Nonlinear Parameter Identification of Partial Rotor Rub Based on Experiment

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To model and understand the physics of partial rub, a nonlinear rotor model is sought by applying a nonlinear parameter identification technique to the experimental data. The results show that the nonlinear terms of damping and stiffness should be included to model partial rotor rub. Especially, the impact and friction during the contact between rotor and stator are tried to explain with a nonlinear model on the basis of experimental data. The estimated nonlinear model shows good agreements between the numerical and the experimental results in its orbit. Also, the estimated nonlinear model could explain the backward whirling orbit and jump phenomenon, which are the typical phenomena of partial rub.

Key Words : Partial Rotor Rub, Nonlinear System Identification, Impact, Friction

Nomenclature

c_x, c_y : Rotor damping coefficients in two stationary orthogonal directions x, y
e : Eccentricity of rotor
f_N : Radial impact force
f_T : Tangential rub force
G : Center of mass of rotor
k_x, k_y : Rotor stiffness coefficients in two stationary orthogonal directions x, y
m : Mass of rotor
$x(t)$, $y(t)$: Rotor lateral displacements in two stationary orthogonal directions
δ : Radial clearance between rotor and stator
μ : Friction coefficient between rotor and stator
ω : Shaft rotational velocity

1. Introduction

The performance of rotating machinery such as motor, pump or steam turbine can be maximized by reducing the clearance between rotor and stator. However, a rotor can contact with a stator if the rotor has a large whirling motion by unbalance and misalignment, etc. The phenomenon that a spinning and precessing rotor contacts with a stator is called rubbing. Once rubbing occurs during the operation of rotating machinery, high frequency and large amplitude happen and rubbing can cause the damage of rotating machine itself. If such rubbing happens in a power plant when high power is required such as summer season, it deals with a difficult situation.

Rubbing phenomenon occurs in various ways depending on the design parameters and operation conditions of rotating machinery. Generally, rubbing phenomenon is divided into two types, full annular rub and partial rub. Full annular rub is characterized by continuous contact between rotor and the interior surface of stator during the entire whirling motion. In contrast, partial rub has the intermittent contact between rotor and stator. There are two kinds of whirling motion depending on the whirling direction of rotor ; forward whirl and backward whirl. However, the researchers involved with rubbing are mostly on full annular rub than partial rub. This is because full annular rub is easier than partial rub to

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analyze since partial rub accompanies intermittent impact and friction. However, partial rub happens more often than full annular rub during the operation of rotating machine.

Numerous investigators have studied on the rubbing of rotating machinery. However, the vibration problem involving friction and impact due to clearance and looseness results in the abrupt change of stiffness and damping values, which makes a strong nonlinear vibration problem. Ehrich (1966) and Black (1968) as the pioneers of rubbing research tried to design a jet engine optimally by investigating the physics when a rotor contacts with a stator. Choy (1989) and Lin (2001) showed that rubbing can bring complicated vibration through numerical analysis. Muszynska (1995) tried to analyze theoretically on rubbing phenomenon with an experiment in laboratory scale. Crandall (1990) and Choi (2000) did an experiment with a simple experimental apparatus and tried to find the physics of rubbing. The result showed that super- and sub-harmonic responses could be found due to rubbing. Choi (2002) did an extensive study on this rubbing problem. He tried to find the whirling motion of full annular rotor rub. In this research, partial rubbing phenomenon is revived in laboratory. And, the nonlinear parameters involving rubbing is estimated using a nonlinear parameter identification technique to make an analytical model of partial rub. Also, the estimated nonlinear analytical model is verified through the comparison with the experimental results of jump phenomenon and backward whirling, and is used for different clearance cases.

2. Mathematical Model

If partial rub happens in rotating machine, impact and friction occur due to the contact between rotor and stator. The impact makes partial deformation and rebounding, and the friction acts in the tangential direction to the whirling orbit. The impact force happens in a very short duration which results in board frequency band, and brings periodic frequency components due to recursive contacts. Also, it has the nonlinear

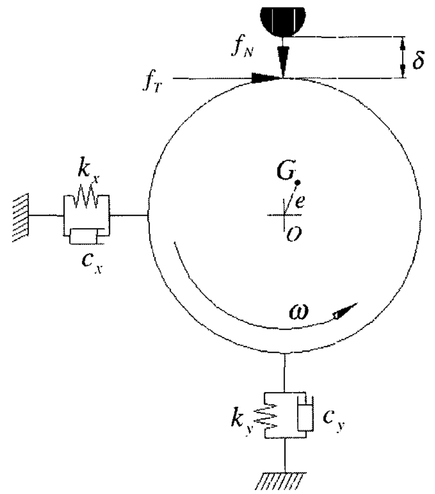


Fig. 1 Schematic of partial rubbing rotor

characteristics by energy absorption and rebound by the impact at the contact point. From the consideration on the effects of partial rub, an analytical model of Fig. 1 can be made. The differential equations of motion for partial rubbing rotor system can then be written as Eq. (1).

$$
\begin{aligned}
m\ddot{x}+c_x\dot{x}+k_x x&=f_x+me\omega^2\cos\omega t\\
m\ddot{y}+c_y\dot{y}+k_y x&=f_y+me\omega^2\sin\omega t
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
f_x&=f_T\Theta(y-\delta)\\
f_y&=-f_N\Theta(y-\delta)\\
f_T&=\mu f_N
\end{aligned}
\tag{2}
$$

where Θ is the Heaviside step function.

$$\Theta(z)=\begin{cases}0, & z\leq 0\\ 1, & z\geq 0\end{cases},\quad z=y-\delta$$

Eq. (2) indicates that when the rotor displacement y is smaller than the clearance δ between rotor and stator, there will be no rub, and f_N and f_T are zero. While the rub happens when the rotor displacement is larger than the clearance. Where x and y is the positions of the rotor geometric center. The center of mass G of the rotor is located with eccentricity e from the axis of rotation, which produces an unbalance force $me\omega^2$. f_x and f_y are the nonlinear force components in the x and y direction, respectively. Since the protrusion of the stator is on the top, f_y becomes normal force f_N, and f_x becomes friction force f_T. It is difficult to define $f_T=\mu f_N$ simply. How to find f_N and μ is the question.

In this research, a nonlinear system identification technique is applied to build an analytical model of partial rub due to impact and friction. The normal force f_N which considers the damping and stiffness during the contact should be estimated from an actual experiment data. In this respect, the rubbing phenomenon should be reproduced in laboratory.

3. Experiment of Partial Rub

In this research, the RK-4 rotor kit manufactured by Bently Nevada Co. as shown in Fig. 2 was used to demonstrate various partial rubbing phenomena. The contact between rotor and stator was accomplished by making a protrusion of brass screw bolt at the top of the stator. If the radial displacement of the rotor exceeds the given clearance between the protrusion and the shaft during the whirling motion of the rotor, the shaft becomes contact with the protrusion. Two gap sensors were used with 90 degree apart to measure the whirling orbit in two dimensional spaces. The whirling orbit can be depicted with an oscilloscope. The rotational speed of the rotor was measured by the speed probe among the attached accessories of RK-4 rotor kit. The speed probe is composed of a photo sensor and a spur gear having 60 teeth. The locations of the sensors are depicted in Fig. 2. The signals measured by the sensors were stored in a computer through analog-to-digital conversion, and analyzed using MATLAB software.

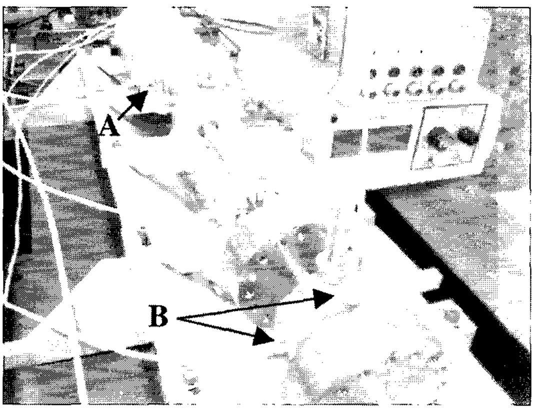


Fig. 2 Experimental apparatus, RK-4 Rotor Kit
A : speed probe B : gap sensors

Figure 3 is the whirling orbits when partial rub does not occur. The whirling orbits become the ellipse by the anisotropic characteristics of the rotor. It means that the natural frequencies of horizontal and vertical direction are different. When the amplitudes of horizontal and vertical direction of the rotor with the variation of rotational speed are observed, the orbit becomes a horizontal ellipse, and goes to vertical ellipse, and goes to a circle since the horizontal natural frequency is smaller than that of vertical one. The stiffnesses of the rotor, k_x and k_y were measured by a static deflection experiment. And the mass was calculated from the natural frequencies. The damping coefficients, c_x and c_y were calculated from the x and y response curves of the rotor. The parameters that extracted from the experiment are : $m=1.52$ kg, $e=0.0706$ mm, $c_x=15.100$ kg/sec, $c_y=14=14.727$ kg/sec, $k_x=38.521\times10^3$ N/m, $k_y=41.017\times10^3$ N/m ; the static clearance between rotor and stator $\delta=0.3$ mm.

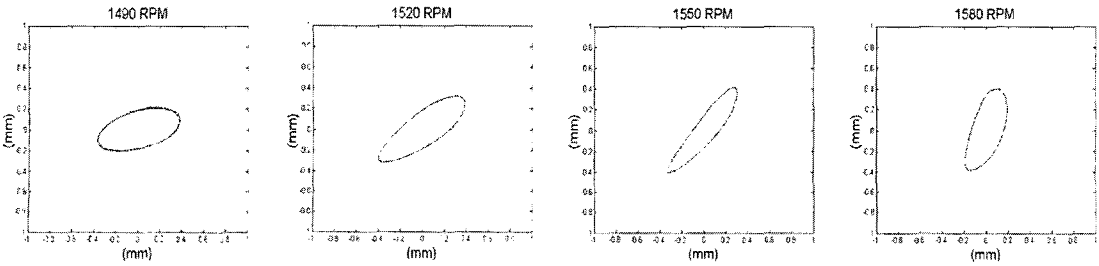


Fig. 3 Whirling orbits without rubbing

4. Nonlinear System Identification

The analytical model of partial rub of Eq. (1) reveals that $f_x = f_y = 0$ at no rub condition. However, f_x and f_y can not be known in advance since f_x and f_y are the function of displacement and velocity. f_x and f_y can be estimated as an optimal function from a postulated model structure with the experimental results. The importance of each term can be decided on the basis of its contribution to the reduction of modeling error. The error reduction ratio ERR is defined as Eq. (3). Where, X_i means the candidate terms, y means the response of the system, and n means the number of candidate term. And, the error reduction ratio defined as the ratio of each term to the entire response.

$$[ERR]_i = \frac{(X_i)^2}{y^2}, \quad 1 \le i \le n \tag{3}$$

After determining the model structure, if $1 - \sum_{i=1}^{n}[ERR]_i$ is smaller than a target error value, the optimal model set can be built. However, if ERR is larger than that, it should be calculated again with other candidate term. This procedure can make an optimal model set through repeating process which confirms whether ERR is within the error bound or not. Figure 4 is the block diagram of the model selection procedure. The parameter estimation of the each term can be carried out using the least square method. In partial rub, the normal force was assumed initially as Eq. (4). With the consideration of maximum 4th order ($L=4$) as an initial model, ERR of each term is calculated, and it brings about Table 1. By the repetition of model selection procedure as Fig. 4 until satisfying $1 - \sum_{i=1}^{n}[ERR]_i \le 0.01$, the terms with the error reduction ratio more than 1 percent are included into the final model set.

$$f_N = \sum_{n=1}^{L} k_n \cdot z^n + \sum_{n=1}^{L} c_n \dot{z}^n \tag{4}$$

$$f_N = k_1 \cdot z + k_3 \cdot z^3 + c_2 \dot{z}^2 \tag{5}$$

The nonlinear system identification is done for the case of 0.3 mm clearance in the y direction

Table 1 ERR of each term

$k_n z^n$	ERR	$c_n \dot{z}^n$	ERR
z	0.39170	$\dot{z}$	0.00217
z^2	0.00871	$\dot{z}^2$	0.02636
z^3	0.56768	$\dot{z}^3$	0.00145
z^4	0.00103	$\dot{z}^4$	0.00090

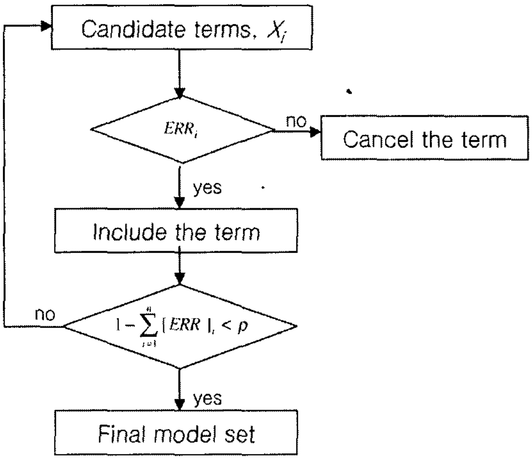


Fig. 4 Model selection procedure

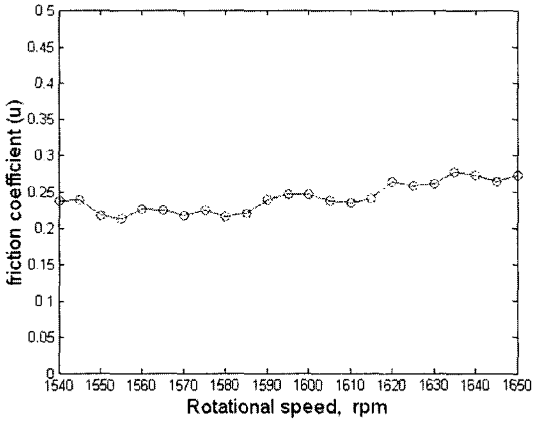


Fig. 5 The variation of the friction coefficient between rotor and stator

rotor displacements. Next, in order to know the relationship between f_N and f_T, the displacements which are measured by the experiment, are plugged into Eq. (1) and (2). And then, f_N and f_T can be calculated. And, the friction coefficient is calculated and depicted in Fig. 5. The average value of 0.24 for friction coefficient is used for the analytical model because it has a small variation as rotational speed changes. It can be said that the assumption of $f_T = \mu f_N$ is satisfied sufficiently. Finally, the model of Eq. (5) is obtained. As shown in Eq. (5) and Table 1 and 2, the first and third of term of stiffness and the second term of damping become important. The nonlinear model of Eq. (5) is consequently estimated and then the fourth-order Runge-Kutta method can be used to integrate this set of equations.

5. Contact Model

The coefficient of restitution and piecewise-linear stiffness are generally used to model the contact between parts of mechanical system like partial rub. The models adopting the coefficient of restitution and piecewise-linear stiffness are comparatively simple to explain the complex phenomenon of mechanical contact. In this research, partial rub phenomena are observed experimentally and the parameters including nonlinear terms are estimated. Here, comparing the orbits between the models of adopting the coefficient of restitution and piecewise-linear stiffness can be made.

The coefficient of restitution is a measure of the elasticity of the collision between parts. Elasticity is a measure of how much bounce there is, or in other words, how much of the kinetic energy of the colliding objects before the collision remains as kinetic energy of the objects after the collision. The analytical model adopting the coefficient of restitution basically neglects tangential force. Only the effect of normal force i.e, contact force is considered by the coefficient of restitution ; the velocity ratio before and after the contact is given as forllows :

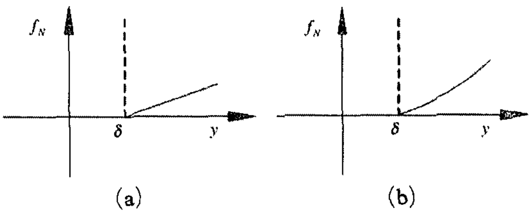
$$\dot{y}_+ = -r\dot{y}_- \tag{6}$$

The value of the coefficient of restitution r in the partial rub experiment using RK-4 rotor kit was estimated as 0.86 on the average. Using the coefficient of restitution and Runge-Kutta algorithm for Eq. (1) except f_x and f_y draws the orbits of Fig. 6, in which the numerical results are compared with the experimental observations. They show more or less good agreements.

Figure 7 illustrates the normal force change by the piecewise-linear model (a) where the nonlinear terms of Eq. (5) are omitted and the piecewise-nonlinear model (b) excluding the nonlinear damping term from Eq. (5). Fig. 8 shows the orbits adopting two piecewise-linear and nonlinear models with the comparison of experimentally observed orbits. It is hardly to find the large difference between them. However, as

Table 2 Estimated parameters

k_1	k_3	c_2
3.946×10^6 N/m	1.408×10^{11} N/m^3	3.720×10^4 kg/m



(a) (b)

Fig. 7 Contact stiffness of (a) piecewise-linear and (b) piecewise-nonlinear

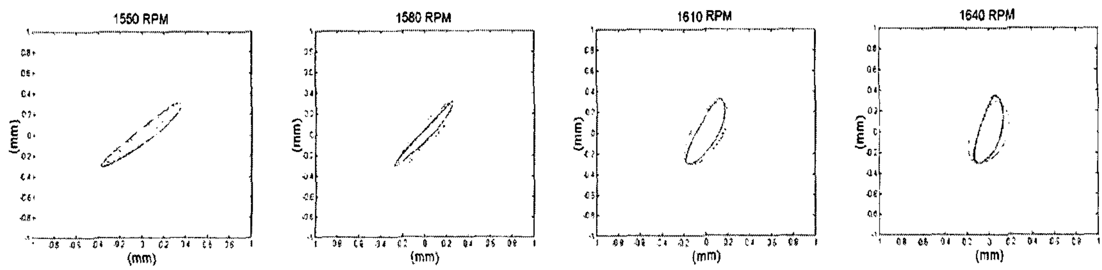


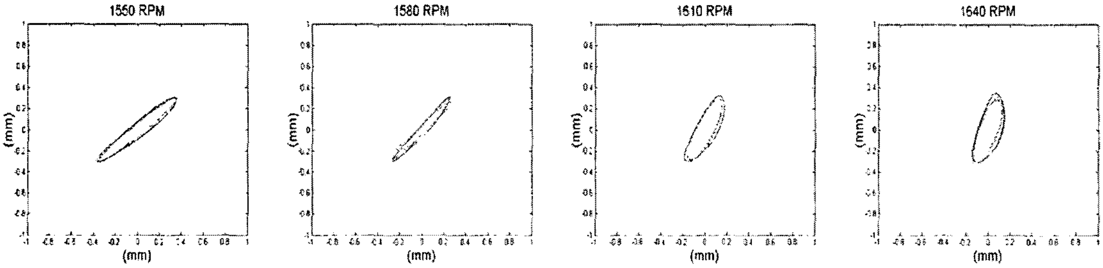
Fig. 6 The orbits by the coefficient of restitution (solid line) and the experimentally observed orbits (dotted line) for 0.3 mm clearance

the speed goes up the piecewise-nonlinear model shows better agreements between numerical and experimental orbits.

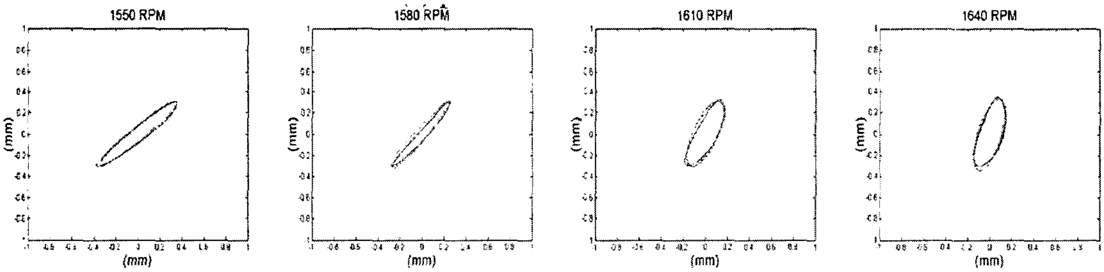
6. Verification of Nonlinear Model

When partial rub happens, the impact due to rubbing affects on the orbit and frequency components. Figure 9(a) is a waterfall diagram for the signals by numerical integration of the analytical model, which shows the occurrence of the rotational component and its harmonic components. This is confirmed by Fig. 9(b) that comes from the experiment.

When partial rub happens, the response shows jump phenomenon, one of a typical nonlinear phenomenon. Figure 10 shows the jump phenomenon in comparison with the experimental results and numerical results. This means that the estimated analytical model is good to explain the

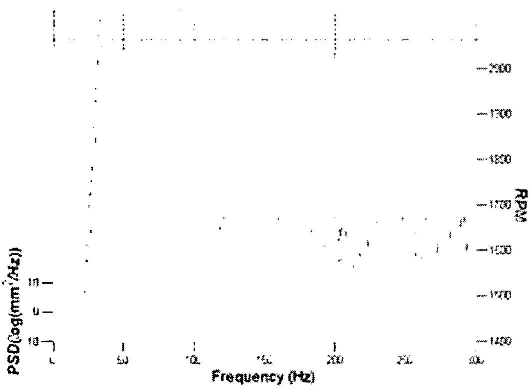


(a) Piecewise-linear model

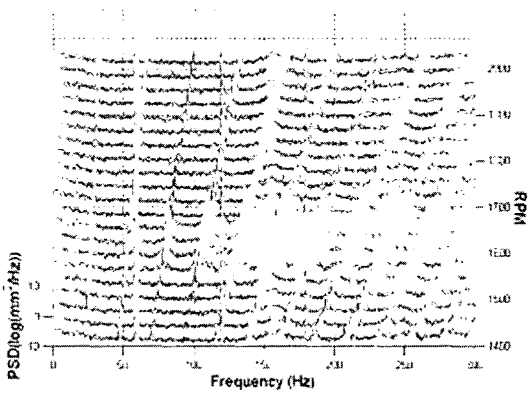


(b) Piecewise-nonlinear model

Fig. 8 Orbits by piecewise-linear and piecewise nonlinear model : experimental (solid line), numerical (dotted line)



(a)



(b)

Fig. 9 Waterfall diagrams of (a) numerical and (b) experimental result nonlinearity due to partial rub.

Rubbing phenomenon can have forward (the same direction as the rotor rotation) and backward whirl (the opposite direction to the rotor rotation) due to impact and friction. It needs to investigate the whirling direction during rubbing. For the classification of forward and backward whirling motion, the directional frequency response function (dFRF) could be used. Rotor response can be generally expressed as Eq. (7).

$$r(t) = x(t) + jy(t) = r_f e^{j\omega t} + r_b e^{-j\omega t} \qquad (7)$$

Where $x(t)$ and $y(t)$ are the real signals which are measured in an orthogonal coordinate system, $j(=\sqrt{-1})$ means the imaginary number,

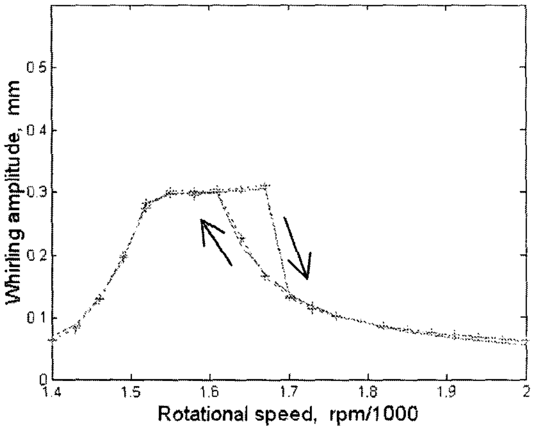


Fig. 10 Jump phenomenon, experimental (dotted line), analytical (solid line)

and the subscripts b and f denote backward and forward components, respectively. Rotor response can be expressed as Eq. (7) by the vectors of forward whirling component with radius r_f and backward whirling component with radius r_b. Figure 11(a) shows the directional spectrum for the response of numerical investigation on the estimated nonlinear analytical model. Figure 11 (b) is the directional spectrum of the experimental results obtained using two gap sensors. It confirms that the backward whirling component exceeds the forward whirling component from 1550 rpm to 1670 rpm, where the rubbing happens. The whirling motion due to partial rub changes from forward whirl to backward whirl at the resonance region. And it goes back to forward whirl again at 1700 rpm where partial rub ends.

Figure 12 is the whirling orbits for the different clearance sizes of 0.2 mm, 0.3 mm and 0.4 mm, respectively near the critical speed including from 1550 rpm to 1670 rpm, where partial rub occurs in comparison with the experimental and numerical results which is calculated from Eq. (5) with the friction coefficient μ of 0.24. Fig. 10 shows that the responses of the estimated analytical model coincide well with the experimental responses. Consequently, the nonlinear parameter identification which is used in this research can examine the nonlinear characteristic of the system, and the estimated model is a suitable model to analyze partial rub phenomenon.

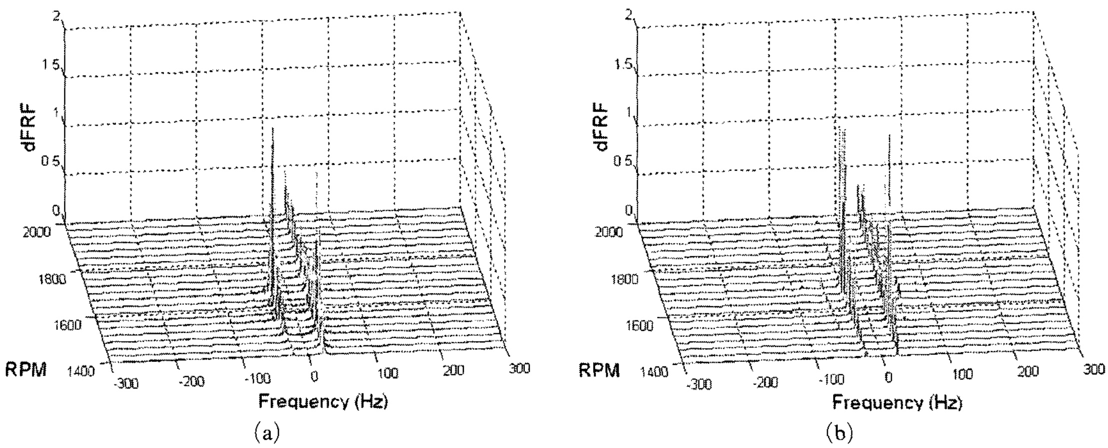


Fig. 11 Directional FRFs of (a) numerical and (b) experimental results

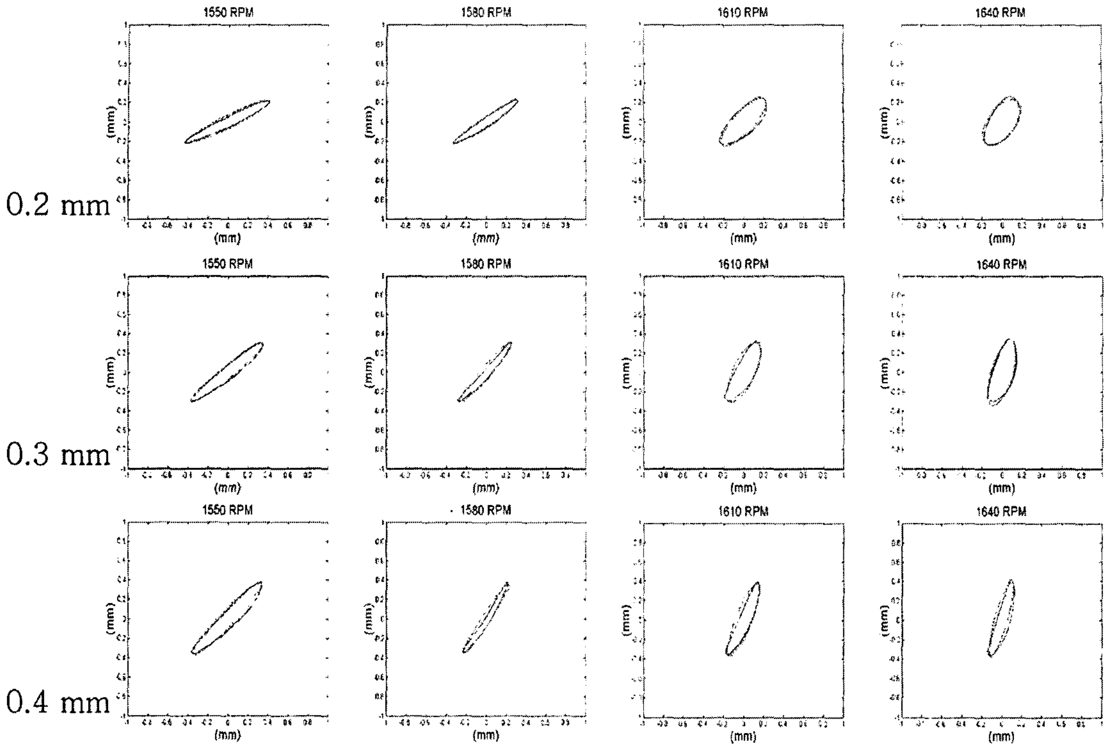


Fig. 12 Comparisons between the experimental (solid line) and numerical (dotted line) results

7. Conclusion

In rotating machines, the partial rub due to unbalance and misalignment can make an excessive vibration by the contact between rotor and stator. The impact and friction due to partial rub make the nonlinear characteristics of distorted whirling orbit and jump phenomenon, etc. Although the theoretical analysis of partial rub is not easy, partial rub can bring large amplitude and often cause a destruction of rotating machine itself. To prevent and detect the partial rub, the physics of partial rub should be investigated. In this research, a nonlinear parameter identification technique is used to estimate the nonlinear parameters involving partial rub. The estimated model includes a nonlinear stiffness component and energy loss term by impact and friction. The nonlinear model including the nonlinear stiffness and damping terms shows better agreements in the orbits when compared to experimental observations than the models adopting the coefficient of restitution or piecewise-linear stiffness. The estimated nonlinear analytical model is confirmed by numerical analysis, and compared with the experimental results. Also, the estimated nonlinear model can explain the backward whirling orbit and jump phenomenon, which were observed experimentally.

Acknowledgment

This research is financially supported by a project from Korea Science and Engineering Foundation (Grant Number R05-2003-000-11632-0).

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