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Bounds on System Reliability by Linear Programming : Recent Developments

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1. Introduction

A system, in general, is a set of possibly interconnected or interdependent components, such that the state of the system depends on the states of its constituent components. In case the failure of a structure is the system event of interest, for example, the failure of structural elements or the failure modes can be considered as its component events. The state of a system composed of a set of components in general can be expressed as a Boolean or logical function of the component states. Considering two-state components and systems, let E_i denote the event of failure of component i, $i = 1, \dots, n$, and $\overline{E}_i$ denote its complement, the survival of the component. Likewise, let E_{system} denote the event of failure of the system and $\overline{E}_{\text{system}}$ denote its survival. For three classes of systems, the relations between the component states and the system state are as follows:

$$\text{Series systems : } E_{\text{system}} = \bigcup_{i=1}^{n} E_i \qquad (1)$$

$$\text{Parallel systems : } E_{\text{system}} = \bigcap_{i=1}^{n} E_i \qquad (2)$$

$$\text{General systems : } E_{\text{system}} = \bigcup_{k=1}^{K} \bigcap_{i \in C_k} E_i \qquad (3)$$

In the case of "general," i.e., non-series and non-parallel, systems, the system failure event is defined in terms of cut sets C_k, $k = 1, \dots, K$, where each cut set is a set of components, whose joint failure constitutes failure of the system.

Once the system state is formulated as in (1)-(3), the system failure probability, or its complement, the system reliability, can be expressed as the probability of the logical function. The exact computation of this probability is often costly or unavailable due to the complexity of the system or lack of complete probability information. For these reasons, theoretical bounding formulas have been derived in terms of marginal or joint component probabilities. These include uni-component bounds for series and parallel systems(Boole 1854), and bi-component(Kounias 1968, Hunter 1976, Ditlevsen 1979) and multi-component(Hohenbichler and Rackwitz 1983, Zhang 1993) bounds for series systems. The latter bounds can also be used for parallel systems after such systems are converted to series systems by use of de Morgan's rule. No theoretical bounding formulas are available for general systems. See Song and Der Kiureghian (2003a) for a detailed review of these bounding formulas.

Recently, Song and Der Kiureghian(2003a) developed a new method for structural system reliability by use of linear programming(LP). The idea of using LP for computing bounds on system failure probability was first explored by Hailperin(1965). Specialized versions of this approach were employed in operations research (Prékopa 1988). However, it appears that this approach has never been used in the field of structural reliability or civil engineering. LP bounds are applicable to any type of system and any level of information regarding the component probabilities. Equally important, these bounds are the narrowest possible bounds that one can obtain for any specified information regarding the marginal or joint component probabilities. Although the resulting LP problem can be large for a system with many components, with the enormous increase in the speed and capacity of computers in recent years, it is believed that the LP approach is viable and a powerful tool for many system reliability problems.

After briefly describing the LP bounds methodology, this article presents two recent developments made for the approach. First, Song and Der Kiureghian(2004a) developed a method to identify the critical components and cut sets by use of LP. It is shown that one can easily compute various importance measures for components and cut sets by use of the LP approach, even when there is statistical dependence between the component failure events. Secondly, as an effort for applying the LP approach to large-scale systems even with limited computing power, an approximate method is developed to reduce the size of the LP problem by treating selected subsets of component as "super-components" (Song and Der Kiureghian 2004b). In this manner, the large LP problem is replaced with a number of smaller LP problems. The developed methodologies are demonstrated by an electrical substation example.

2. Bounds on System Reliability by Linear Programming

LP solves the problem of minimizing (maximizing) a linear function, whose variables are subject to linear equality or inequality constraints. LP gained worldwide interests when G.B. Dantzig developed the simplex method in 1947 (Dantzig 1951). Since then, encouraged by dramatic improvements in computing technology, many powerful algorithms have been developed and a profound mathematical understanding of the problem has been gained.

Among various equivalent forms of LP's, the compact formulation of LP appropriate for our analysis has the following form:

$$\text{minimize (maximize)} \quad \mathbf{c}^{T}\mathbf{p} \tag{4a}$$

$$\text{subject to} \quad \mathbf{a}_1\mathbf{p} = \mathbf{b}_1 \tag{4b}$$

$$\mathbf{a}_2\mathbf{p} \geq \mathbf{b}_2 \tag{4c}$$

In the above, $\mathbf{p} = (p_1, p_2, \cdots)$ is the column vector of "decision" or "design" variables, $\mathbf{c}^T\mathbf{p}$ with $\mathbf{c}$ a vector of coefficients is the linear "objective" or "cost" function, and $\mathbf{a}_1$, $\mathbf{b}_1$, $\mathbf{a}_2$ and $\mathbf{b}_2$ are coefficient matrices and vectors that respectively define equality and inequality constraints. In (4c), the inequality between the vectors must be interpreted component-wise. A vector $\mathbf{p}$ is called "feasible" if it satisfies all the given constraints. The solution of the LP problem is a feasible $\mathbf{p}$ that minimizes (maximizes) the objective function.

Hailperin(1965) showed that the problem of finding bounds on the probability of a Boolean function is a LP problem. He first divided the sample space of the n component events into 2^n mutually exclusive and collectively exhaustive (MECE) events, each consisting of a distinct intersection of the component events E_i and their complements $\overline{E}_i$, $i=1, \ldots, n$. Let us name these the 'basic' MECE events and denote them by e_i, $i=1, \ldots, 2^n$ As an example, Figure 1 shows the basic MECE events for the case with $n=3$. Let $p_i = P(e_i)$, $i=1, \ldots, 2^n$, denote the probabilities of the basic MECE events. These probabilities serve as the design variables in the LP problem to be formulated. According to the basic axioms of probability theory, p_i, $i=1,\ldots,2^n$, should satisfy the following linear constraints:

$$\sum_{i=1}^{2^n} p_i = 1 \quad (5a)$$

$$p_i \geq 0, \ \forall i \quad (5b)$$

The constraint (5a) is analogous to (4b) with a_1 being a row vector of 1's and $b_1 = 1$, whereas (5b) is analogous to (4c) with a_2 being an identity matrix of size 2^n and b_2 a 2^n-vector of 0's.

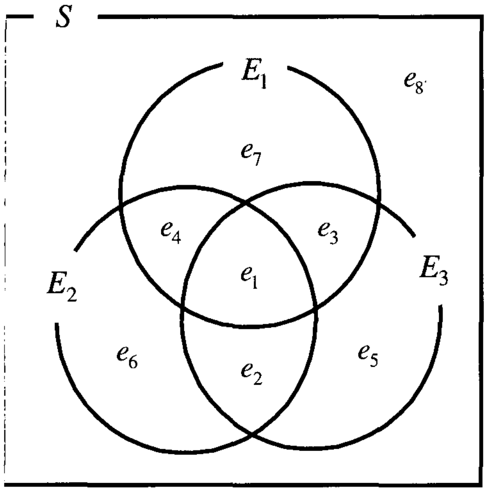

Fig 1. Basic MECE events e_i for a 3-event sample space

Due to mutual exclusivity of the basic MECE events, the probability of any subset made of these events is the sum of the corresponding probabilities. In particular, the probability of any component event E_i is the sum of the probabilities of the basic MECE events that constitute that component event. Similarly, the probability of any intersection of the component events is given as the sum of the probabilities of the basic MECE events that constitute the intersection event. Therefore, we can write

$$P(E_i) = P_i = \sum_{r: e_r \subseteq E_i} p_r \quad (6a)$$

$$P(E_iE_j) = P_{ij} = \sum_{r: e_r \subseteq E_iE_j} p_r \quad (6b)$$

$$P(E_iE_jE_k) = P_{ijk} = \sum_{r: e_r \subseteq E_iE_jE_k} p_r \quad (6c)$$

and so on.

In most system reliability problems, the uni-, bi- and sometimes tri-component probabilities (P_i, P_{ij}, and P_{ijk}) are known or can be computed. In that case, the above expressions provide linear equality constraints on the variables p in the form of (4b) with $_1$ a matrix having elements of 0 or 1 and b_1 a vector listing the known component probabilities. If, instead, inequality constraints on component probabilities are given, such as $P_i \leq 0.01$, $0.01 \leq P_{ij} \leq 0.03$ or $P_i \geq P_j$, then the above expressions provide linear inequality constraints on the variables p in the form of (4c).

Any Boolean function of the component events can also be considered as being composed of a subset of the basic MECE events. It follows that the probability of the system event E_{system} can be written in the form $P(E_{\text{system}}) = \mathbf{c}^T\mathbf{p}$, where c is a vector whose elements are either 0 or 1. The lower bound of the system probability is obtained by minimizing the objective function, and the upper bound is obtained by maximizing the same function. For a system with n component events, the number of design variables is 2^n; one equality and 2^n inequality constraints result from the probability axioms (5a) and (5b), respectively, n equality or inequality constraints result from knowledge of uni-component probabilities or their bounds as in (6a), $n!/[2!(n-2)!]$ equality or inequality constraints result from knowledge of bi-component probabilities or their bounds as in (6b), and so on. Obviously the size of the LP problem quickly grows with the number of component events.

The bounds by LP have many advantages over the existing theoretical bounding formulas. First, LP is guaranteed to provide the narrowest possible bounds, if a feasible solution exists for the given constraints (Hailperin 1965). This is not the case for the theoretical bounds for series systems based on the multi-component probabilities, even for the best ordering of the component events. (Note that the LP formulation is independent of the ordering of the component events.) Second, the LP formulation is uniformly applicable to all systems, including general systems characterized by unions and intersections of component events(and their complements). Third, the LP formulation can incorporate general forms of information about the component probabilities.

Specifically, any linear equality or inequality expression involving uni- or multi-component probabilities can be used. Finally, it is not necessary to have the complete set of probabilities for all the components at a particular, e.g., uni-, bi- or tri-component, level. Any partial set of the component probabilities can be used. Of course, incomplete information will lead to wider bounds.

The LP bounds methodology was demonstrated by application to three example structural systems(Song and Der Kiureghian 2003a): a statically determinate truss structure(series system), a bundle of ideally brittle wires(parallel system) and a structural system consisting of both brittle and ductile components (general system). Song and Der Kiureghian(2003b) demonstrated the usefulness of the LP approach also in estimating and improving the seismic reliability of an electrical substation which is a complex set of interconnected equipment items.

3. Identification of Critical Components and Cut Sets

An important objective in system reliability assessment is the identification of critical components and cut sets. These are defined as components or cut sets which make significant contributions to the system failure probability for a specified system performance criterion and load hazard. When upgrading the system reliability is an objective, the identified critical components and cut sets should be considered as prime candidates for reinforcement and strengthening on a preferential basis, especially when the system upgrade is subject to cost or other constraints.

Various importance measures(IM) have been defined for evaluating and ranking the contributions of components and cut sets to the failure probability of a system. When the component failure events are statistically independent of each other, the IM's can be easily computed by use of the marginal component failure probabilities(Henley and Kumamoto 1981; Anders 1990). However, when there is dependence between the component states, it is a daunting task to compute the probabilities required for these measures, including the system failure probability. An incomplete set of component probabilities or inequality type information on component probabilities would make the task even more difficult.

The LP approach provides a convenient framework for a systematic identification of critical components and cut sets. The proposed method allows us to easily compute various IM's for components and cut sets, even when there is statistical dependence between the component failure events. Inheriting all the advantages of the LP bounding methodology, this method is fairly flexible in gathering information such that it can incorporate incomplete sets of probabilities or inequality -type constraints.

3.1 Importance Measures by LP Bounds

Suppose the bounds on the failure probability of a system are obtained by solving the LP problem in (4) for given information on component probabilities. Let $\hat{\mathrm{p}}$ denote the solution of $\mathbf{p}$ at the upper bound of the system failure probability. The vector $\hat{\mathrm{p}}$ stores the probabilities of all the basic MECE events that contribute to the upper bound of the system failure probability. Therefore, the contribution of any event of interest to the upper bound system failure probability can be determined by simply adding the components of $\hat{\mathrm{p}}$ for the basic MECE events that are contained within the specified event. This can be done by simple algebraic manipulation of vectors and matrices. The same can be done with the solution of $\mathbf{p}$ for the lower bound of the system failure probability. In general, the set of critical components and cut sets and the corresponding IM's obtained based on the two system bounds may be different. However, as the probability information increases and the two bounds approach each other, the ordering of the critical components and cut sets and the corresponding IM's based on the two bounds also tend to approach each other. In the following, the formulas are described in general terms, where either bound can be used. This subsection introduces several well-known component and cut set

IM's and shows how they can be easily determined by use of the LP bounds formulation.

Fussell-Vesely Importance Measure

The Fussell-Vesely (FV) IM evaluates the fraction of the system failure probability, which is contributed by cut sets containing the component of interest (Fussell 1973). For component i, the FV IM is defined as

$$FV_i = P(\bigcup_{k:E_i \subseteq C_k} C_k)/P(E_{\text{system}}) \tag{7}$$

This measure quantifies the contribution of each component to the system failure probability. In the LP formulation (4), the system failure probability in the denominator is computed in terms of its lower or upper bound. With the solution $\hat{\mathrm{p}}$ available, the corresponding probability in the numerator is computed as the vector product

$$P(\bigcup_{k:E_i \subseteq C_k} C_k) \cong a_i^{FV}\,\hat{\mathrm{p}} \tag{8}$$

where a_i^{FV} is a row vector whose j-th element is 1 if the j-th basic MECE event is included in the union of the cut sets including the i-th component, and 0 otherwise. It is noteworthy that the proposed method computes the FV IM for general systems, regardless of whether statistical dependence between the component events exists or not. Furthermore, these measures can be computed for the lower and upper bounds of the system failure probability, even when the available information on component probabilities is incomplete or is in terms of inequalities.

Risk Achievement Worth

The failure of important components tends to increase the failure probability of the system. The Risk Achievement Worth (RAW) IM quantifies the importance of a component by the increase in the system failure probability when the component is removed from the system (Borgonovo and Apostolakis 2001), i.e., it is assumed to be perfectly unreliable. The RAW of the i-th component is defined as

$$RAW_i = P(E_{system}^{(i)})/P(E_{system}) \tag{9}$$

where $P(E_{system}^{(i)})$ denotes the failure probability of the system with component i removed. The system failure probability in the denominator of (9) is obtained from the original LP problem in (4). The probability in the numerator is obtained by solving a new LP problem formulated for the system with the i-th component removed. This requires changes in the original LP problem. For the details of these changes and a simpler definition of RAW IM in the case of statistically independent components, see Song(2004).

Risk Reduction Worth

The Risk Reduction Worth (RRW) IM measures the decrease in the system failure probability when the component of interest is replaced by a perfectly reliable component, i.e., a component having zero probability of failure(Vinod et al. 2003). Thus, the RRW of the i-th component is

$$RRW_i = P(E_{system})/P(E_{system}^{\overline{(i)}}) \tag{10}$$

where $P(E_{system}^{\overline{(i)}})$ denotes the failure probability of the system when the i-th component is replaced with a perfectly reliable component. This probability is placed in the denominator of (10) so that a higher value of RRW indicates higher importance of the corresponding component. For the probability in the denominator, one needs to solve another LP problem formulated for the system with the i-th component replaced with a perfectly reliable component. This is accomplished by making changes to the original LP problem. For the details of these changes and a simpler definition of RRW IM in the case of statistically independent component events, see Song(2004).

Boundary Probability

The Boundary Probability(BP) of a component measures the change in the probability of failure of the system that is solely due to a change in the state of the component (Anders 1990).

$$BP_i = P(E_{system}^{(i)}) - P(E_{system}^{\overline{(i)}}) \tag{11}$$

As shown for RAW and RRW, the probabilities $P(E_{system}^{(i)})$ and $P(E_{system}^{\overline{(i)}})$ can be obtained by solving LP problems for systems derived from the original system by removing or replacing component i.

Fussell-Vesely Cut Set Importance Measure

The critical cut sets can be identified by measuring the contributions of the individual cut sets to the system failure probability. For this purpose, the Fussell-Vesely Cut Set (FVC) IM for the k-th cut set is defined as (Fussell 1973)

$$FVC_k = P(C_k)/P(E_{system}) \tag{12}$$

The probability in the numerator is computed in terms of the LP solution $\hat{p}$:

$$P(C_k) \cong a_k^{cut}\hat{p} \tag{13}$$

where a_k^{cut} is a row vector whose j-th element is 1 if the j-th basic MECE event is included in the cut set C_k, and 0 otherwise.

3.2 Application to Electrical Substation Systems

As a numerical example, consider the two-transmission-line substation system described in Song and Der Kiureghian(2003b) and Figure 2. The failure events of the i-th equipment item is formulated as $E_i = \{\ln R_i - \ln A - \ln S_i \le 0\}$, where R_i denotes the capacity of the item, A is the bed-rock peak ground acceleration and S_i denotes a factor representing the local site response. For the probabilistic information of the random variables, see Song and Der Kiureghian(2003b). The connectivity failure of the system is a general system event composed of 12 components and having 25 minimum cut sets:

$$\begin{aligned} E_{system} = {} & E_1E_2 \cup E_4E_5 \cup E_4E_7 \cup E_4E_9 \cup E_5E_6 \cup E_6E_7 \cup E_6E_9 \cup E_5E_8 \cup \\ & E_7E_8 \cup E_8E_9 \cup E_{11}E_{12} \cup E_1E_3E_5 \cup E_1E_3E_7 \cup E_1E_3E_9 \cup E_2E_3E_4 \cup \\ & E_2E_3E_6 \cup E_2E_3E_8 \cup E_4E_{10}E_{12} \cup E_6E_{10}E_{12} \cup E_8E_{10}E_{12} \cup E_5E_{10}E_{11} \cup \\ & E_7E_{10}E_{11} \cup E_9E_{10}E_{11} \cup E_1E_3E_{10}E_{12} \cup E_2E_3E_{10}E_{11} \end{aligned} \tag{14}$$

Due to the correlation between the equipment capacities within the same category and the presence of a common random variable A in the limit-state functions of all components, significant statistical dependence between the component failure events is present. Under the uni-, bi- and tri-component probability constraints, the upper LP bound is estimated as 0.0942. In the following, IM's are computed with respect to this bound.

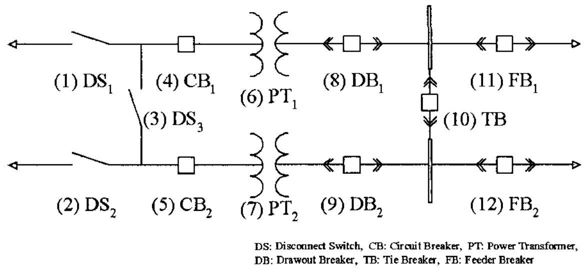

Fig 2. Two-transmission-line substation system

Simple post-processing of the upper-bound LP solution yields the vectors $\hat{p}$ and a_i^{FV}. The FV IM is obtained by substituting these results together with the upper bound probability estimate(0.0942) into (7) with (8). For the probabilities $P(E_{system}^{(i)})$ and $P(E_{system}^{\overline{(i)}})$, a total of $2 \times 12 = 24$ additional LP problems are solved according to the rules described in Song(2004). Substituting these probabilities and the upper bound LP solution in (9), (10) and (11), the RAW, RRW and BP IM's are, respectively, obtained for each component. Table 1 lists the various IM's of the components in the substation system. According to the FV, RRW and BP measures, the importance ranking of the components is in the order (CB1,2)→(PT1,2)→(DB1,2)→(DS1,2)→(DS3)→(FB1,2,TB). The ranking according to RAW is (CB1,2, PT1,2, DB1,2)→(DS1,2)→(DS3)→(FB1,2,TB), which is identical to the order by the other measures, except that CB, PT and DB have the same order of

Table 1. Component importance measures (maximum IM's are highlighted)

I	FV_i	RAW_i	RRW_i	BP_i
1 (DS1)	0.183	1.18	1.11	0.0264
2 (DS2)	0.183	1.18	1.11	0.0264
3 (DS3)	0.168	1.09	1.06	0.0142
4 (CB1)	0.601	1.44	1.57	0.0760
5 (CB2)	0.601	1.44	1.57	0.0760
6 (PT1)	0.283	1.44	1.18	0.0565
7 (PT2)	0.283	1.44	1.18	0.0565
8 (DB1)	0.280	1.44	1.15	0.0539
9 (DB2)	0.280	1.44	1.15	0.0539
10 (TB)	6.12×10^{-4}	1.00	1.00	3.90×10^{-5}
11 (FB1)	8.19×10^{-4}	1.00	1.00	5.20×10^{-5}
12 (FB2)	8.19×10^{-4}	1.00	1.00	5.20×10^{-5}

importance. As a result, one can say that the two circuit breakers (CB1,2) are the most critical com ponents in the system, followed by the other equipment items as listed above. To identify the critical minimum cut sets, the FVC IM is computed by (12) with (13) using the upper bound LP solution. The cut sets are sorted in the descending order of FVC and the first 12 cut sets are listed in Table 2. The joint failure of the two circuit breakers is the most critical cut set. The joint failures of the drawout breaker (DB) and circuit breaker on different lines are ranked next. It is noteworthy that the top five most critical cut sets all include at least one circuit breaker. This further reinforces the importance of these components for the system.

Table 2. Fussell-Vesely cut set importance measures

Order	Cut Set	FVC_i
1	(4, 5)	0.463
2	(5, 8)	0.200
3	(4, 9)	0.200
4	(5, 6)	0.179
5	(4, 7)	0.179
6	(6, 7)	0.155
7	(1, 2)	0.138
8	(8, 9)	0.138
9	(2, 3, 4)	0.0976
10	(1, 3, 5)	0.0976
11	(7, 8)	0.0951
12	(6, 9)	0.0951

4. Reliability of Large-Scale Systems by use of "Super-Components"

Suppose there is only limited computing power and advanced LP algorithms for large-scale systems such as the column generation method(Hansen and Jaumard 1996) are not available. In this case, the LP bounds approach can be applied to large-scale systems by treating selected subsets of components as "super-components." First, using the LP bounds approach, one computes bounds on the probability of each super-component as well as bounds on the joint probabilities of pairs, triples, etc., of the super-components. If there are individual components that are not included in the super-components, one must also compute bounds on the joint probabilities of each such individual component and all selected super-components. The probability bounds on the entire system are then computed by LP in terms of the computed probability bounds on the super-components and the given information on the individual components. The LP problems for the failure event of the system (represented by super-components) and each super-component have significantly reduced number of decision variables. Therefore, in essence, the large LP problem is decomposed into a number of smaller LP sub-problems. Of course, some information in the process of decomposition is lost, as a result of which the computed LP bounds are usually wider than the bounds computed if the original large LP problem is solved. At the cost of a wider probability bound, this approach enables one to solve the reliability problem for large-scale systems even with limited computing power.

As an example, let us consider again the two-transmission-line substation system shown in Figure 2. Since the system event is defined by $n=12$ components, the corresponding LP has $2^{12}=4096$ decision variables. Based on 12 equality constraints for the uni-component probabilities and 66 equality constraints for the bi-component probabilities, the bounds on the system failure probability are estimated as 0.0436 and 0.146. As shown in (14), the connectivity failure of the system is a general system event composed of 12 components and having 25 minimum cut sets.

As illustrated in Figure 3, let us now represent the assembly of CB, PT and DB in each transmission line with a super-component, denoted with bracketed numbers. The new system event E'_{system} has only 8 components: 1, 2, 3, [1], [2], 10, 11, and 12. The failure event of the new system is defined by 9 minimum cut sets:

$$E'_{\text{system}} = E_1E_2 \cup E_1E_3E_{[2]} \cup E_1E_3E_{10}E_{12} \cup E_2E_3E_{[1]} \cup E_2E_3E_{10}E_{11} \cup E_{[1]}E_{[2]} \cup E_{[1]}E_{10}E_{12} \cup E_{[2]}E_{10}E_{11} \cup E_{11}E_{12} \quad (15)$$

The number of the decision variables for the corresponding LP is significantly reduced to $2^8 = 256$ Among 36 equality constraints (8 uni-component probabilities and 28 bi-component probabilities) required for the bi-component bounds, 21 constraints on the original components, i.e., {1, 2, 3, 10, 11, 12}, are already available. The 15 remaining probabilities are

$$\begin{aligned}
&P_{[1]} = P(E_4 \cup E_6 \cup E_8),\ P_{[2]} = P(E_5 \cup E_7 \cup E_9),\\
&P_{1[1]} = P(E_1E_4 \cup E_1E_6 \cup E_1E_8),\ P_{1[2]} = P(E_1E_5 \cup E_1E_7 \cup E_1E_9),\\
&P_{2[1]} = P(E_2E_4 \cup E_2E_6 \cup E_2E_8),\ P_{2[2]} = P(E_2E_5 \cup E_2E_7 \cup E_2E_9),\\
&P_{3[1]} = P(E_3E_4 \cup E_3E_6 \cup E_3E_8),\ P_{3[2]} = P(E_3E_5 \cup E_3E_7 \cup E_3E_9),\\
&P_{10[1]} = P(E_{10}E_4 \cup E_{10}E_6 \cup E_{10}E_8),\ P_{10[2]} = P(E_{10}E_5 \cup E_{10}E_7 \cup E_{10}E_9),\\
&P_{11[1]} = P(E_{11}E_4 \cup E_{11}E_6 \cup E_{11}E_8),\ P_{11[2]} = P(E_{11}E_5 \cup E_{11}E_7 \cup E_{11}E_9),\\
&P_{12[1]} = P(E_{12}E_4 \cup E_{12}E_6 \cup E_{12}E_8),\ P_{12[2]} = P(E_{12}E_5 \cup E_{12}E_7 \cup E_{12}E_9),\\
&P_{[1][2]} = P(E_4E_5 \cup E_5E_6 \cup E_5E_8 \cup E_4E_7 \cup E_6E_7 \cup E_7E_8 \cup E_4E_9 \cup E_6E_9 \cup E_8E_9)
\end{aligned} \quad (16)$$

The inequality constraints on each of the above probabilities are available by solving a LP based on the constraints on the original components. By this approach, therefore, we solve one LP with 8 components(for the entire system), two LP's with 3 components, 12 LP's with 4 components, and one LP with 6 components. The resulting bounds on the system failure probability are 0.0436 and 0.147. It is seen that the proposed approach leads to slightly relaxed bounds computed with several smaller LP problems (of size equal to or smaller than $2^8 = 256$), instead of one large LP problem (of size $2^{12} = 4{,}096$).

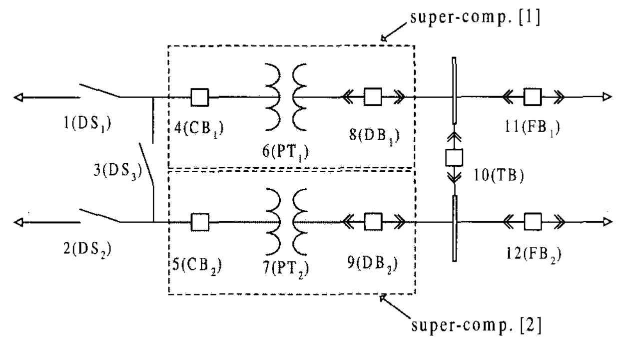

Fig 3. A two-transmission-line substation system and selected super-components

When a system has a hierarchical structure consisting of many typical sub-systems, e.g., a transmission network with many local substations, one may wish to estimate the marginal or joint failure probabilities of the sub-systems as well as the reliability of the entire system. The proposed method is particularly useful in such cases, because the probabilities of the sub-systems represented as super-components are immediately available from the solution of the intermediate LP problems. Moreover, the proposed approach simplifies the derivation of the failure event of a complex system by decomposing the event, as can be seen by comparing (14) and (15)-(16). The proposed method was successfully applied to series and general systems with a total of 44 components in Song and Der Kiureghian (2004b).

References

Anders, G.J. (1990), *Probability Concepts in Electric Power Systems*, Wiley, New York.

Boole, G. (1854), *Laws of Thought*, Amer. Reprint of 1854 ed., Dover, New York.

Borgonovo, E. and G.E. Apostolakis (2001), "A New Importance Measure for Risk-informed Decision Making," *Reliability Engineering and System Safety*, 72: 193-212.

Dantzig, G.B. (1951), "Application of the Simplex Method to a Transportation Problem," *Activity Analysis of Production and Allocation*, T.C. Koopmans (ed.): 359-373.

Ditlevsen, O. (1979), "Narrow Reliability Bounds for Structural Systems," *Journal of Structural Mechanics*, 7(4): 453-472.

Fussell, B.J. (1973), "How to Hand-calculate System

Reliability Characteristics," *IEEE Trans. on Reliability*, 24(**3**).

Hailperin, T. (1965), "Best Possible Inequalities for the Probability of a Logical Function of Events," *American Mathematical Monthly*, 72(**4**), 343-359.

Hansen, P. and B. Jaumard (1996), *Probabilistic Satisfiability*, Les Cahiers du GERAD G-96-31, Montréal, Canada.

Henley, E.J. and H. Kumamoto (1981), *Reliability Engineering and Risk Assessment*, Prentice-Hall, Inc., Englewood Cliffs.

Hohenbichler, M. and R. Rackwitz (1983), "First-order Concepts in System Reliability," *Structural Safety*, 1(**3**). 177-188.

Hunter, D. (1976), "An Upper Bound for the Probability of a Union," *Journal of Applied Probability*, 13, 597-603.

Kounias, E.G. (1968), "Bounds for the Probability of a Union, with Applications," *Annals of Mathematical Statistic*, 39(**6**): 2154-2158.

Prékopa, A. (1988), "Boole-Bonferroni Inequalities and Linear Programming," *Operations Research*, 36(**1**). 145-162.

Song, J. (2004), *Seismic Response and Reliability of Electrical Substation Equipment and Systems*, Ph.D. dissertation, Berkeley, Calif.: University of California.

Song, J., and A. Der Kiureghian (2003a), "Bounds on System Reliability by Linear Programming," *Journal of Engineering Mechanics*, ASCE, 129(**6**): 627-636.

Song, J., and A. Der Kiureghian (2003b), "Bounds on System Reliability by Linear Programming and Applications to Electrical Substations," *Proc., 9th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP)*, San Francisco, July 6-9.

Song, J. and A. Der Kiureghian (2004a), "Identification of Critical Components and Cutsets Using Linear Programming Bounds on System Reliability," *ASCE - The 9th Joint Special Conference on Probabilistic Mechanics and Structural Reliability (PMC04)*, Albuquerque, NM, July 26-28.

Song, J. and A. Der Kiureghian (2004b), "Bounds on System Reliability by Linear Programming: Application to Large-Scale Systems," *The 3rd Asian-Pacific Symposium on Structural Reliability and its Applications (APSSRA'04)*, Seoul, Korea, August 18-20.

Vinod, G., H.S. Kushwaha, A.K. Verma, and A. Srividya (2003), "Importance Measures in Ranking Piping Components for Risk Informed In-service Inspection," *Reliability Engineering and System Safety*, 80: 107-113.

Zhang, Y.C. (1993), "Higher-order Reliability Bounds for Series Systems and Application to Structural Systems," *Computers and Structures*, 46(**2**): 381-386.