Journal of the Korea Society of Mathematical Education Series D:
Research in Mathematical Education
Vol. 8, No. 3, September 2004, 145–155

韓國數學敎育學會誌 시리즈 D:
<數學敎育硏究>
제 8 권 제 3 호 2004 년 9 월, 145–155

Development of Creativity through Mathematical Applications[1]

Donaldson, John D.
School of Mathematics and Physics, Faculty of Science, Engineering and Technology,
University of Tasmania, Australia; Email: john.donaldson@utas.edu.au
Current Address: Division of Applied Mathematics, School of Natural Science, KAIST,
Guseong-dong, Yuseong-gu, Daejeon 305-701, Korea

(Received August 30, 2004)

Mathematics, by its nature, is a creative activity. Creativity can be developed either through considering its intrinsic beauty or by examining the role that it plays in applications to real world problems. Many of the great mathematicians have been vitally interested in applications and gained inspiration in developing new mathematics from the mathematical descriptions of physical phenomena. In this paper we will examine the processes of applying mathematics by looking at how mathematical models are formed and used. Applications from sport, the environment and populations are used as illustrations.

Keywords: creativity, physical phenomena, population, applications
ZDM Classification: C40
MSC2000 Classification: 97C20

1. INTRODUCTION

In May 2004, the second Abel Prize for Mathematics was awarded to Michael Atiyah of the University of Edinburgh and Isadore Singer of MIT for their work on the Index Theorem. It relates to the number of solutions of a class of mathematical problems making it unnecessary to look for others. It was found that the theorem has wide applications and answered some important questions in areas such as high-energy physics.

In a subsequent talk, Singer emphasized that mathematics in itself is intrinsically beautiful; it is the study of patterns and their interconnections. This should not be

[1] This article will be presented at the Ninth International Seminar of Mathematics Education on Creativity Development at Korea Advanced Institute of Science and Technology, Daejeon, Korea, October 9, 2004.

overlooked at the expense of the pressure for applications

More generally, science is a study of patterns and mathematics is the language of science. As a result applications of mathematics are extremely important to the future of our world. In fact the greatest mathematicians have been vitally concerned with the real world (Hansen 1971). Newton's work on the orbits of the planets and Gauss' motto, 'Thou, nature, art my Goddess: to thy laws my services are bound' provide profound confirmation.

It is observed that truly creative contributions to our world come from the study of mathematics as an entity in itself as well as in its applications.

Buck (1973)[2], helped overcome her daughter's lack of interest in mathematics by explaining that mathematics is the symbolic language of relationships and relationships contained the essential meaning of life.

Bogomolny (2000) describes a discussion in an internet column of a question from a parent who is concerned that her child who loves the creativity of music has become disinterested in mathematics since there appears to be no creativity in it and so why study mathematics. The parent asks for resources that show that mathematics can be 'imaginative, creative and even fun.'

The Mathematical Sciences Education Board of the National Research Council of the USA has expressed a need for the development of imaginative programmes reflecting vitality and uses of mathematics and which 'stimulate creative approaches to mathematics curricula in the next century' (Steen 1990). They argue that the present and future practice of mathematics — at work and in research — should shape education in mathematics. Steen provides five examples, which illustrate what is possible.

In this paper we will consider the process of applying mathematics to patterns in real world phenomena. We will consider what type of information may be expected from developing a mathematical model and the advantages of using it. The process requires considerable creativity on behalf of the user in a variety of different ways. Kim & Lee (2001) give a definition of creativity as related to mathematics as the interaction of understanding, intuition, insight and generalization, which reflects the steps in forming, analyzing and using a mathematical model.

2. NEED FOR VARIETY

We have noted already that there are two different types of problems, intrinsic mathematical problems and mathematical applications, each of which may have an appeal to an individual. We shall be concerned with applications and here the list of possible

[2] Pearl S Buck, the American Nobel Laureate author.

areas of interest is continually increasing. Examples can be drawn from Art, Astrology, Beauty, Music, Poetry, Religion, Ethics, Economics, Government, Games, Culture, Literature, Philosophy, Science, Nature, the Environment, Sport and many others. Chick & Watson (1994) contain articles on Mathematics in art, music and nature in addition to presenting topics, which illustrate the power and beauty of mathematics.

2.1. Interests of Student and Teacher

A major issue is bringing together the interests of the teacher and the student. Naturally students will have less experience and their areas of interest will be limited by comparison with their teachers. One student may have an interest in music, another in art and yet another in sport. It is incumbent on the teacher to exploit that interest by providing the material to allow the student to be creative in applying mathematics to their favourite topic.

Unfortunately, teachers also have their own special areas of interests and they may not coincide with that of the student. Teachers must be given the opportunity to develop their knowledge of mathematics in their areas of interest as an example of what can be achieved in developing creativity. Then, they can be introduced to a wider range of areas so that they can transfer that knowledge to students and encourage them to be creative in their approach to their areas.

An introduction to the processes involved in applying mathematics would provide a firm basis for obtaining knowledge in specific areas. In particular, in forming a mathematical model there is a well-established step by step approach which should be followed. Each step requires the modeller to be creative in producing responses to questions, which do not necessarily have unique or specific answers. In the next section we will consider the list of the usual steps taken in forming a mathematical model and indicate what advantages there are in using them.

3. MATHEMATICAL MODELLING

The steps taken in forming a mathematical model are well-defined and are outlined in many applied mathematics reference books (Lin & Segal 1994; Logan 1987). Once the physical situation to be analysed is chosen then the process involves

- identifying the pattern,
- choosing the variables for describing the pattern,
- obtaining relationships satisfied by the variables, these may be functional relations or equations governing the way in which they change,

• obtaining further information from the relationships (this may require more information about the function or it may involve solving equations, new mathematics may be required)
• and comparing the new information with the physical situation.

In the last step, if the information is not in agreement with the physical situation then the model is questionable.

Then, a return is made to one of the previous steps and the model is redeveloped. The process is often iterative and several returns may be needed, each requiring a creative alternative approach.

If the comparison is valid, the model is used for:

• Further understanding the physical situation — discovering new patterns,
• predicting what can happen,
• controlling the physical situation
• and extending the problem to a more complex physical situation.

Each part of the process is not unique. There are often different ways to handle each step allowing for the possibility of different outcomes within the ultimate goal of confirming that the model is realistic.

4. THE ADVANTAGES OF A MODEL

Using a mathematical model allows us to perform trials and tests relating to the physical situation under many different conditions. It is expensive and time consuming to conduct actual experiments under a wide variety of circumstances. If we wished to test airfoils or ship's hulls, it would be impractical to build full-size replicas and subject them to a series of experiments to assess their performance.

Similar mathematics is found for quite different models and behaviour patterns of one physical situation can be interpreted or searched for in another.

The equation governing heat flow can also be used to describe random walk or prices of stocks on the financial markets.

An added advantage of making a mathematical model is that it often gives us new insights and spawns new ideas and new lines of thought on the physical situation.

Recently, mathematical models of the denaturizing of RNA have shown why there are sudden changes in conformation, the folding and unfolding of molecular bonding. It is interesting to note that the same models are being used to describe the stability of nano-clusters.

5. COMPUTERS

Obtaining mathematical solutions in closed form is often impossible and one may have to rely on numerical or computational solutions. The numerical methods must be checked; too often computer output is accepted without question.

- The numerical output should compare well with the pattern
- It should be compared with known solutions.
- Experimenting with the parameters, the numerical output may reveal trends and ideas about the physical situation allowing for a closer look at mathematical solutions.

Computers are in fact having a considerable impact on how we investigate mathematical models and there is a never-ending source of creativity through experimentation with numbers. Numerical methods allow us to explore much more detailed phenomena. Newton's methods deal with the motion of only two particles, say the sun and the earth. The computer allows us to examine the effects on the motion of the other planets and even the moon in our calculations.

With computers, mathematics can now handle much more complex models such as weather patterns and the long-term effects on the climate of pollution, the hot-house effect. Computers have assisted in the development of the intriguing world of chaos giving us the Mandlebrot set and the period doubling effects in population studies.

6. EXAMPLES

In the final section we will illustrate the mathematical modeling using an example from sport and one from population modelling indicating the creative decision making nature from an examination of the mathematical model. The models are chosen keeping in mind the level of mathematical skills of school age children

6.1. Mathematics and Sport

There are many interesting applications of mathematics in Sport and two readily accessible to those with limited background are associated with high jumping and long jumping.

Consider the High jump. The objective is to clear a bar at the highest height. The pattern is that the jumper will leave the ground at a certain speed and at the highest point of the jump the speed will be zero. At first instance it looks like there is a straightforward answer given by the energy. The variables are the speed, v, and the height, h. Energy considerations gives us the equation relating the variables,

$$\frac{1}{2}mv^2 + mgh = c,$$

where m is the mass, v is the speed, g is the acceleration due to gravity, h is the height and c is a constant. So we have the answer

$$h = (c - \tfrac{1}{2}mv^2)/(mg).$$

We can test this answer by making trials of throwing a ball vertically in the air. Let us now try to be creative by looking a little more closely. What is h? It is the height of the centre of gravity. This indicates that how high we can raise our centre of gravity, C of G, is more relevant than the height of the bar.

Different types of jumps allow for differences in the maximum height of the centre of gravity. In the Fosbury Flop, Figure 1b, the body of the jumper flows over the bar with part of the body always below it. In the old-fashioned scissors jump, Figure 1a, all parts of the body will be above the bar at the maximum height and so the centre of gravity is above the bar at its peak position. It is clear that the former jump will achieve a greater height for the same initial speed.

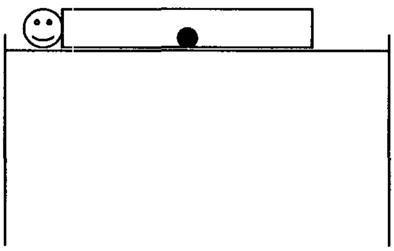

Figure 1a. Scissors: C of G above bar

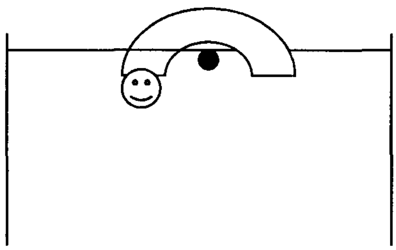

Figure 1b. Fosbury Flop: C of G below bar

In the long jump, the jumper will reach the take-off point with a certain speed, v, and leave it at a take-off angle, say b. The objective is to attain the maximum horizontal distance, x. This is the measurement variable and to achieve this we take off with a speed v at an angle b to the horizontal (see Figure 2).

Then the initial speed in the horizontal direction is $v \sin b$ and in the vertical direction it is $v \cos b$. Applying the laws of accelerated motion, after a time t, the jumper will be at position, $x = v \cos bt$ and due to gravity, $y = v \sin bt - 1/2gt^2$. The jumper returns to the ground when $y = 0$ or when $t = 2v \sin b/g$ and the horizontal distance is

$$x = \frac{2v^2 \sin b \cos b}{g}.$$

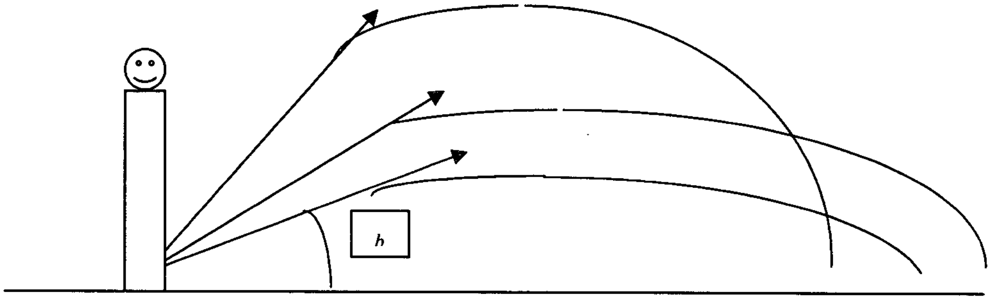

Figure 2. Long jump: take-off angle: maximum distance at $b = \pi/4$.

The jumper has the choice of angle for his given speed. This is maximized when $b = \pi/4$ or 45 degrees. However, this angle is not attained by athletes since, the vertical take-off speed is limited, and they can reach values of about 23 degrees. But the solution to the problem indicates that the higher the angle of take-off, up to 45 degrees, the better.

In each case the student can ask questions about the effects of changing gravity or air resistance. Computer methods are needed to test the more complex model.

6.2. Population Models Leading to Chaos

Population models are a continuing source of interest and serve to illustrate very effectively the creative activity of mathematical modeling.

Firstly we recognize the pattern of a population in a single species model grows quite quickly in its early stages and begins to level off to steady state.

The variable describing this pattern is the population, p, and we can write $p = p(t)$ since the population is changing with time, t. We are interested in how p changes. We can obtain this information from its rate of change, dp/dt, or the difference in a time interval, dt, $p(n+1) - p(n)$, where $t = n\,dt$ is the nth step in time, t, of size, dt.

These will depend on many factors but they will obviously depend on $p(t)$, or $p(n)$, itself. The relationships or equations satisfied by $p(t)$ or $p(n)$ then take the forms

$$\frac{dp}{dt} = f(p) \quad \text{or} \quad p(n+1) - p(n) = g(n)dt,$$

where the change in the second equation will depend also on the time interval.

In their simplest forms, the functions, f and g in the lastequations, will have a linear growth term, proportional to the population. Then, there will also be a competitive factor, which is proportional to the square of the population indicating that each member of the population is competing eventually with every other for resources. So, assuming a quadratic approximation for the forms of the functions, f and g, these equations can be written in the forms

$$\frac{dp}{dt} = ap - bp^2 \text{ or } p(n+1) - p(n) = (cp(n) - dp(n)^2)dt.$$

If we ignore the p^2 term and solve for $a > 0$, it is found that the population in both cases increases without bound. For $a < 0$ the population dies away. In Figure 3, the value of $a =$ 0.003 in the increasing case while in the decreasing case $a = -0.003$, a very small change indeed. However the outcome is quite different. A small change has made a great difference in the behavior of the population level.

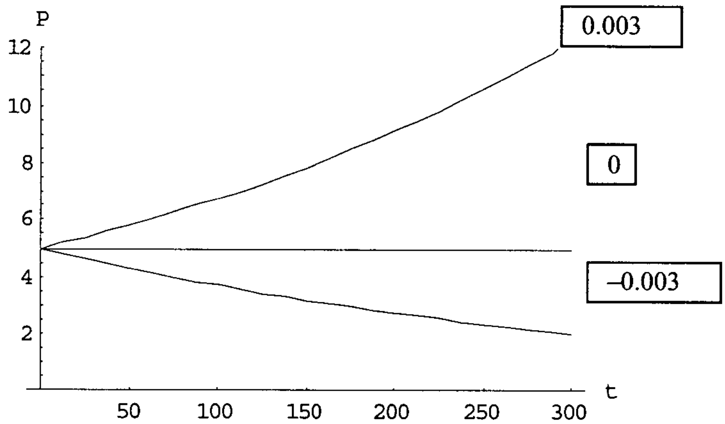

Figure 3. A small change in population growth rate has a big effect on the outcome

For the case $a > 0$, the real world tells us that an exponential growth is not sustainable so we should include the competitive factor, the p^2 term. The resulting solutions reflect the observed patterns of population growth, see Figures 3a and 3b. The graphed outcomes are almost identical.

At this stage we can be creative in asking further questions about the problem. For example, what happens to populations of birds? Their growth rate is cyclic on a yearly basis. What if there is competition with other species-consider when there are populations of birds and worms in the same environment? What can we say about spatial limitations-the species may be confined to an island?

These last questions may be too difficult for students without calculus. But we can explore the second form since it is a recurrence equation or an equation giving a sequence of numbers. With a simple change in the variables, it can be written in the form

$$P(n+1) = kP(n)(1 - P(n)),$$

where k is related to the growth rate and $P(n)$ can be regarded as a proportion of the population with $0 < P(n) < 1$. If we know $P(0)$ we can determine $P(1)$, $P(2)$, $P(3)$,....

Now let us consider what happens when we experiment by taking various values of k. The following diagrams indicate a remarkably diverse set of outcomes. It is noted that without the use of high-speed computational devices such experimentation would be virtually impossible

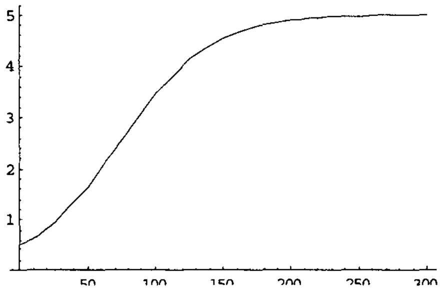

Figure 3a. Solution of differential

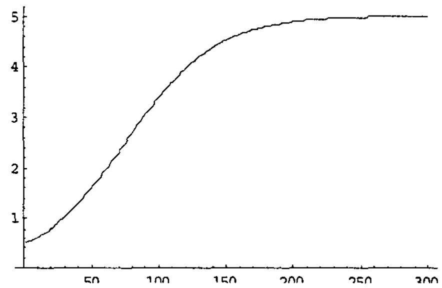

Figure 3b. Solution of recurrence

For $k < 3.1$, we find what we expect, the population settles into a steady state value which is independent of the initial value, $P(0)$ (see Figures 3c and 3d).

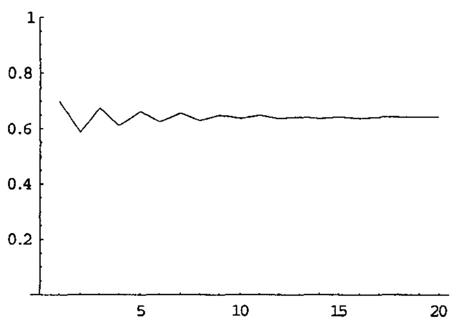

Figure 3c.

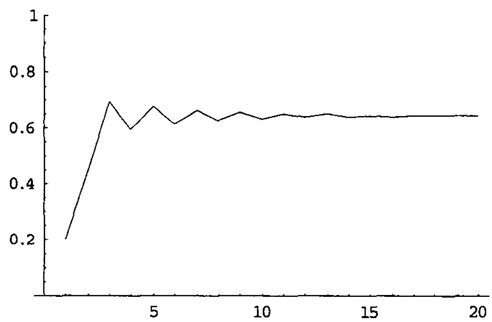

Figure 3d.

With k=2.8 steady state is reached independent of initial value.

For $k = 3.1$ there is an oscillation of the population between two levels (see Figures 3e and 3f), and for $k = 3.5$ there is an oscillation between 4 population levels (see Figure 3g). This is referred to as period doubling. For $k = 3.56$ there is a further doubling to 8 population levels (see Figure 3h). Each of these is independent of the initial value (see Figures 3e and 3f) for the 2-period case.

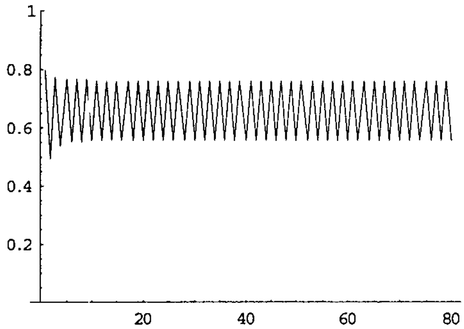

Figure 3e.

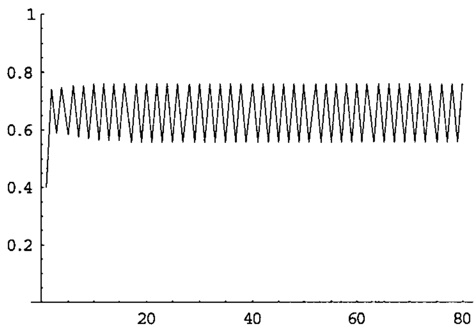

Figure 3f.

k= 3.1; 2-periodic solution independent of initial value.

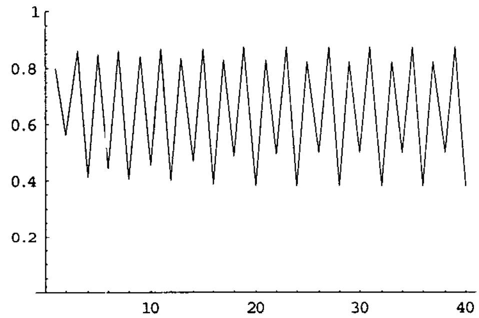

Figure 3g. *k*=3.5 Period doubling 4-period

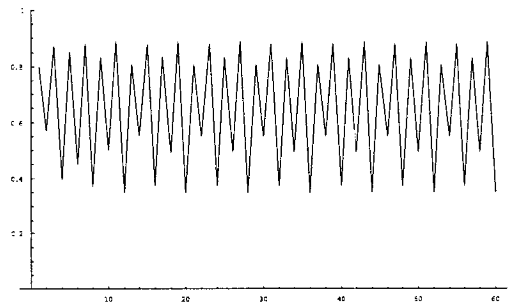

Figure 3h. *k*=3.56; 8-period

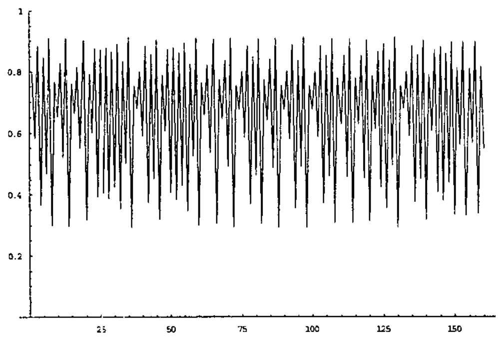

Figure 3i.

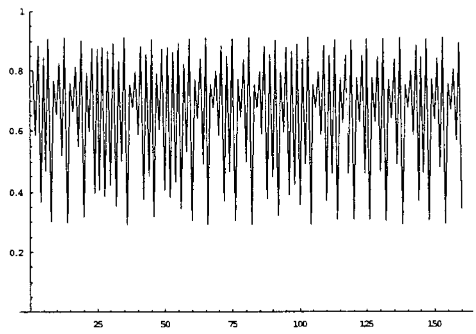

Figure 3j.

k=3.65; Chaotic outcome; Initial value changes from 0.8 to 0.80000001.

When k= 3.7 there is no longer a recognizable repetition of the pattern. The behavior is chaotic and the longer-term outcome is no longer independent of the initial value. In fact for very small changes in the initial value, from $P(0) = 0.8$ to $P(0) = 0.80000001$ the outcome is quite different (see Figures 3i and 3j).

7. CONCLUSION

Mathematics is indeed a source of creative activity for students at all levels. As the language of patterns it intrinsically provides a rich supply of experimental activity with numbers and forms for many students. For other students motivation is through applications of mathematics to areas of their own special interests like sport, art or music. Introducing students to such applications will give them confidence that mathematics is indeed useful and it is worth spending their time on experimenting judiciously or creatively exploring with the models to discover new patterns which they may be able to interpret to their own advantage or for their own satisfaction.

REFERENCES

Bolgomolny, B. (2000): What is your answer to that question? http://www.cut-the-knot.org/ctk/Magic.shtml

Buck, P. S. (1973): *The Goddess*, Part 1. New York: Pocket Books.

Chick, H. L. & Watson, J. M. (Eds.) (1994): *Mathematics and Teaching.* Adlelaide, Australia: Australian Association of Mathematics Teachers/Hobart, Tasmania, Australia: Moores Business Systems.

Hansen, D. W. (1971): The Dependence of Mathematics on Reality. *Mathematics Teacher* **64**, 715–719.

Kim, B. Y. & Lee, J. S. (2001): A study of the Development of Creativity in the Secondary Mathematics in Korea. *J. Korea Soc. Math. Ed. Ser. D Res. Math. Ed.* **5(1)**, 45–48. MATHDI **2001f.**05195

Lin, C. C. & Segal, L. A. (1994): Mathematics Applied to Deterministic Problems in the Natural Sciences. NewYork: Macmillan Pub. Co.

Logan, J. D. (1987): Applied Mathematics: A Contemporary Approach. NY: Wiley.

Steen, L. A. (Ed.) (1990): *On the shoulders of Giants — new approaches to numeracy.* Washington, DC: National Acad. Press. MATHDI **1990j.**01110