On the Selection of Demand Used in Planning for the Distribution Networks

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Abstract

This paper first addresses a distribution planning method on centrally controlled supply chain. The distribution channels are assumed to be network of arborescence form. For such distribution networks, this study proposes a distribution planning scheme when the demands for retail sites are provided for a given planning horizon. As the planning horizon rolls forward, for a new horizon, forecasted demand distributions of periods in the horizon are updated. An idea of controlling customer service level by the selection of demand to be used in the planning (Demand Used in Planning, DUP) from the forecasted values is also discussed.
Keyword : distribution networks, distribution planning scheme, service level

1. Introduction

A typical distribution network of manufacturing firms has the form of arborescence, and it is particularly true when the network is spread over on a large area. Starting from the root node, which is usually the production site, the tree shaped network branches toward leaf nodes at the end of branches. The leaf nodes are demand site where the delivering of goods to the customers takes place. The nodes in the middle stream are transshipment depots, and any of such depots can act as demand site as well. The process of delivering produced goods through the distribution network never stops as long as a manufacturing firm is in operation, and what dictates the flows through the network is the actual or forecasted customer demands arising on the demand sites. What a manufacturing firm does in the planning of production and distribution is to find a plan that best supplies commodities to the customers, and it does the planning in the rolling horizon basis. In most of such planning practices, the estimated demands, which are forecasted for each individual period in a forecasting window, are regarded as

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deterministic or assumed to follow a certain distribution. However, in reality, the forecasted demands for the periods of a given horizon are neither deterministic nor distributed identically, when they are stochastic. The forecasted demand values for a given period, in general, is distributed over a certain range, and the distribution itself is subjected to update as the time advances. Safety stock is a choice to be used to handle the variability in customer demands when the demand distributions of upcoming periods are identical. However, in the case presented in this study, the behavior of forecasted demand distributions does not allow one to devise a scheme that obtains reasonable safety stock levels. Instead of using safety stock, this study considers a direct control of service level by choosing the demand used in planning (DUP) from the distributed forecasted demand values for the corresponding period.

This study regard a forecasted demand actually used in a distribution planning as the value best selected by the decision maker from the range of the forecasts being distributed over. Obviously the way of selecting DUP for periods in planning horizon influences the service level of the distribution network as well as the cost incurred. The realization of a demand usually does not coincide with the selected one, and therefore, the discrepancy between each DUP and realization affects inventory level of depots in the network. The issue of demand value selection is discussed in a later part of this study.

For the investigation of the effect of DUP selection on service level, a proper scheme of distribution planning is needed first. In the case of distribution planning, what a manufacturing firm wants is a proper scheme with which it updates an existing distribution plan using the demands newly forecasted. This study addresses the problem of determining the amount of commodity flow on the branches of the distribution tree, for each of time periods in a given planning horizon. The objective of the problem is to minimize the sum of inventory and backlog costs over the distribution network. This also can be viewed as a multi-echelon inventory problem under a central planner with complete information where demands are dynamic and reforecasted periodically.

We found it difficult to encounter studies on controlling service level by DUP in supply chain literatures. However, there are numerous reported studies on distribution networks in terms of multi-echelon inventory problem. Multi-echelon inventory theory provides means of modeling supply chains, thereby enabling quantitative analysis and characterization of optimal control policies [6, 8]. Since Wagner and Whitin [12] used dynamic programming for the single echelon dynamic lot size problem, many studies on single echelon time varying demand problem have been appeared on literatures [2, 10]. The Wagner-Whitin algorithm was generalized to the backlogging case [13]. Clark and Scarf [5] first studied multi-echelon inventory models. They address a system of several stages in series operating under periodic review ordering policies. Clark and Scarf model was extended to general arborescence structures in [3]. Many others are also found in the recent surveys on optimal policies for different types of multi-echelon inventory system in [9]. In the cases of deterministic but time varying demand, order sizes are obtained and approximated [1, 4]. The papers on arborescence distribution systems are limited mostly in two echelon systems due to the complexities arise in the modeling of such systems. In practice, however, large distribution networks are frequently encountered and, therefore generalization of two echelon policies is needed [6, 7, 11]. This study attacks this problem.

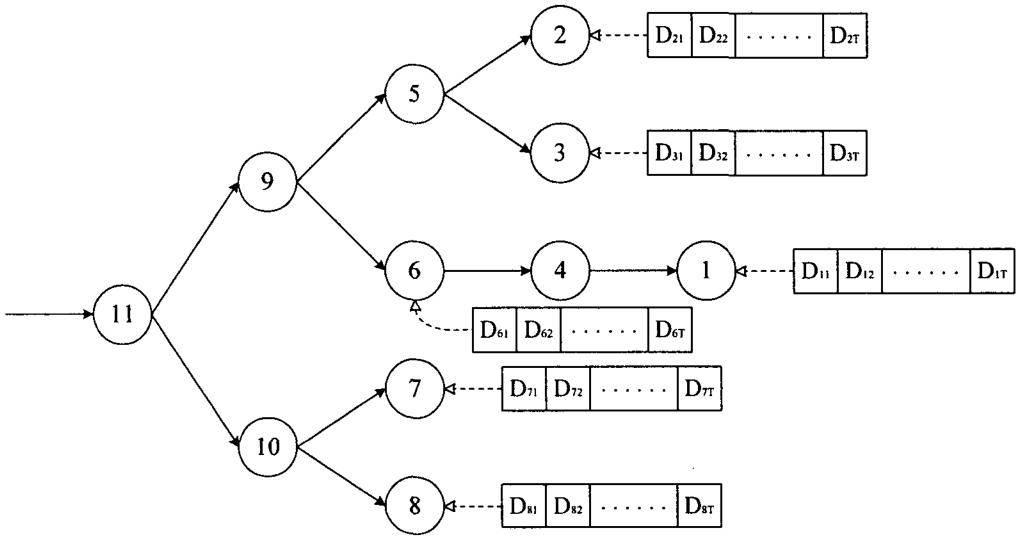
The method presented in this paper assigns an amount of commodity flow on each arc of a distribution tree for a given planning horizon, which keeps advancing in time. Therefore, at a moment of a new decision making, there are items in transit state moving through the arcs of the network. These outstanding orders have been activated by the result of the prior decision and they are not to be controlled by the current decision making. Without loss of generality, it is assumed that those items already in moving stage arrive at their destinations on time. Following this introduction the problem to be studied is defined along with the presentation of the solution algorithm in section 2. In sections 3 and 4 the scheme to use the distribution planning algorithm and customer service level are explained respectively, and a discussion is made in section 5.

2. Distribution planning algorithm

The problem of distribution planning for a given planning horizon is addressed here. As depicted in Figure 1, the distribution network has the form of arborescence.

The site at the most upstream receives ordered quantity from an outside source with no capacity limit. All the sites at the downstream ends are demand site, i.e., ordered items are delivered to the customers from the site. Also any transshipment depots in the midstream can be demand site as well. For a given planning horizon, at each of demand sites, demands are forecasted and are known. The associated lead times (or transportation times) with the branches are deterministic and given. And we assume that the lead times are integral number of periods. The distribution plans are continuously updated on the rolling horizon base. Therefore, at an initiation of a planning, on each of branches in the network, there are items flowing. The amounts of these flows have been decided at the previous planning.

All initiated flows are expected to arrive at their destinations after consuming assigned lead times. Demand at a demand site is time varying, and forecasted demand is given for each of the periods in the planning horizon. The objective of the problem is to minimize the total cost, which is the sum of inventory and backlog costs of all sites.



< Figure 1 > An example of distribution network.

This study assumes all the activities related to the inventory management are taken places at the beginning of a new planning horizon, and the sequence of activities at a site assumed throughout this study is as follow.

- Arrivals of outstanding orders due to arrive at current period.
- The backlogged order should be replenished, if necessary.
- Plan the distribution with the current inventory on hand and with the newly updated demand forecast for the planning horizon.
- External demand for the period is realized and put the planned amount on the move.
- Incurred cost is assessed.

Note that a realized demand may not coincide with the forecasted one. This becomes the source of inventory and backlog. Also a plan to be meaningful the length of a planning horizon must be longer than the longest of the path lead times. A path lead time is defined as the accumulation of lead times on the branches in the path that connects a leaf node to the root node.

To formulate the problem into a model, notations are introduced as the following.

<Notations>

N : Number of sites

T : Number of periods

D_{it} : External demand at site i for period t

X_{it} : Quantity in transit toward site i at the beginning of period t

I_{it} : Stock at site i at the end of period t

s_i : Set of immediate successor sites of site i

h_i : Holding cost (per item per period) at site i

b_i : Backlog cost (per item per period) at site i

l_i : Lead time (transportation time) from the predecessor node to site i

L_i : Cumulative lead time from the root node to site i

Using the defined notations, the problem is formulated as follows.

$$\min \sum_{t=1}^{T}\sum_{i=1}^{N}[h_i \max(I_{it},0)+b_i \max(-I_{it},0)] \quad (1)$$

$s.t.$

$$I_{it}=I_{i(t-1)}+X_{it}-D_{it}-\sum_{j\in s_i}X_{j(t+l_j)},\ \ i=1,\ldots,N,\ \ t=1,2,\ldots,T+L_i \quad (2)$$

$$I_{it}\geq 0,\ \ \textit{if } i \textit{ is not demand site.} \quad (3)$$

$$X_{it}\geq 0,\ \ i=1,\ldots,N,\ \ t=1,2,\ldots,T+L_i \quad (4)$$

The objective function (1) is the sum of inventory holding and backorder costs for all sites over the planning horizon. Constraint (2) expresses the flow conservation constraints for site i. It defines the inventory level existing at site i at the end of period t.

The decision variables are $X_{it}(i=1,\ldots,N,\ t=1,2,\ldots,T+L_i)$, i.e., the amount to be arrived to site i at time t over all sites and time periods. This model is basically a linear programming model. However, utilizing the special structure of the network and the presented model an efficient solution procedure is introduced in the following. The idea behind the algorithm is to satisfy customer demands as much as possible starting from the highest backlog cost sites at the leaf nodes, and the flows in branching nodes are allocated to support those fulfillments of the demands.

To explain the solution procedure, definition of stage is introduced. We define stage m sites as the nodes that are apart from the root node by m arcs, thus the root node is at stage 1 and the nodes that are immediate successor of the root node are stage 2 sites. We also denote the nodes most apart (by the number of arcs) from the root node as stage M sites, J_k as the set of all sites belong to stage k, S_i as the set of all successors of site i. Also m_i represents the last stage among all successors of site i, and l_{ij} is the cumulative lead time from site i to j. The detail solution procedure is as follows.

<Solution Procedure>

STEP 1:

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FOR k = M, M-1, ..., 1
  FOR i ∈ J_k
    IF S_i = ∅, THEN
      Compute I_it from I_it = I_i(t-1) + X_it - D_it for t = 1, ..., l_i.
    END IF
    IF S_i ≠ ∅, THEN
      FOR t = 1, ..., l_i
        FOR k' = m_i, m_i - 1, ..., k+1
          FOR j ∈ S_i and j ∈ J_k'
            Set t' = t + l_ij.
            IF I_j(t'-1) ≥ 0, THEN
              Set X*_jt' = D_jt' and Î_j(t'-1) = I_j(t'-1).
              Sort X*_jt' by descending order of backorder costs for all
              j' ∈ S_j and site j.
              Move first X*_jt'.
              WHILE( Î_j(t'-1) ≠ 0 ) DO
                IF X*_jt' ≥ Î_j(t'-1), THEN
                  Set X*_jt' = Î_j(t'-1) and Î_j(t'-1) = 0.
                END IF
                IF X*_jt' < Î_j(t'-1), THEN
                  Set Î_j(t'-1) = Î_j(t'-1) - X*_jt' and X*_jt' = 0
                END IF
                Move next X*_jt'
              END WHILE
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            END IF
            IF $I_{j(t'-1)} < 0$, THEN
                Set $X^*_{jt'} = D_{jt'} - I_{j(t'-1)}$
            END IF
        END FOR $j$
        Set $\hat{X}_{jt'} = X^*_{jt'} + \sum_{j' \in S_j} X^*_{j'(t'+l_{jj'})}$.
    END FOR $k'$
    Compute $I_{it}$ from $I_{it} = I_{i(t-1)} + X_{it} - D_{it} - \sum_{j \in s_i} \hat{X}_{j(t+l_j)}$.
    IF $I_{it} \geq 0$, THEN
        Set $X_{jt'} = \hat{X}_{jt'}$ for all $j \in S_i$.
    END IF
    IF $I_{it} < 0$, THEN
        IF $I_{i(t-1)} \geq 0$, THEN
            Set $X^*_{it} = D_{it}$.
        END IF
        IF $I_{i(t-1)} < 0$, THEN
            Set $X^*_{it} = D_{it} - I_{i(t-1)}$.
        END IF
        Sort $X^*_{jt'}$ by descending order of backorder costs for all $j \in S_i$
        and site $i$.
        Set $\hat{I}_{i(t-1)} = I_{i(t-1)} + X_{it}$ and move first $X^*_{jt'}$.
        WHILE( $\hat{I}_{i(t-1)} \neq 0$ ) DO
            IF $X^*_{jt'} \geq \hat{I}_{i(t-1)}$, THEN
                Set $X^*_{jt'} = \hat{I}_{i(t-1)}$ and $\hat{I}_{i(t-1)} = 0$.
            END IF
            IF $X^*_{jt'} < \hat{I}_{i(t-1)}$, THEN
                Set $X^*_{jt'} = X^*_{jt'}$ and $\hat{I}_{i(t-1)} = \hat{I}_{i(t-1)} - X^*_{jt'}$.
            END IF
            Move next $X^*_{jt'}$.
        END WHILE
        Set $X^*_{jt'} = 0$ for remaining $X^*_{jt'}$.
        Compute $X_{jt'}$ from $X_{jt'} = X^*_{jt'} + \sum_{j' \in S_j} X^*_{j'(t+l_{jj'})}$ for $j \in S_i$.
    END IF
    Compute $I_{it}$ from $I_{it} = I_{i(t-1)} + X_{it} - D_{it} - \sum_{j \in s_i} X_{j(t+l_j)}$.
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Compute $I_{jt'}$ from $I_{jt'} = I_{j(t'-1)} + X_{jt'} - D_{jt'} - \sum_{j' \in s_j} X_{j'(t'+l_{j'})}$ for all $j \in S_i$.

END FOR t

END IF

END FOR i

END FOR k

STEP 2:

FOR $k = M, M-1, \ldots, 1$

FOR $i \in J_k$

FOR $t = (L_i + l_i + 1), (L_i + l_i + 2), \ldots, (L_i + T)$

Compute X_{it} from $X_{it} = \max(D_{it} + \sum_{j \in s_i} X_{j(t+l_j)} - I_{i(t-1)}, 0)$.

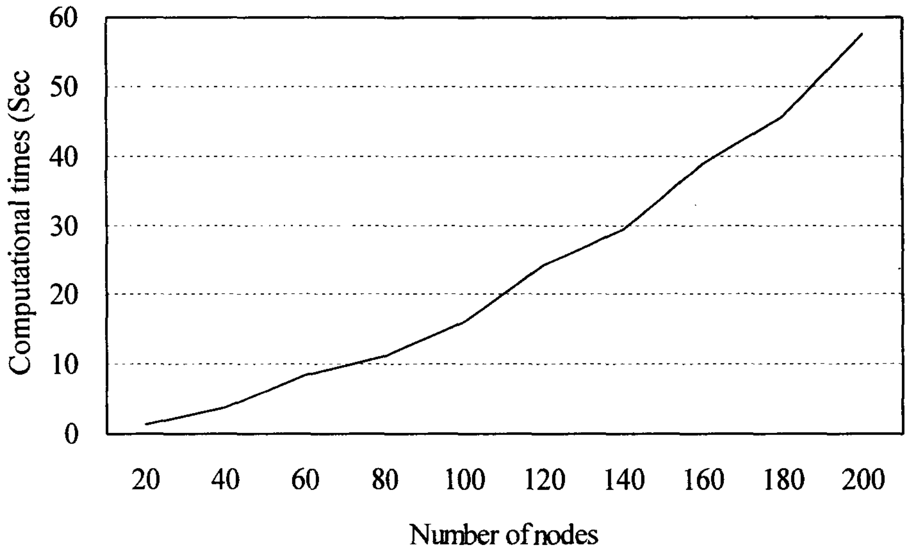
Compute I_{it} from $I_{it} = I_{i(t-1)} + X_{it} - D_{it} - \sum_{j \in s_i} X_{j(t+l_j)}$.

END FOR t

END FOR i

END FOR k

The solution procedure presented above is simple. And, as shown in Figure 2, this solution procedure requires practically small enough amount of computational times even for a fairly big problem (such as 200 nodes) with Pentium IV desktop computer.



< Figure 2 > Computational times.

3. Scheme to use the distribution planning algorithm

As mentioned previously, the aim of this study is to develop an usable planning scheme for the distribution networks. To apply the method presented in this paper for the real distribution networks, the following steps of using the distribution planning algorithm where demands are assumed to be deterministic values on hand is suggested.

Step 1: At a beginning of a new planning horizon, records for the existing inventories are updated. This is necessary to reflect the arrivals of outstanding orders due to the first period arrive as well as the realization of customer orders.
Step 2: Updated demand distributions.
Step 3: For each period of the planning horizon demand values to be used in the planning is selected. Customer service level and network management costs are the concern of this stage.
Step 4: Decide the quantities to be moved on the branches of the networks using the distribution planning algorithm developed, and the results are implemented.

4. Service level and DUP

It is generally true that as an item goes further downward in a distribution network it expects occurrence of higher holding costs. This makes a planner advantageous to keep inventories as upstream side as possible. On the other hand, shortage costs are realized only at demand sites and this encourages the planner keep stocks at downstream side of a distribution network for quicker response to excessive demands. In the model presented in section 2, there is no explicit consideration on safety stock. Stocks on sites are realized only by the discrepancy between the prediction and the realization of demands. In reality the forecasted demands for a period in a planning horizon is distributed over a certain interval, and the distributions of demand on periods are generally not identical. From the viewpoint of the delivery to customers on time, the use of safety stock is not feasible due to the variability among the demand distributions. The deterministic valued demand for a period, which is used in a distribution planning, is the value chosen among the distributed values by a decision maker, such as a central planner or marketing personnel. In this case, the higher quantiles of distributions are chosen as the demands of periods, the more stocks are expected at demand sites.

Therefore, customer service level can be controlled by the choice of demand to be used, and it is not too hard to see the selection of quantile also have effect on the total costs incurred in a planning horizon. To see these we performed a series of simulation experiments and the results are summarized in the Table 1. The network used is a two echelon tree with two demand sites and a root node. The distributions of customer demands on demand sites are assumed to follow normal distribution, and greater variances are assigned to the demand distributions of periods further into future. The values at the most left hand side column are the ratios of shortage and holding cost on the root node, and the costs in the table are relative ones to the cost of 0.5th quantile case, i.e., the selected DUP is the 0.5th quantile of the demand distribution. For each case the cost per period is obtained after running the simulation for 10,000 periods. As shown in the Table 1, it seems that the cost per period behaves as a convex function of quantile of the demand distribution used in the planning, and the minimum of the cost per period is achieved at a higher ratio of shortage and holding cost.

< Table 1 > Cost and service level with regard to DUP selection.

Ratio \ Quantile	0.5^{th}	0.6^{th}	0.7^{th}	0.8^{th}	0.9^{th}	0.95^{th}
1	1	0.88	0.99	1.28	1.86	2.43
10	1	0.76	0.67	0.68	0.84	1.05
50	1	0.75	0.64	0.61	0.73	0.90
200	1	0.74	0.63	0.60	0.71	0.87
Service level	72.5	80.1	87.3	92.0	95.2	97.5

Since, as long as the shortage cost is higher or equal to the holding cost, the rule is to meet all the forecasted demands as much as possible, the customer service level is decided only by the demands, and the cost has no role here. In general, customer service level (percent of immediately fulfilled demand) is higher than the selected quantile. For example when 0.5th quantile is used for the selection of demand value, the service level is 72.5. This is because when a realized demand (say 0.75) is higher than the estimated one (say 0.5), on the average estimated/realized $(0.5/0.75)\times 100\%$ of a demand are satisfied by the prepared items. Although all discussed thus far in this section are only preliminary results, it suggests that the selection of demand is important with regard to the cost per period and customer service level. The aim of the study is to find ways of selecting DUPs for demand sites, and the objective of such selections is to minimize total cost while keeping the service level above a certain value.

5. Conclusions

This study presented a realistic distribution planning scheme for a widely presenting distribution networks. The model of the problem appeared in the study is relatively simple. This is due to the absence of ordering cost in the model unlike many other inventory models of the literatures. However, when a distribution network is for a single manufacturing firm, ordering costs are negligible, and the main concern of the firm is to serve customers with a least possible cost using a simple to use method or tool. This study serves this purpose. This study also presents the issue of satisfying customer service level by the selection of demand level. We believe this issue is to be investigated further, especially in terms replacing the use of safety stocks. The problems with capacity constraints and multi-items are also to be followed this study.

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