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Development of a New Max-Min Compositional Rule of Inference in Control Systems

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Abstract

Generally, Max-Min CRI (Compositional Rule of Inference) method by Zadeh and Mamdani is used in the conventional fuzzy inference. However, owing to the problems of Max-Min CRI method, the inference often results in significant error regions specifying the difference between the desired outputs and the inferred outputs. In this paper, I propose a New Max-Min CRI method which can solve some problems of the conventional Max-Min CRI method. And then this method is simulated in a D.C.series motor, which is a bench marking system in control systems, and showed that the new method performs better than the other fuzzy inference methods.

Key word : fuzzy inference system, similarity measure

1. Introduction

The most important thing in fuzzy system models is that the behaviors of system models are defined by an approximate reasoning, i.e. CRI (Compositional Rule of Inference) method, which is used for inference in an approximate reasoning[1]. To efficiently usage of CRI method, Mamdani proposed an inference method using 'max' and 'min'. This is called Max-Min CRI method, which has mostly been used technique in the fuzzy inference.

However, Max-Min CRI method has problems: subjective formulation of membership functions, error-prone weighting strategy, and inefficient compositional rule of inference. Therefore, the inference results by Max-Min CRI method produce error regions. The error regions make system misoperating, so the reducing techniques of error regions are very important to implement the efficient fuzzy systems, especially fuzzy control systems.

In this paper, I will propose a New Max-Min CRI (NCRI) fuzzy inference method to reduce the error regions. Also, I will propose a new similarity measure using in the NCRI method. The similarity measure can be reduced the error regions at least. I will apply the NCRI method to D.C.series motor to show how this method is better than any other similarity measures, which were proposed by Takefuji and Turksen before.

This paper is composed of four sections. In section 2, Max-Min CRI method and D.C.series motor are explained. In section 3, the Max-Min CRI method and its serious problems are explained. Also, an algorithm of the New Max-Min CRI method including a new similarity measures is proposed. And finally, I will discuss some conclusions and further studies in section 4.

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2. Conventional Max-Min CRI Method and D.C.series Motor

2.1 Definition

The Max-Min CRI method is a conventional fuzzy inference method which uses 'max' and 'min' operators proposed by Mamdani. It is often called a direct method or Zadeh's inference method.

The Max-Min CRI method can be represented by possibility distribution theory [2], and the inference procedures of the method are the followings. In these expressions, fuzzy implication statements are represented by possibility distribution values (equation (1)). A' is an input fact, B' is an inferred result and R is a fuzzy relation and Π_x, Π_y are possibility distribution values of x,y.

$$IF\ X\ is\ A\ THEN\ Y\ is\ B \rightarrow IF\ \Pi_x = \mu_A\ THEN\ \Pi_r = \mu_B \quad (1)$$

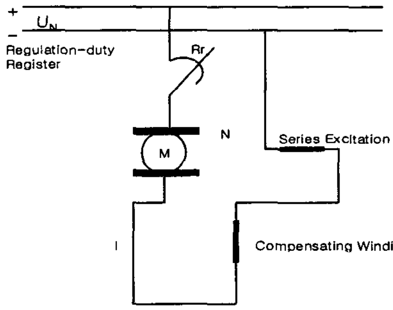
It can be represented by CRI method such as the equation (2). In here, '$\circ$' is a compositional operator, '$\rightarrow$' is a fuzzy implication operator.

$$\begin{aligned} B' &= A' \circ (A \rightarrow B) \\ &= A' \circ R \end{aligned} \quad (2)$$

D.C.series motor is a simple bench marking vehicle to examine the performance of FLC (Fuzzy Logic Controller) D.C.series motor in Figure 1.

Table 1. Fuzzy Model and Data in D.C.series Motor

Fuzzy System Model					Data					
	IF	I = null	THEN	N = very large	I	N	I	N	I	N
ALSO	IF	I = zero	THEN	N = medium	0.0	2000	3.5	600	7.0	1600
ALSO	IF	I = small	THEN	N = zero	0.5	1800	4.0	400	7.5	1800
ALSO	IF	I = medium	THEN	N = medium	1.0	1600	4.5	600	8.0	2000
ALSO	IF	I = large	THEN	N = very large	1.5	1400	5.0	800	8.5	1800
ALSO	IF	I = very large	THEN	N = medium	2.0	1200	5.5	1000	9.0	1600
					2.5	1000	6.9	1200	9.5	1400
					3.0	800	6.5	1400	10.0	1200



Components	Performance Ranges
Power rating	$P_N = 0.7Kw$
Voltage rating	$U_N = 110V$
Rated current	$I_N = 8.84A$
Rated speed	$N_N = 1500rpm$

Figure 1. D.C.series Motor and Performance Range

In the motor, electronic incomes are proportional to current I , and the relation between current I and speed N are in a nonlinear relation. As a result, the actual states of D.C.series motor are evaluated by the expression $N = f(I)$ which represents the relation between current I and speed N. In the expression, f means a nonlinear function.

The Max-Min CRI method has a fuzzy implication operator R_8 shown in the equation (3). The R_8 is a fuzzy implication operator having a max-min operation and union operator for '*ALSO*'. "$\vee$" means 'max', "$\wedge$" means 'min' in here.

$$R_8(i,n) = \vee(\mu_A(i) \wedge \mu_N(n)) \qquad (3)$$

Kiszka [3] proved the rightness of R_8 by adopting many fuzzy implication operators to D.C.series motor and showing some results.

The procedures of the Max-Min CRI method can be performed by the equation (4). In here, $u \in U$ is a input fact, $v \in V$ is a output result. A, A', B are fuzzy sets.

$$\begin{aligned}\mu_{B'}(v) &= \underset{u\in U, v\in V}{Max} Min(\mu_{A'}(u), \mu_A(u) \rightarrow \mu_B(v)) \\ &= \underset{u\in U, v\in V}{Max} Min(\mu_{A'}(u), \mu_R(u,v))\end{aligned} \qquad (4)$$

Kandel [4] simulated the performance of R_8 by adopting D.C.series motor and the fuzzy system model which has the rule base such as Table 1.

Because the Mamdani's fuzzy implication operator R_8 is better than any other fuzzy implication operators[4], most engineers have used the Max-Min CRI method in fuzzy control systems to implement hardware chips or other devices.

2.2 Problems

The Max-Min CRI method has an advantage such that it is easy to implement, but it has some problems in inference techniques, that is to say, even though the condition part of a rule is the same as input facts in a rule, the inferred output is not the same as the conclusion parts of a rule. That is to say, the actual output is not the same as the desired output by the Max-Min CRI method. However, the problem is related to the ordering of composition in GMP(Generalized Modus Ponens) type, so if it is used the composition ordering which is proposed by Takefuji [5], then it can be better somewhat. This is called the GMP method regarding compatibility degree (equation (5)). The compatibility degree 'α', which is proposed by Takefuji, can be estimated by the following equation (6).

$$\begin{aligned}\mu_{B'}(v) &= \underset{u\in U, v\in V}{Max} Min(\mu_{A'}(u), \mu_R(u,v)) \\ &= \underset{u\in U, v\in V}{Min}(Max \; Min(\mu_{A'}(u), \mu_A(u)), \mu_B(v)) \\ &= \underset{v\in V}{Min}(\alpha, \mu_B(v))\end{aligned} \qquad (5)$$

$$\alpha = Max \;\; Min(\mu_{A'}(u), \mu_A(u)) \qquad (6)$$

If the equation (5) and (6) are used in an inference, the Max-Min CRI inference method can be represented by the compatibility degree α. This is a practical inference procedure in the Max-Min CRI method. Here,

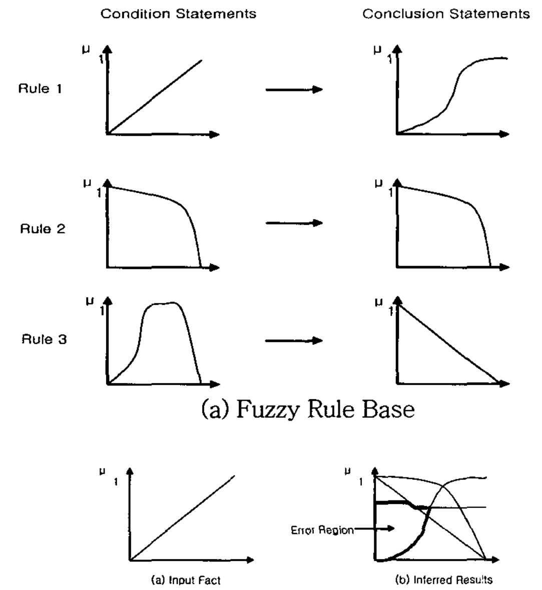
A, A', B are fuzzy sets. A is a conditional part of a rule, A' is an input fact , and B is a conclusion part of a rule.

In spite of all, the Max-Min CRI method has some drastic problems. Firstly, there has some error-pone weighting strategy in the Max-Min CRI method. For example, let's make an image a rule base of FLC (Fuzzy Logic Controller) having the following style rule base such as Table 2. '*ALSO*' means Max in equation (3).

Table 2. Rule Base of FLC

Rule Base					
Rule 1		IF	X_1 is A_1	THEN	Y_1 is B_1
Rule 2	ALSO	IF	X_2 is A_2	THEN	Y_2 is B_2
Rule 3	ALSO	IF	X_3 is A_3	THEN	Y_3 is B_3

In here, A, B are two fuzzy sets. The rules can be represented by Figure 2(a).



(b) Inferred Results

Figure 2. The Graphical Diagram of Fuzzy Rule Base and Inferred Results

In this case, if an input fact (Figure 2(b)) is entered to this system, then the inferred result by the Max-Min CRI method can be obtained in Figure 2(b). As it can be seen, even though the input fact is the same as the condition part of Rule 1, the inferred output is not the same as the conclusion part of this Rule 1. In here, this is called an error region. The error region is the area characterized by the difference between the fuzzy conclusion of the inference and the consequence of one of the fuzzy rules in case if the input fact is exactly the same as its antecedent. In Figure 2(b), the bold black part is the error region, the bigger this area, and the farer inferred results from desired output. The reason why this region has taken place is that the inefficient process of compositional rule of inference was used and the relative weights were disregarded in the Max-Min CRI method.

Secondly, because the method composites all of the rules even though having no relative, there can be derived some insufficient actual outputs in the Max-Min CRI method. This situation is called an inefficient compositional rule of inference. Therefore, the computing time takes much higher, the complexity is higher as well as the reliability of the inferred results is much lower. So, the inefficient or irrelative rules have to be omitted when inference is activated. Because of the compositional rule of inference, the error region has taken place here.

Finally, the membership functions in FLC are determined by a subjective manner. The subjectivity is different with man, but it can be inducted some common properties from a lot of subjectivity. So it needs to compensate the common properties in the formulation of membership functions. To do this, Gorzalczany[6] presents neuro-fuzzy techniques for the controllers. The technique can effectively deal with two main types of knowledge which usually describe the control strategy for complex systems, that is, a qualitative, linguistic, fuzzy knowledge usually expressed in the form of linguistic rules, and quantitative, nonfuzzy information in the form of measurements and other numerical data. Kasabov[7] developed to a new concept of evolving neuro-fuzzy systems, with respective algorithms for learning, aggregation, rule insertion, rule extraction. Lin[8] also proposed a robust neuro-fuzzy controller with tuning mechanism of membership functions and neural weights to achieve the tracking control of composite multivariable systems. A self-organizing neural-network-based fuzzy system is proposed by Wang[9]. The system which can partition the input spaces in a flexible way, based on the distribution of the training data in order to reduce the number of rules without any loss of modeling accuracy.

3. New Max-Min CRI Method

3.1 Similarity Measure

There has been studied some similarity measures. Sessa[10] proposed a similarity-based SLD resolution for an approximate reasoning, so approximate inferences are possible since similarity relation allows us to manage alternative instances of entities that can be considered "equal" with a given degree.

However, many papers have discussed the topic from different points of view. That is to say, in some sense, similarity measure must have a generality not a specialty. So, in this case it has to be proposed to compensate the inaccurateness of the Max-Min CRI method. The above proposed similarity measures can not apply to the Max-Min CRI method, because these are more popular to estimate the language similarity. Also, the measuring equation is too complicated, so, a simple measure should be proposed in real time system.

Therefore, to resolve some problems of the Max-Min CRI method in real time, a New Max-Min CRI inference method (NCRI) is proposed here. In NCRI method, the similarities between fuzzy sets are estimated by the proposed similarity measure, and adopted to the Max-Min CRI method. So, the error regions which are occurred in the Max-Min CRI method can be reduced at least. The similarity measure which is proposed in this paper is in the following Definition 3.1.

Definition 3.1: Similarity Measure

For all $u \in U$, the similarity degrees between fuzzy sets are determined by the following equation. In here, τ is a measured similarity degree or value, '$\wedge$' means '**min**' and '$\vee$' means '**max**' operation.

$$SM(A,A') = \tau = \frac{f(\mu_A(u) \wedge \mu_A'(u))du}{f(\mu_A(u) \vee \mu_A'(u))du}, for \ \ all \ \ u \in U$$

For all Universe of discourse ($u \in U$), the shadowed part is the intersection part of two fuzzy sets and the white part is the union part of two fuzzy sets.

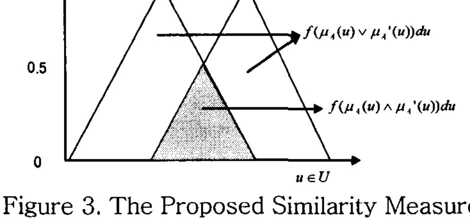


Figure 3. The Proposed Similarity Measure

If there are two fuzzy sets which have the triangle-type membership function, the similarity degree between two fuzzy sets can be estimated using the proposed similarity measure very easily. From the estimated similarity degree, it can be deducted that the smaller the relative distance in the universe of discourse between two fuzzy sets, the bigger the similarity degree between two fuzzy sets, and vice versa.

3.2 Fuzzy Inference Algorithm by NCRI method

The NCRI algorithm having an inference procedure using the proposed similarity measure consists of three necessary procedures: fuzzification procedure, inference procedure, and defuzzification procedure .

The fuzzification membership functions in a fuzzy rule base are triangular-typed ones defined by equation (7) with $a,b,u \in U$. The fuzzy rule base composes of Multi Input Single Output typed rule base. Each fuzzy membership function in the fuzzy rule base has a membership value area [0,1], and it should be normalized in this area.

$$y = \begin{cases} \frac{2}{b-a}(x-a), \ a \le u \le b, \ u \in U \\ 0, \ otherwise \end{cases} \tag{7}$$

This interval includes all possible values for the variable in universe of discourse(U). All fuzzy sets in a fuzzy rule base have the same support interval $[a,b]$. The equation (7) can be represented all types of fuzzy membership functions both fuzzy and non-fuzzy membership functions.

The main process of NCRI is the equation (8). Compare to the Max-Min CRI method (see equation (5) and (6)), NCRI makes use of the proposed similarity measure in Definition 3.1 between input facts and fuzzy sets in the condition part of a rule. The similarity degree which is estimated by the similarity measure is represented as τ in (8). For all rule i, the following steps are performed for inference.

$$\begin{aligned} \mu_{B'}(v) &= \underset{u \in U, v \in V}{Max} \ Min(\mu_{A'}(u), \mu_R(u,v)) \\ &= \underset{u \in U, v \in V}{Min}(Max \ \ Min(\mu_{A'}(u), \mu_A(u)) \times \tau, \mu_B(v)) \\ &= \underset{u \in U, v \in V}{Min}(\alpha \times \tau, \mu_B(v)) \end{aligned} \tag{8}$$

In defuzzification procedure of NCRI method, for the '*ALSO*' operation, '**max**' operation is adopted like the Max-Min CRI method. From the results of equation (8), the defuzzification procedure can be activated by equation (9).

$$\mu_B = \underset{v \in V}{Max} \, \mu_{Bi}'(v), \ for \ \ all \ \ rule \ \ i, \ 0 \le i \le n \tag{9}$$

And then, the center of gravity method is adopted for producing the defuzzification value.

$$Defuzzification \ \ value = \frac{\int \mu_B(v) \cdot vdv}{\int \mu_B(v)dv} \tag{10}$$

For example, if there is an input fact like Figure 2(a), the results could be produced in Figure 4 by NCRI method. The error region is in an acceptable range, so, the system's behavior works well, if NCRI method is applied in the application areas.

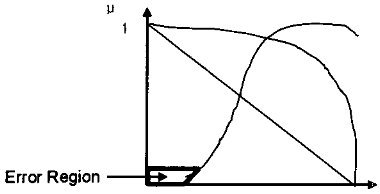


Figure 4. The Result of Figure 2(a) by NCRI Method

From this algorithm, the similarity degrees between fuzzy sets are applied to inference results in NCRI method. That is to say, the bigger a similarity degree, and the higher an adaptation degree. If NCRI method is used in fuzzy inference, the error region which was shown in Figure 5 can be reduced over 93% in the case of the above example. There are more general simulation results in section 3.3.

3.3 Simulations and Results

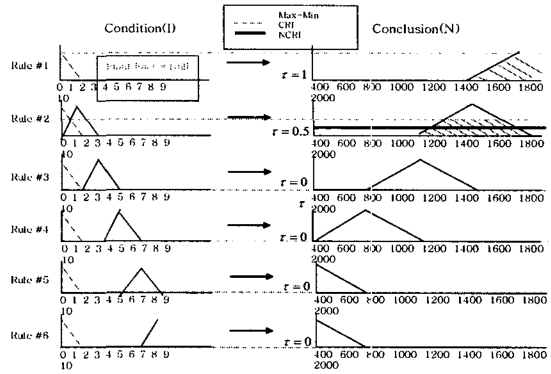
The similarity measure(SM) in NCRI method, I compare with other similarity measures which have already proposed by Turksen and Takefuji for the Max-Min CRI method. To do that, I apply these similarity measures to D.C.series motor. The similarity measures by Turksen[11] and Takefuji[5] are equation (11) and (12), respectively

$$\begin{aligned} SM &= \frac{1}{1+DM} \\ where, DM &= 1 - \underset{u \in U}{Max}\, \mu_{A \cap B}(x) \\ &= 1 - \beta \end{aligned} \tag{11}$$

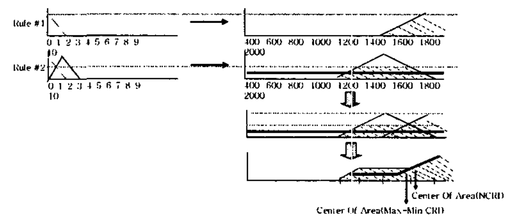
$$\begin{aligned} \sigma &= 1 - \frac{d}{a+a'} \\ where, d &= \sum_{i=1}^{m} Max(A_1(x_i), A'(x_i), A'(x_i)) \\ a &= \sum_{i=1}^{m} Max(A_1(x_i)) \\ a' &= \sum_{i=1}^{m} Max(A_1'(x_i)) \end{aligned} \tag{12}$$

In this paper, these following rule base and data are adopted to compare with other similarities. In here, I use a rule base which Kiszka used already [3]. The environment which D.C.series motor activates is in the rule base such as Table 1. Rule base have fuzzy sets and data which has a motor current I and a rotation speed per hour N.

According to the data in a rule base, the rules in a fuzzy rule base are represented in Figure 5. In the fuzzification procedure, if input fact N is entered in this fuzzy rule base, according to NCRI method, τ is computed by the proposed similarity measure (Definition 3.1) such as Figure 5(a) in the inference procedure. And finally, the defuzzification procedure is performed in Figure 5(b). In Figure 5, the dotted line means the process by the Max-Min CRI method, and the solid line means the process by NCRI method having a similarity measure τ.



(a) Inference Procedure by NCRI method



(b) Defuzzification Procedure by NCRI Method

Figure 5. NCRI Inference Procedure

To reduce error regions which produce a serious problem in the Max-Min CRI method, some systems in the environments such as D.C.series motor having fuzzy value and nonfuzzy value as input facts are simulated and compared the desired outputs with actual outputs in a stable state. Even though D.C.series motor is not a general case, it is a good bench marking system for performance estimating of control. Therefore, if the performance of the applying method is good for the system, the method can be regarded as a nice method.

Figure 6 is the simulated outputs of Max-Min CRI method, Turksen method, Takefuji method, and NCRI method. In the figure, "+" is desired outputs, "×" is actual outputs. *N(Null), Z(Zero), S(Small), M(Medium), L(Large), VL(Very Large)* are entered eventually as fuzzy input facts to the system in order to simulate. In the figure, x axis represents fuzzy input values, and y axis represents inferred results N.

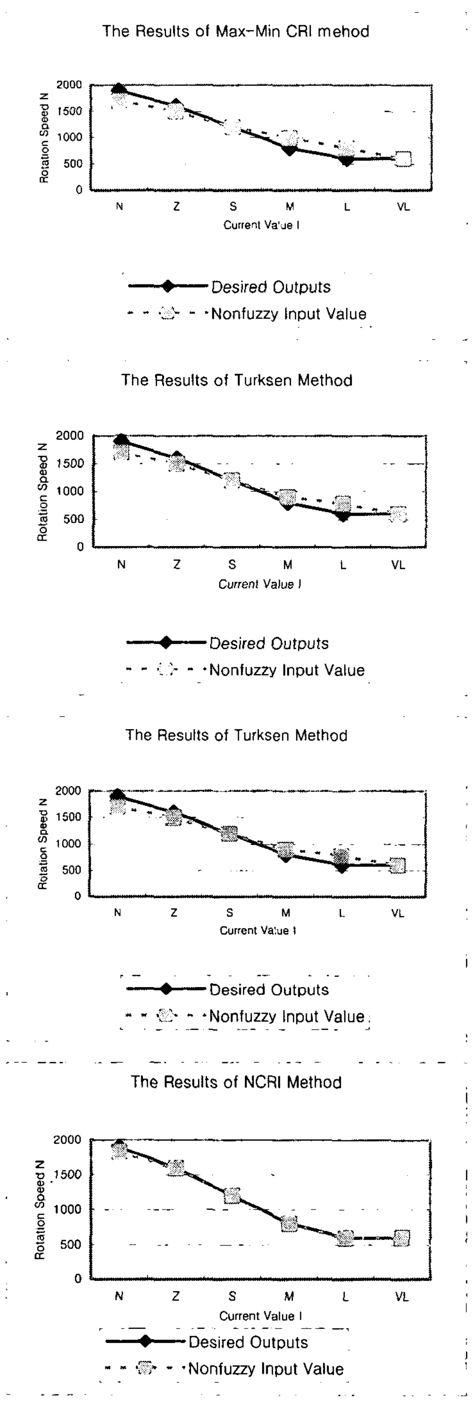


Figure 6. The Results of Fuzzy Inference Methods

From the Figure 6, the smaller range between actual outputs and desired outputs produce the smaller error regions. That is to say, the smallest error regions makes fuzzy controller more efficiently. Therefore, NCRI method makes error regions reduced to an minimum acceptance range. In this paper, the error region rate is computed by the equation (13), where '*correct region*' means that the conclusion part of the rule whose membership functions of the condition part are the same as the input fact, the '*error region*' means that the difference between the desired output and the actual inferred output by the conclusion part of the same rule as the condition part of the input fact.

$$error\ region\ rate(\%) = \frac{error\ region}{correct\ region} \times 100 \quad (13)$$

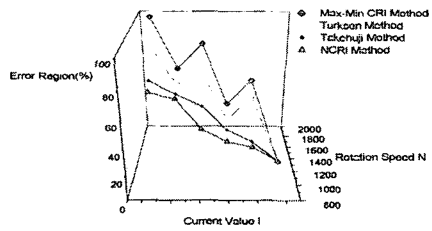


Figure 7. The Error Regions According to Inference Methods

In Figure 7, it is compared with the error region rate among four methods. For example, if an input fact is N, the rate of error regions of the Max-Min CRI is in 100%, Turksen in 61.1%, Takefuji in 27.8%, NCRI in 16.0%. As a result, NCRI method can be reduced error regions to 89% at most compared with the Max-Min CRI method. Those values are 75% reduced values compared with the Max-Min CRI method, 41% compared with Turksen, 8.1% compared with Takefuji. However, in the case of VL (very large) the rate of error region is almost 0. From the results about the case of fuzzy input value N which can occur the maximum rate of error region, the Max-Min CRI method has the largest error regions(51.9%) compared with other fuzzy inference methods, NCRI method has the smallest error region(5.8%) compared with other regions. Therefore, NCRI method is the most improved fuzzy inference method in D.C.series motor compared with other fuzzy inference methods. Even though some fuzzy quantifiers such as "very, more or less, not" combine with any fuzzy input facts, NCRI method is better than any other fuzzy inference methods.

4. Conclusions

The Max-Min CRI method is a conventional method in fuzzy inference, but it has problems, so that the inference results from the method have error regions. To solve those problems, an improved fuzzy inference method, NCRI method, compared with the conventional fuzzy inference method is proposed in this paper. To reduce the error region to acceptance range which has taken place in the Max-Min CRI method, I proposed a similarity measure and applied it to D.C.series motor as an example.

For simulation, it is compared with the error region rate among four methods in this paper. The proposed NCRI method is the most improved fuzzy inference method in D.C.series motor compared with other fuzzy inference methods.

However, to reduce the error region completely or to be always in a stable acceptance range, some improved neural networks should be developed in this method. Also, a multi-level rule base should be developed for complicated application area as well as many application areas should be applied to the NCRI method.

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