[논 문] - 비파괴검사학회지
Journal of the Korean Society for Nondestructive Testing
Vol. 24, No. 2 (2004. 4)

Guided Wave Calculation and Its Applications to NDE

Takahiro Hayashi

Abstract This paper describes the calculation technique for guided wave propagation with a semi-analytical finite element method (SAFEM) and shows some results of numerical calculation and guided wave simulation for plates, pipes and railway rails. The SAFEM calculation gives dispersion curves and wave structures for bar-like structures. Dispersion curve software for a pipe is introduced, and also dispersion curves for a rail are given and experimentally verified. The mode conversions in a plate with a defect and in a pipe with an elbow or a defect are shown as examples of our guided wave simulations.

Keywords: guided wave, semi-analytical finite element method, dispersion curve, simulation, animation

Introduction

When applying vibration on the surface of plates, pipes and elongated structures as railway rails, acoustic modes generate and propagate over distances of the order of 100 times or 1000 times wavelength in the longitudinal direction. This acoustic wave, called guided wave, has been recently attracted as a means for rapid nondestructive evaluation of large structures. Especially, guided wave application to pipe inspection is highly expected as a rapid NDE for a buried and/or coated pipe since guided waves are experimentally verified to travel up to about one hundred meters (Alleyne, 1997, Cawley, 2003, Wilcox, 2003, Rose, 1999, Rose, 2002, Kwun, 2003). Guided waves, however, are a kind of resonant modes with very complex characters. Many modes can be excited with different wave velocities and dispersion in the same frequency range. As a result, analyzing received signals requires profound theoretical insight, and so even researchers on guided wave often have different opinion on the received signals. In such cases, numerical analysis and simulation on guided wave propagation play a very important role.

The calculation techniques widely used in such dynamic elastic problems as ultrasonic wave propagation are finite difference method (FDM), finite element method (FEM) and boundary element method (BEM) (See Brebbia and Walker, 1980, Kobayashi, 2000, Zienkiewicz, 2000). The problems of calculation time and computational memory, however, arise in guided wave calculation using the conventional techniques. In guided wave inspection, guided waves are usually used with propagating over distances of 100 or 1000 times of wavelength. Any conventional techniques as FDM, FEM and BEM generally need at least four nodes for one wavelength to express waveform accurately. Thus, large number of nodes in the propagation direction is necessary to express guided waves propagating in large structures.

Recently, the conventional FEM and general-purpose FEM software have given calculation results with sufficient accuracy

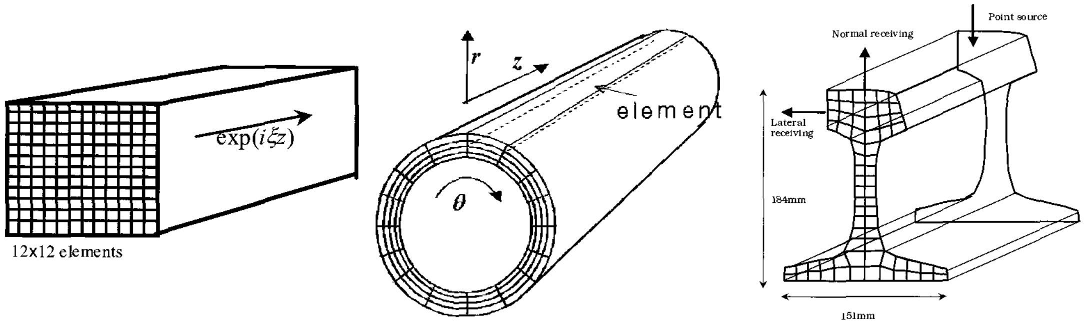
(Received: February 2, 2004, Accepted: March 17, 2004) Faculty of Engineering, Nagoya Institute of Technology, Gokiso Showa Nagoya, 466-8555, Japan (E-mail: hayashi@nitech.ac.jp)

(Demma, 2001, Sanderson, 2002). On the other hand, special techniques for guided wave propagation such as a hybrid technique and a semi-analytical FEM (SAFEM) have been developed to avoid large calculation time and memory. The hybrid technique, the combination of theoretical solutions and the FEM or BEM, was developed and applied to NDE by Koshiba et.al. (Koshiba et.al., 1984), Al-Nassar et.al. (Al-Nassar et.al, 1991) and Cho and Rose (Cho and Rose, 1996). Since normal expansion theory is adopted for intact regions with no irregularities in the longitudinal direction and FEM or BEM is used only for irregular regions, calculation costs have been largely reduced. The SAFEM was developed as a special solution of FEM by Cheung (Cheung, 1968, 1976). Expressing wave distribution in the longitudinal direction with orthogonal functions enables to reduce one dimension for any kinds of bar-like structures including plates and pipes with constant material properties and geometrical structures in the longitudinal direction. The SAFEM was firstly applied to NDE for obtaining dispersion curves for layered plates by Dong and Huang (Dong and Huang, 1985), Kausel (Kausel, 1986) and Datta et.al. (Datta et.al., 1988), and then for transient solutions by Liu et.al (Liu and Achenbach, 1995, Liu and Xi, 2001) and for guided waves in pipes by Rattanawangcharoen et.al. and Zhuang et.al (Rattanawangcharoen et.al., 1994, Zhuang, 1999).

The author has studied on the hybrid technique (Hayashi, 1999) and the SAFEM for guided wave calculation (Hayashi, 2002-2004). Especially, the SAFEM is applicable to wide range from a layered plate to a bar with an arbitrary cross-section with a crack or a curved region, and from numerical analysis to simulation of guided wave propagation. This paper briefly describes calculation technique by the SAFEM and shows calculation results on Lamb wave in a plate, guided wave in a pipe and dispersion curves for a railway rail.

Calculation Technique by a Semi- analytical Finite Element Method

In the SAFEM, the cross-sections of plates, pipes or bars are discretized into small sections as shown in Fig.1. Instead of dividing the region in the longitudinal direction as in ordinary FEM, the orthogonal function $\exp(i\xi z)$ is used for expressing the distribution of the displacement field in the longitudinal direction. Similarly to ordinary FEM, the virtual work principle or minimization of potential energy in the entire volume of an object rewrites the governing equations into an integration form. Discretizing the integration form then gives an eigensystem with respect to the wave number ξ for a certain frequency. Eigenvalues and eigenvectors obtained from the eigensystem correspond to wave numbers and wave structures for resonance



(a) Square rod (b) Straight pipe (c) Rail

Fig. 1 Sub-divisions in the semi-analytical finite element method guided wave modes in the bar-like object, where wave structure means displacement distribution on the cross-section. Phase velocity dispersion curves can be drawn by obtaining wave number ξ for frequency steps. And then for given boundary conditions, displacement fields can be obtained as a summation of guided wave modes. Wave propagation can be simulated by collecting these displacement data for all frequency steps in the frequency bandwidth. Since the SAFEM can handle only with a bar-like structure with constant material properties in the longitudinal direction, the combination technique of some SAFEM regions and the hybrid technique with the ordinary FEM and BEM are needed to calculate the guided wave propagation in a bar-like object with irregularities such as cracks and curved regions.

Table 1 shows the application range of guided wave calculations by the SAFEM. Calculation technique is more sophisticated and difficult in the direction from the left bottom of the table to the right top. Guided wave simulation has become technically feasible for any kinds of bar-like structures with an arbitrary cross-section with or without cracks and curved regions. And we are now solving problems one by one following the practical demands.

Calculation Results

Dispersion Curves

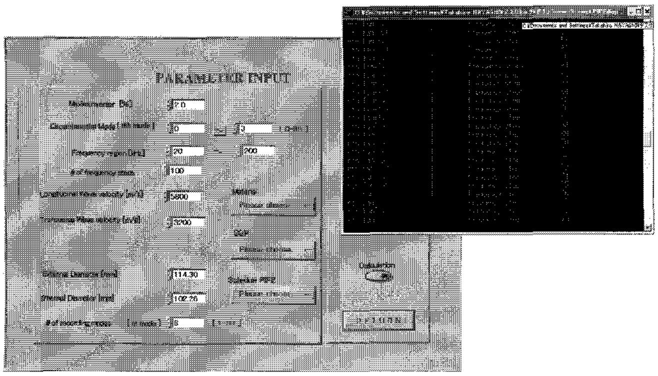
Dispersion curves, representing frequency dependence of guided wave velocities, are indispensable for such guided wave NDE. Dispersion curves present fundamental information on guided waves such as wavelength and dispersion as well as phase and group velocities at a certain frequency. Such fundamental information plays an important role in determining incident and receiving angles of angle beam transducers and spacing of comb type transducers such as EMATs and PVDF films and in estimating the traveling time of each mode. Therefore, dispersion curves are

Table 1 Feasible application by the semi-analytical finite element method
Higher technique is needed in the direction from the left bottom to the right top.

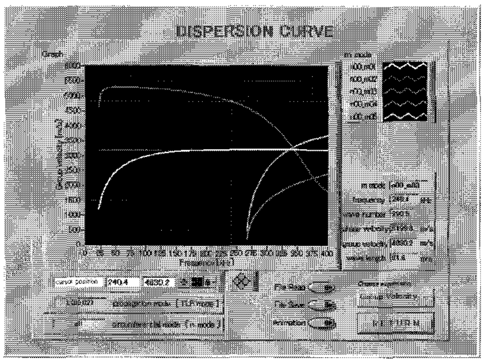
	Plate	Pipe	Bar with an arbitrary cross-section
Hybrid technique	Plate with arbitrary shape cracks (Koshiba et.al., 1984) (Al-Nassar et.al., 1991) (Cho and Rose, 1996) (Hayashi et.al., 1999)	Pipe with arbitrary shape cracks (Hayashi et.al., 2004b) Curved pipe (Hayashi et.al.2003e)	Wave propagation in any types of bar or plate like structures with or without defects
Simulation of guided wave propagation	Plate with non-volume cracks and delaminations (Liu and Achenbach, 1995) (Liu and Xi, 2001) (Hayashi, 2002) Simulation of Lamb wave propagation in a plate (Hayashi, 1999) (Hayashi, 2002)	Pipe with a non-volume crack (Hayashi et.al. 2004a) Straight pipe (Hayashi. e.al. 2003a,c,d)	Guided wave simulation in a bar with an arbitrary cross section
Dispersion curve	Dispersion curves for a plate	Dispersion curves for a pipe	Dispersion curves for a bar with an arbitrary cross section (RAIL, Square rod) (Sanderson, 2002) (Hayashi et.al. 2003f)

theoretically obtained initially followed by actual guided wave measurements. Dispersion curves have been analytically obtained for a homogeneous plate, but not for a layered plate. Since the SAFEM has layered elements, it can calculate dispersion curves for a layered plate and a surface coated plate by changing material properties in layered elements. And also the SAFEM gives wave structures, displacement distribution in the thickness direction, for a certain mode at a certain frequency.

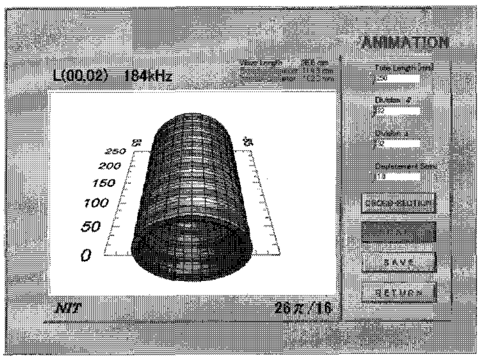
Similarly to Lamb wave dispersion curves, dispersion curves for a pipe can be calculated by dividing the cross-section of a pipe into cylindrical elements and describing the distribution in the circumferential direction θ as a summation of orthogonal functions $\exp(in\theta)$. Since it is not easy to derive analytical solutions of dispersion curves for a pipe, we have developed software that calculates and shows dispersion curves. This software also shows wave structure for all modes at a certain frequency, which is given by the SAFEM calculation at the same time as a wave number. Fig.2 shows the input and output window of the dispersion curve software "PIPE DISPERSION". Calculation is carried out after inputting such parameters as frequency range, the number of frequency steps, pipe dimensions, material properties of a pipe and circumferential mode parameter n to be calculated as shown in Fig.2 (a). Then, dispersion curves are displayed as shown in Fig.2 (b). When a guided wave mode and frequency are chosen by the cursor on the dispersion curves Fig.2 (b), data set of the displacement distribution in the thickness and circumferential



(a) Parameter setting page and calculation window



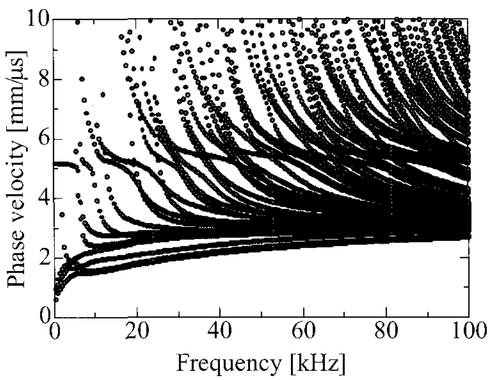
(b) Representation of dispersion curves



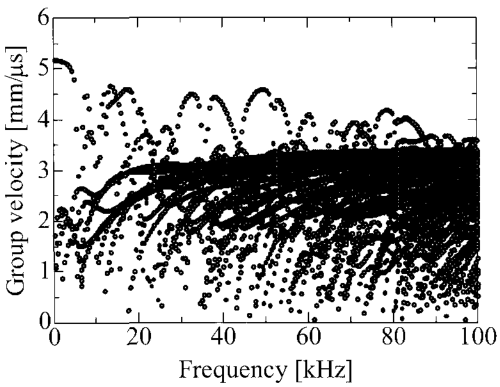
(c) Animation showing a wave structure at a given point in the dispersion curve (b)

Fig. 2 Software for dispersion curves and wave structures of a pipe directions stored with the dispersion curve data are recalled and displayed as Fig.2 (c). The number of cylindrical elements strongly affects to the accuracy of the calculation, but this software has a function for determining the optimum number of elements for easy use and accurate calculation.

Theoretical dispersion curves for a bar with an arbitrary cross-section cannot be derived. For such bar-like structures, the calculation by the SAFEM enables us to obtain the dispersion curves without any difficult mathematical treatment. The cross-section of the bar is descretized into two dimensional small sections as shown in Fig.1 (c). Fig.3 shows phase velocity and group velocity dispersion curves for a rail as Fig.1 (c). As there are too many dots in Fig.3 showing all possible resonant modes in the rail, it is difficult to estimate the physical meaning of the curves. The dispersion curves show a lot of unnecessary modes that are not included in received signals due to experimental conditions like transducer position and frequency range. For example, dispersion curves for shear horizontal plate wave are generally not shown in Lamb wave dispersion curves. Though both SH plate wave and Lamb wave are resonant wave modes in a plate, different transducers are used for excitation and reception due to their different vibration direction. Therefore, SH plate wave is not detected in the inspection by Lamb waves and the resonant modes of SH plate waves are omitted in Lamb wave dispersion curves. Similarly, the dispersion curves for the rail can be simplified by reducing unnecessary modes. Fig.4 shows the dispersion curves for the rail plotted in gray scale depending on the intensity of the displacement under a boundary condition determined by an experimental set-up. In the case of Fig.4, it is one example when the excitation force is applied on the upper surface of the railhead and the signals are detected on the upper surface of the railhead. Dominant modes under this transducer set-up are highlightened. The dispersion curves obtained by the SAFEM calculation were compared with experimental ones to verify the validity of the calculation results. Experimental dispersion curves are given by collecting many signals at many points and applying two-dimensional FFT to the signals. Fig.5 shows the experimental dispersion curves when the excitation contact transducer of 50kHz center frequency is on the upper surface of the railhead and the receiving air-coupled transducers are over the railhead. The calculation results shown in Fig.4 (a) agree well with the experimental dispersion curves of Fig.5. For example, two distinct curves in the range from 40 to 60kHz and from 4mm/μs to 6mm/μs can be seen in the calculation result.

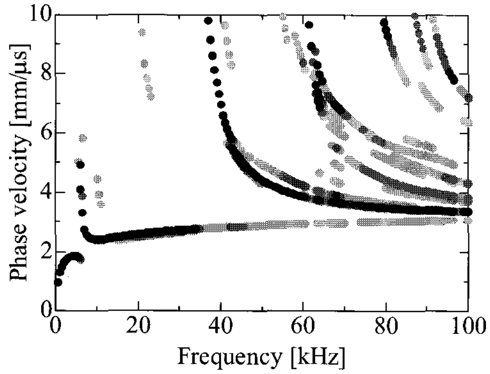


(a) Phase velocity

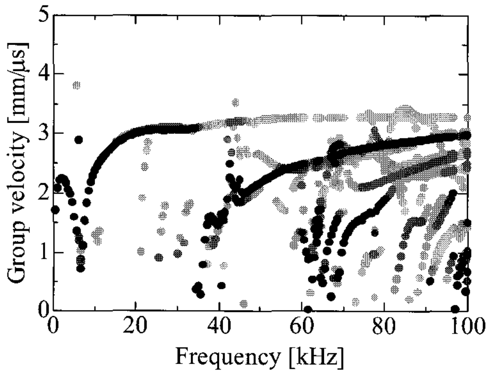


(b)Group velocity

Fig. 3 Dispersion curves for a rail obtained by the SAFEM calculation
All possible modes are shown.



(a) Phase velocity



(b) Group velocity

Fig. 4 Dispersion curves for a rail by the SAFEM calculation. Dominant modes are highlightened.

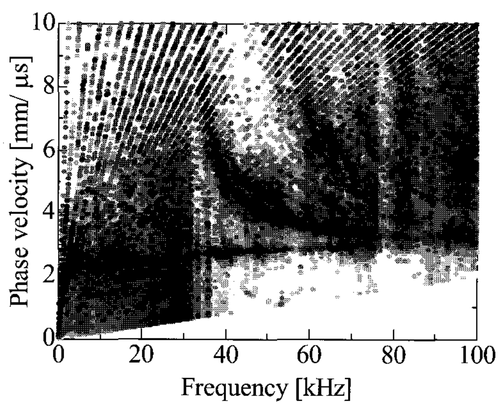
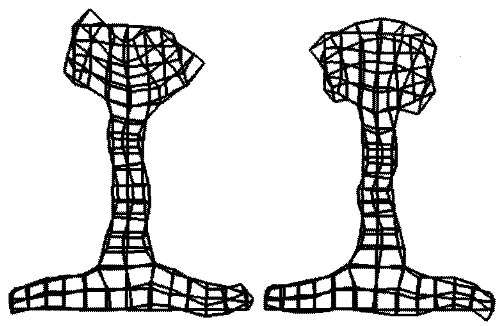


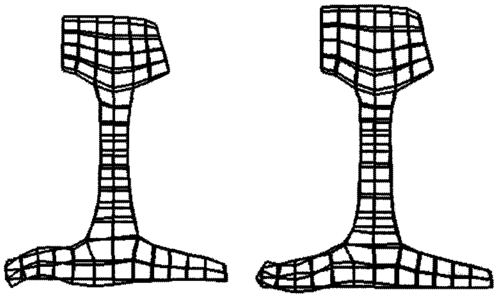
Fig. 5 Experimental dispersion curves for a rail

The SAFEM calculation also gives wave structures of these characteristic modes. Fig.6 shows wave structures of these two modes at 50 kHz. (a) and (b) show the wave structures at a certain time and after half period T/2. Both original grid lines and grid lines after vibration are shown in one figure to see wave motion easily. From these wave structures, we can estimate what kind of defects strongly affects the modes. In these cases, since vibration energy distributes all parts of the cross-section, these two modes can be used to detect defects in all regions. But vibration directions are different; (a) is dominated by flexural vibration, but (b) is dominated by dilatational and compressional vibration in the vertical direction. Therefore, these two modes are expected to be sensitive to different type of defects.



t=0 t=T/2

(a) phase velocity = 4.4 mm/μs at 50 kHz



t=0 t=T/2

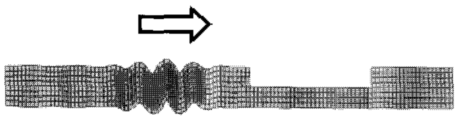
(b) phase velocity = 4.9 mm/μs at 50 kHz

Fig. 6 Wave structures for two characteristic modes shown in Fig. 4(a) and Fig. 5. Both original grid lines and grid lines after vibration are shown in one figure.

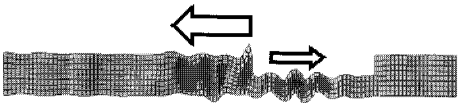
Simulation of guided wave propagation

Dispersion curves and wave structures are fundamental information for guided wave measurements, but simulation of guided wave propagation is necessary to know the reflection and transmission characteristics at defects and mode conversions at curved regions. Fig.7 is the calculation results when an A0 mode of Lamb wave is excited in a plate with a rectangular notch. An incident A0 mode reflects back from both front and rear edges of the notch. When wavelength is as long as the width of the notch, two wave packets cannot be seen distinctly in the received signals, but resonance by two walls can be seen in frequency spectrum of the received signals. The multiple reflections from the two edges are widely confirmed in notch type defects experimentally, and it becomes a kind of guideline to obtain defect length in the longitudinal direction.

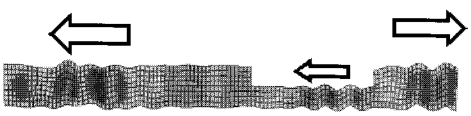
Next, simulation results for guided wave propagation in a pipe are shown. Guided wave NDE for a pipe generally uses axisymmetric L(0,2) and T(0,1) modes that are dominated by in-plain displacements. The L(0,2) mode has wide frequency range with small dispersion, and the T(0,1) is a perfectly non-dispersive mode with constant velocity for all frequency. And since both of them emit little energy to the outside of the pipe due to the in-plane vibration nature, they show very good characteristics for long range NDE. Nevertheless, if we use an axisymmetric mode as input signals for a small defect with the reduction rate of 5% to the cross-section of the pipe that is to be detected in practical testing, we can estimate that very small reflection echo can be seen about 5% of incident energy. Therefore, guided wave focusing technique has been developed to improve the signal to noise ratio of reflected waves from such a small defect. Many excitation transducers



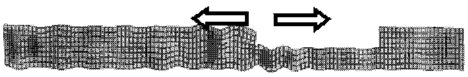
(a) t = 140 μs



(b) t = 160 μs

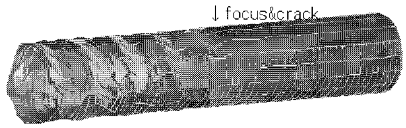


(c) t = 180 μs

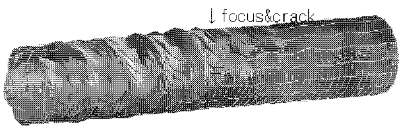


(d) t = 200 μs

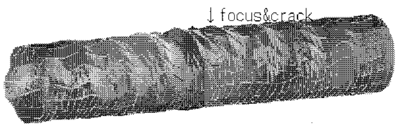
Fig. 7 Lamb wave propagation around a notch in a plate



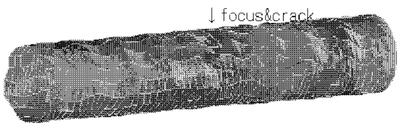
(a)



(b)



(c)



(d)

Fig. 8 Torsional mode focusing and reflection from a defect. Focusing point is on the crack. Color indicates displacement in the circumferential direction.

located in the circumferential direction emit the different waveforms to focus at a desired point like phased array system. Fig.8 shows torsional mode focusing and reflection from a notch at the top center of a pipe. Eight transducers lining up in the circumferential direction with the same spacing excite shear horizontal vibration with different time delays and amplitudes to focus at the desired point. Fig.8 (b) shows good focusing at the top center of a pipe. And (c) shows large reflection from a notch at the focusing point. Since focusing points can be changed over an entire surface of a pipe, defect location and size are estimated by changing focusing points and measuring reflected echoes.

Fig.9 shows mode conversions at an elbow of a pipe for axisymmetric mode incidence. The axisymmetric mode retains the waveform until the elbow (Fig.9(a)), but is largely distorted at the elbow. After large energy is seen at the extrados side of the elbow, the subsequent waveforms become small compared to incident wave, which prevent from detecting defects beyond an elbow.

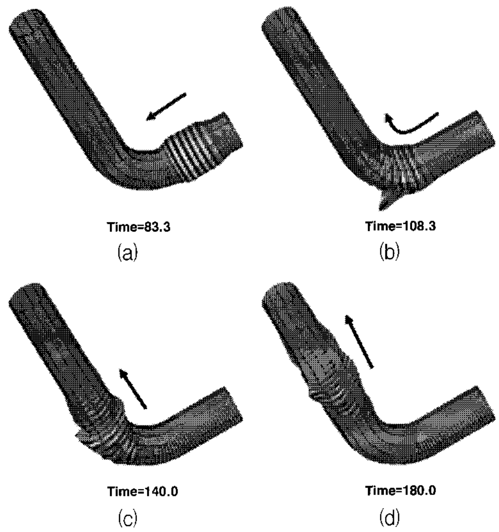


Fig. 9 Guided wave propagation in a pipe with an elbow

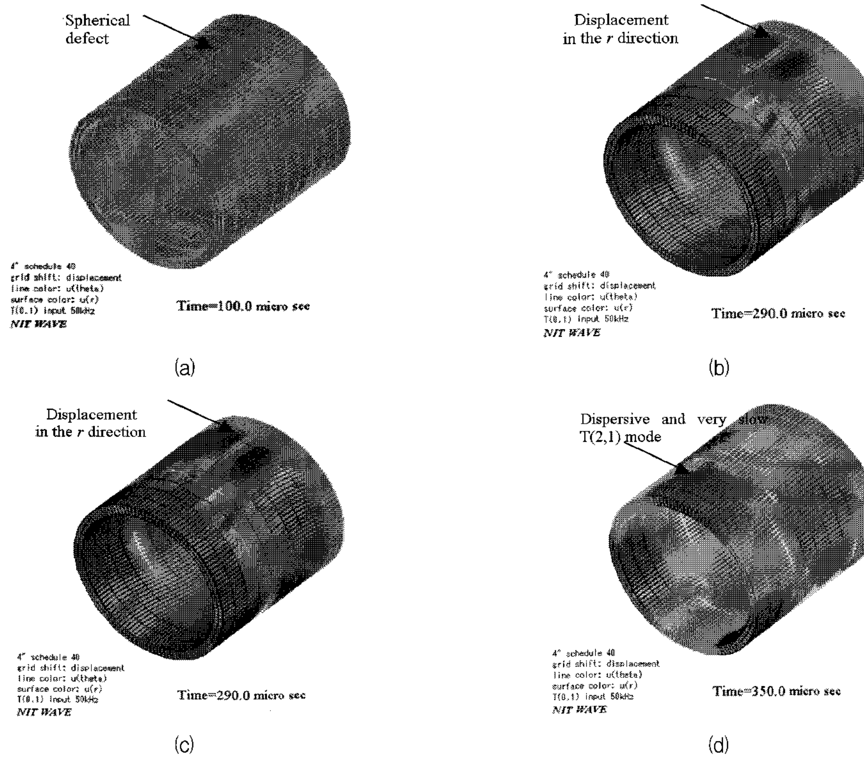


Fig. 10 Scattering wave from a spherical defect on a pipe. Color of grid lines and surfaces show displacement in the circumferential and thickness directions, respectively.

Next, guided wave scattering from an arbitrary shape defect is calculated by the hybrid technique of the SAFEM with the FEM. Fig.10 shows scattering from a spherical defect on a pipe when an axisymmetric torsional mode T(0,1) of 50 kHz center frequency as input. Fig.11 shows the dimensions of the pipe used in this calculation. The color on the surfaces and the grid lines mean displacements in the r and θ directions, respectively. The spherical defect is located at the top of the pipe in Fig.10 (a). The input T(0,1) mode propagates into the crack region as shown in Fig.10(b), and after the displacement in the r direction is excited in Fig.10(b), three dark regions are seen in the circumferential direction, showing the T(2,1) mode that has two nodes in the circumferential direction. Since the T(2,1) mode is very dispersive and slow at the frequency region used here, the T(2,1) mode can separate from the other modes as T(0,1) and T(1,1).

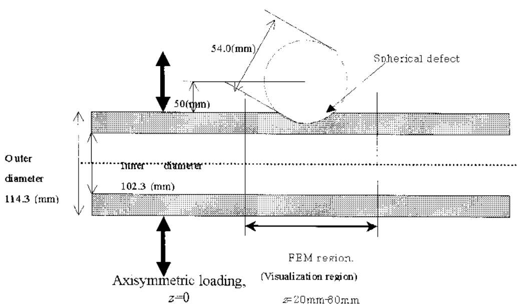


Fig. 11 Dimension of a pipe with a defect

Conclusions

Guided wave calculation technique by a semi-analytical finite element method (SAFEM) was briefly described, and the results of guided wave calculation and simulation done by the authors were introduced in this paper. The guided wave calculation technique by the SAFEM is now being developed from practically higher demands such as guided wave inspections for plates and pipes. Since the SAFEM calculation is technically feasible to any kinds of bar-like structures to be inspected with the guided waves, we expect that the guided wave calculation enlarges the application range of guided wave inspection with more experimental studies.

Acknowledgements

The author wishes to thank Professor Dr. Koichiro Kawashima, Nagoya Institute of Technology, and Professor Dr. Joseph L. Rose, Penn State University, for their encouragements and helpful comments for this work. This work was partly supported by Japan Institute of Construction Engineering.

References

Alleyne D. N. and Cawley P. (1997) Long range propagation of Lamb waves in chemical plant pipework, Materials Evaluation, Vol. 55, pp. 504-508

Al-Nassar Y. N., Datta S. K. and Shah A. H. (1991) Scattering of lamb waves by a normal rectangular strip weldment, Ultrasoncs, Vol. 29, No. 2 pp. 125-132

Brebbia C. A. and Walker S. (1980) Boundary element techniques in engineering, Butterworth & Co.

Cawley P., Lowe M. J. S., Alleyne D. N., Pavlakovic B. and Wilcox P. (2003) Practical long range guided wave testing: Application to pipes and rails, Materials Evaluation, Vol. 61, No. 1, pp. 66-74

Cho Y. and Rose J. L. (1996) A boundary element solution for a mode conversion study on the edge reflection of Lamb waves, The Journal of the Acoustical Society of America, Vol. 99, No. 4, pp. 2097-2109

Cheung Y. K. (1968) The finite strip method in the analysis of elastic plates with two opposite simply supported ends, Proceedings of American Society of Civil Engineering, Vol. 94, EM6, pp. 1365-1378

Cheung Y. K.(1976) Finite strip element method in structural analysis, Pergamon Press

Datta S. K., Shah A. H., Bratton R. L. and Chakraborty T. (1988) Wave propagation in laminated composite plates. The Journal of the Acoustical Society of America. Vol. 83, No. 6, pp. 2020-2026

Demma A, Cawley P. and Lowe M. J. S. (2001) Mode conversion of longitudinal and torsional guided modes due to pipe bends, in: D.O. Thompson and D.E. Chimenti (Eds.), Review of Progress in Quantitative Nondestructive Evaluation, Vol. 20A, American Institute of Physics, Melville, New York, pp. 172-179

Dong S. B. and Huang K. H. (1985) Edge vibration in laminated composite plates. Journal of Applied Mechanics, Vol. 52, pp. 433-438

Hayashi T., Biwa S. Choi Joon-Chul and Endoh S. (1999) Large-scale simulation of Lamb wave propagation by hybrid boundary element method, Transactions of the Japan Society of Mechanical Engineers, Series A, Vol. 65, No. 630 pp. 210-217, in Japanese

Hayashi T. and Kawashima K. (2003) Single mode extraction from multiple modes of Lamb wave and its application to defect detection, JSME international journal, Vol. 46, No. 4, Series A, pp. 620-626

Hayashi T. and Kawashima K.(2002) Multiple reflections of Lamb waves at a delamination, Ultrasonics, Vol. 40, pp. 193-197

Hayashi T., Kawashima K., Sun Z. and Rose J. L. (2003) Analysis of flexural mode focusing by a Semi-analytical finite element method, The Journal of the Acoustical Society of America., Vol. 113, No. 3, pp. 1241-1248

Hayashi T. and Rose J. L. (2003): Guided wave simulation and visualization by a Semi-Analytical Finite Element Method, Materials Evaluation, Vol. 61, No. 1, pp. 75-79

Hayashi T., Kawashima K., Sun Z. and Rose J. L.(2003) Semi analytical finite element analysis for ultrasonic focusing in a pipe, in: D.O. Thompson and D.E. Chimenti (Eds.), Review of Progress in Quantitative Nondestructive Evaluation, Vol. 22A, American Institute of Physics, Melville, New York, pp. 250-255

Hayashi T., Kawashima K., Sun Z. and Rose J. L. (2003) Guided wave focusing mechanics in pipe, Proceedings of ASME Pressure Vessels and Piping Division Conference 2003, pp. 35-42

Hayashi T., Kawashima K., Sun Z. and Rose J. L. (2003) Guided wave propagation mechanics across a pipe elbow, Proceedings of ASME Pressure Vessels and Piping Division Conference 2003, pp. 43-47

Hayashi T., Song W-J and Rose J. L. (2003) Guided wave dispersion curves for a bar with an arbitrary cross-section, a rod and rail example, Ultrasonics Vol. 41 pp. 175-183

Hayashi T., Kawashima K., Sun Z. and Rose J. L. (2004) Calculation of guided wave scattering at a defect in a pipe, to be published in: D.O. Thompson and D.E. Chimenti (Eds.), Review of Progress in Quantitative Nondestructive Evaluation, Vol. 23, American Institute of Physics, Melville, New York

Hayashi T., Kawashima K and Rose J. L. (2004) Calculation for guided waves in pipes and rails, to be published in proceedings of 11th Asian Pacific Conference of Nondestructive Testing

Kausel E.(1986) Wave propagation in anisotropic layered media. International Journal for Numerical Methods in Engineering., Vol. 23, pp. 1567-1578

Kobayashi S. (ed.) (2000) Hadoukaiseki to Kyokaiyousohou (Analysis of wave propagation and boundary element method), Kyoto university press, in Japanese

Koshiba M, Karakida S. and Suzuki M. (1984) Finite-element analysis of Lamb wave scattering in an elastic plate waveguides, IEEE Transactions on Ultrasonics ,Ferroelectrics and Frequency control, Vol. 31, No. 3, pp. 18-25

Kwun H., Kim S. Y. and Light G. M., (2003) The magnetostrictive sensor technology for long range guided wave testing and monitoring of structures, Materials Evaluation Vol. 61, No. 1, pp. 80-84

Liu G. R., Achenbach J. D. (1995) Strip element method to analyze wave scattering by cracks in anisotropic laminated plates, Journal of Applied Mechanics, Vol. 62, pp. 607-613

Liu G.R. and Xi Z. C. (2001) Elastic Waves in Anisotropic Laminates, CRC Press LLC

Rattanawangcharoen N., Shah A. H. and Datta S. K. (1994) Reflection of waves at the free edge of a laminated circular cylinder, Journal of Applied Mechanics, Vol. 61, pp. 323-329

Rose J. L. (2002) Guided wave ultrasonic pipe inspection-The next generation, Proceeding of the 8th European Conference for Nondestructive Testing, Barcelona, Spain

Rose J. L. (1999) Ultrasonic waves in solid media, Cambridge University Press

Sanderson R. and Smith S. (2002): The application of finite element modelling to guided ultrasonic waves in rails, Insight, Vol. 44 No. 6 pp. 359-363

Wilcox P., Evans M., Pavlakovic B., Alleyne D., Vine K., Cawley P. and Lowe M. (2003), Guided wave testing of rail, Insight, Vol. 45, pp. 413-420

Zhuang W., Shah A. H. and Dong S. B. (1999) Elastodynamic Greens function for laminated anisotropic circular cylinders, Journal of Applied Mechanics, Vol. 66, pp. 665-674

Zienkiewicz O. C. and Taylor R. L. (2000) The finite element method solid mechanics, Butterworth-Heinemann, 5th edition