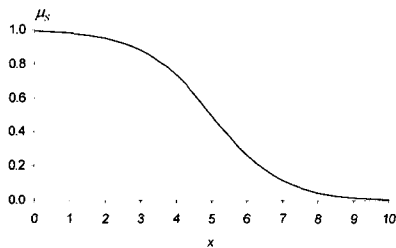


Fig. 3. The membership function of the safety property

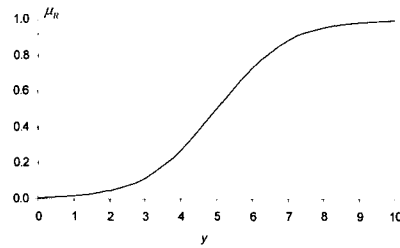


Fig. 4. The membership function of the remoteness property

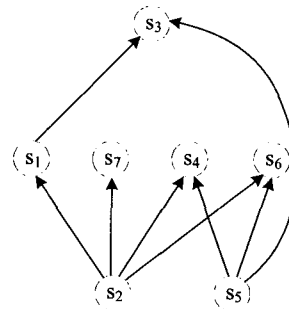


Fig. 5. Hasse diagram

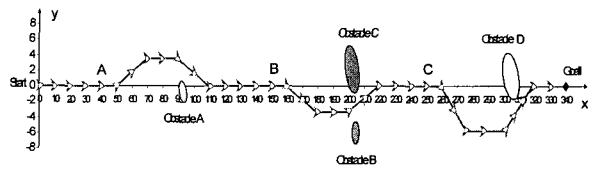


Fig. 6. Scenario in 2-Dimensions

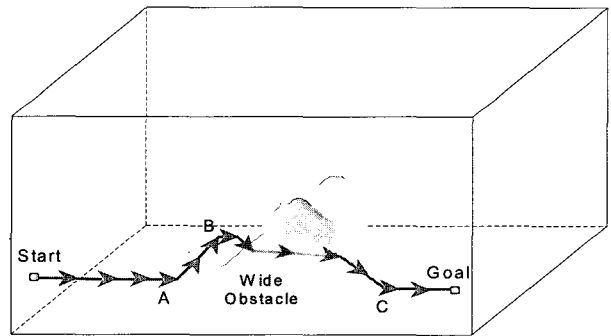


Fig. 7. Scenario in 3-Dimension

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