A Uniform Formulation for Scattering by Very Thin Dielectric Strips for TM Wave Incidence

TM 파가 입사할 경우 얇은 유전체 Strip의 산란에 대한 Uniform 해

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Abstract

In this paper a new approximate formulation for bistatic scattering by a thin dielectric strip for TM wave incidence is obtained. This formulation is uniform, which is valid for any incident angles, observation points, and any properties of the strip such as dielectric constants, width, and thickness. The accuracy of the proposed solution is examined by comparisons with a method of moments(MoM) for various situations.

요 약

본 논문에서 TM 파가 입사할 때 얇은 유전체 strip의 산란특성에 대한 새로운 근사식을 구했다. 구한 식은 uniform하다. 즉 이 식은 어떠한 입사각 또는 관측점, 유전률, strip의 두께 등 모든 경우에 사용할 수 있고 다른 수치해석 방법, MoM을 이용하여 구한 식의 정확도를 여러 경우에 관해 검사해 보았다.

Key words : Scattering, Dielectric Strip, Diffraction

Ⅰ. Introduction

Scattering from thin dielectric sheets is encountered in many practical situations. To simplify the problem, a thin dielectric structure is usually modeled with a resistive sheet[1]. Among the canonical problems, scattering by a resistive strip has been intensively studied by a number of researchers. Many approximate solutions have been proposed for the problem, which all are based on the known exact solution for a resistive half plane[1]. Uniform solutions valid at the transition regions between the shadow and reflection boundaries for the half-plane is needed to include higher order multiple interactions between two edges of a finite strip. One such method is the Extended Spectral Ray Method (ESRM)[2], which can be applied to a general multiple scattering problems with some analytical complexity. This method takes account for multiple scattering in a spectral domain. For example, at one edge diffraction effect is calculated with a non-uniform formulation, and the calculated diffracted field is considered as an incidence field on the other edge in a spectral domain. Hence a resulting double diffracted field can be represented in terms of Fourier transform. Using a steepest descent method(SDM), a uniform solution can be obtained. Other method is self-consistent GTD formulation[3]. In the formulation, multiply scattered surface waves between two edges are simply estimated by

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a similar approach to the generalized S-matrix method. The method is simple, but it is known that the method is not valid at transition regions. As explained, all existing methods directly use the formulation for the half-plane, and hence the resulting solutions contain a higher transcendental function known as Maliuzhinets function.

Recently an approximate solution for a very thin dielectric object with any size and shape was proposed [4]. The solution is represented in terms of a spectral integral whose integrand contains only elementary functions. Based on this formal solution, a uniform solution for bistatic scattering by a very thin dielectric strip is formulated for a TM wave incidence. Through comparisons of results calculated by a numerical method such as MoM, the new formulation is verified for several cases. In Section 2, the approximate solution is formulated, and Section 3, the validity of the proposed solution is investigated. For this paper, $e^{-i\omega t}$ is assumed.

Ⅱ. Formulation

A thin dielectric strip is located in a xy plane, and its normal is z-axis as seen in Fig. 1. The dielectric constant, width, and thickness of the strip are ε_r, w, and t, respectively. When a TM wave is incident on the thin dielectric strip, the scattered field in the far-field region is represented as $\vec{E}^s \sim \hat{y}\sqrt{\frac{2}{\pi k_0 \rho}}e^{i(k_0\rho-\pi/4)}P_e(\theta_i,\theta_s)$. Here $P_e(\cdot)$ is known as the far-field amplitude[1], and can be written in terms of a single integral as $P_e(\cdot)=\frac{1}{\pi\eta}I$. Recently an approximate expression of I is formulated as [4]

$$I=\int_{-\infty}^{\infty}dk_x\frac{k_z}{k_z-\alpha k_0}\frac{\sin\left(\frac{k_x^i-k_x}{2}k_0w\right)\sin\left(\frac{k_x^s-k_x}{2}k_0w\right)}{(k_x^i-k_x)(k_x^s-k_x)}$$
$$=-\frac{I_1}{2}\cos\left(\frac{k_x^i-k_x^s}{2}k_0w\right)+\frac{I_2}{4}e^{-ik_0(k_x^i-k_x^s)w/2}+\frac{I_3}{4}e^{ik_0(k_x^i-k_x^s)w/2} \tag{1}$$

where $k_z=\sqrt{1-k_x^2}$, and $\alpha=\frac{i}{2}t(\varepsilon_r-1)$. I_1, I_2, and I_3 are written as

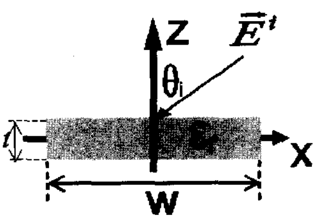


Fig. 1. A thin dielectric strip.

$$I_1=\int_{-\infty}^{\infty}dk_x\frac{k_z}{k_z-\alpha k_0}\frac{1}{\left(k_x^i-k_x\right)\left(k_x-k_x^s\right)},$$

$$I_2 \text{ or } I_3=\int_{-\infty}^{\infty}dk_x\frac{k_z}{k_z-\alpha k_0}\frac{e^{\pm ik_0k_xw}}{\left(k_x^i-k_x\right)\left(k_x-k_x^s\right)}$$

Here "+" is chosen for I_2, and "−" for I_3. By substituting $k_x=-k_x$ in I_3, it can be easily shown that $I_2=I_3$. In (1) I_1 is a new approximate diffraction coefficient of a resistive half plane. I_2 and I_3 are interpreted as diffracted waves at the two edges which propagate in the direction to the opposite edge in a spectral domain. Therefore the first term in (1) represents singly diffracted waves at the two edges, and the rest terms are for multiply diffracted waves between two edges.

When I_1 is carried out analytically, one difficulty is an evaluation of $\int_{-\pi/2}^{\pi/2}\frac{d\theta}{\sin\theta-a}$ with $|a|<1$, and a real. Since the integrand contains a pole in the integral range, the integral should be defined in a sense of Cauchy principle-value. Using $z=\tan\frac{\theta}{2}$, the integral is decomposed into two integrals:

$$\int_{-\pi/2}^{\pi/2}\frac{d\theta}{\sin\theta-a}=-\frac{2}{a}\frac{1}{a_1-a_2}\left[\int_{-1}^{1}\frac{dz}{z-a_1}+\int_{-1}^{1}\frac{dz}{z-a_2}\right]$$

where $a_{1,2}=\frac{1}{a}\mp\sqrt{\frac{1}{a^2}-1}$ $\begin{cases}- & \text{for } a_1\\ + & \text{for } a_2\end{cases}$. If the first integrand in the above integral contains a pole($a>0$), the integral can be performed as

$$\int_{-1}^{1}\frac{dz}{z-a_1}=\int_{-1}^{a_1}\frac{dz}{z-a_1}+\int_{a_1}^{1}\frac{dz}{z-a_1}=\ln(-1-a_1)+\ln(1-a_1)+\pi i=\ln\frac{1-a_1}{1+a_1}$$

Since the other integrand is analytic, the integral is simply evaluated as $\int_{-1}^{1}\frac{dz}{z-a_2}=\ln\frac{1-a_2}{1+a_2}$. Therefore

$$\int_{-\pi/2}^{\pi/2}\frac{d\theta}{\sin\theta-a}=-\frac{2}{a}\frac{1}{a_1-a_2}\left[\ln\frac{1-a_1}{1+a_1}-\ln\frac{1-a_2}{1+a_2}+\pi i\right]=0$$

For $a<0$, is can be simply shown that the above result holds. After lengthy algebraic manipulation with a help of the above equation, I_1 can be evaluated analytically and given by

$$k_0 I_1=-\frac{4i\eta}{\sqrt{1-\eta^2}}\tanh^{-1}\sqrt{\frac{1-\eta}{1+\eta}}+f_1 \tag{2}$$

f_1 is expressed as

$$f_1=-\eta^2\frac{f_2(k_x^i)-f_2(k_x^s)}{k_x^s-k_x^i}$$

$$f_2(k_x^i)=\frac{g_1 k_x^i k_z^{i2}+\dfrac{2ik_z^i}{\eta}\sin^{-1}k_x^i}{1-\eta^2 k_z^{i2}}$$

where $g_1=\frac{2i\eta}{\sqrt{\eta^2-1}}\left[\cos^{-1}\eta-2\tan^{-1}\frac{\sqrt{\eta^2-1}}{i\eta+1}\right]$, $\alpha k_0=-\frac{1}{\eta}$, and $k_z^{i2}=k_0^2-k_x^{i2}$.

However, I_2 cannot be carried out analytically, and so an asymptotic technique such as steepest descent method(SDM) can be used to obtain an analytical formulation. Since the integrand in I_2 has four poles at $k_x=k_x^i$, k_x^s, and $\pm\sqrt{1-\alpha k_0}$, first, the integrand can be transformed into a more conventional form for applying the SDM approximation:

$$I_2=\frac{\eta}{k_0}\int_{-\infty}^{\infty}dk_x\left[\frac{1}{k_z}-\frac{1}{k_z-\alpha k_0}\right]e^{ik_0k_xw}$$
$$+\frac{k_0}{k_x^s-k_x^i}\int_{-\infty}^{\infty}dk_x\left[\frac{1}{k_z}-\frac{1}{k_z-\alpha k_0}\right]\left[\frac{k_z^{i2}}{k_x-k_x^i}-\frac{k_z^{s2}}{k_x-k_x^s}\right]e^{ik_0k_xw}$$

Taking a partial expansion of the integrand in the second integral again, each integrands in the resulting integrand contain only one pole. Hence by using the standard SDM procedure[5], the pole contribution can be more easily taken account into to obtain a uniform solution. The first term of the above integral is a contribution for edge-on incidence, and written as

$$I_4=\frac{\eta}{k_0}\int_{-\infty}^{\infty}dk_x\left[\frac{1}{k_z}-\frac{1}{k_z-\alpha k_0}\right]e^{ik_0k_xw}$$

In this paper, an asymptotic procedure for the first integral is given. For the rest portions of the integral can be evaluated asymptotically using the same procedure. The first part in the above integral represents Hankel function of the first kind of order 1. Hence only the second term should be asymptotically evaluated, and is denoted as I_2^e. Using $k_x=k_0\sin\theta$, I_2^e is written as

$$I_2^e=\int_\Gamma\frac{\cos\theta}{\cos\theta-\alpha k_0}e^{ik_0w\sin\theta}d\theta$$

where Γ is a SDM path[5]. By another substitution, $\sin\theta=1+is^2$,

$$I_2^e=e^{ik_0w}\int_{-\infty}^{\infty}e^{-k_0ws^2}\frac{\cos\theta}{\cos\theta-\alpha k_0}\frac{2i}{\sqrt{s^2-2i}}ds$$

To remove the pole in the above integral, the integrand is decomposed into two parts: one contains the pole, and the other is regular as

$$\frac{\cos\theta}{\cos\theta-\alpha k_0}\frac{2i}{\sqrt{s^2-2i}}=\frac{b}{s-s_1}+T(s)$$

The residue b can be calculated as $b=\lim_{s\to s_1}(s-s_1)\frac{\cos\theta}{\cos\theta-\alpha k_0}\frac{2i}{\sqrt{s^2-2i}}=\frac{1}{\sqrt{\eta^2-1}}$. Therefore the final integral becomes

$$\int_{-\infty}^{\infty}\left[\frac{b}{s-s_1}+T(s)\right]e^{-k_0ws^2}ds=b\int_{-\infty}^{\infty}\frac{e^{-k_0ws^2}}{s-s_1}ds+\int_{-\infty}^{\infty}T(s)e^{-k_0ws^2}ds$$

The first term of the above integral can be calculated using a known identity $w(z)=\frac{i}{\pi}\int_{-\infty}^{\infty}\frac{e^{-t^2}}{z-t}dt=e^{-z^2}erfc(-iz)$ [6]. Here $erfc(\cdot)$ is the complementary error function, which is related to the GTD transition function[1]. Since the second term doesn't contain any pole, the first order approximation of the integral is sufficient. Finally I_4 is expressed as

$$I_4\sim\pi H_0^{(1)}(k_0w)+e^{ik_0w}\left[T_0\sqrt{\frac{\pi}{k_0w}}+i\pi bw(\sqrt{k_0w}s_1)\right]$$

where $T_0=T(0)=b/s_1$ and $s_1=\sqrt{i(1-\sqrt{1-1/\eta^2})}$. Following the same procedure, each integrals in I_2 can be carried out asymptotically as

$$I_2 \sim \eta\left[I_4 + \frac{e^{ik_0 w}}{k_x^i - k_x^s}\{f_3(k_x^i) - f_3(k_x^s)\}\right] \tag{3}$$

Here f_3 is given by

$$f_3\left(k_x^i\right) = \frac{\eta k_z^{i2}}{1-\eta^2 k_z^{i2}}\left[\eta k_z^{i2}\left\{T_1^i\sqrt{\frac{\pi}{k_0 w}} + i\pi b^i w\left(\sqrt{k_0 w}\, s_1\right)\right\}\right.$$
$$\left. + P_i - T_2^i\sqrt{\frac{\pi}{k_0 w}} + 2i\pi k_z^i w\left(\sqrt{k_0 w}\, s_2^i\right)\right]$$

where $b^i = -1 - \frac{\eta k_x^i}{\sqrt{\eta^2 - 1}}$, $T_1^i = \eta(1-i)(1+k_x^i) + b^i / s_1$, $T_2^i = \sqrt{2}(1+k_x^i) - 2e^{-i\pi/4}\sqrt{1+k_x^i}$, $P_i = i\pi k_z^i(\eta k_z^i - 1)e^{ik_0 k_x^i w}$, and $s_2^i = \sqrt{i(1-k_x^i)}$.

For forward scattering direction($k_x^i = k_x^s$), the expressions for I_1 and I_2 have a removable pole. Therefore for this case we need to take limit of I_1 and I_2 as $k_x^i \rightarrow k_x^s$. For I_1, only f_1 should be modified as

$$f_1 = -\frac{\eta^2}{1-\eta^2 k_z^{i2}}\left[f_1' + g_1(2k_x^{i2} - k_z^{i2}) + \frac{2i}{\eta}\left(\frac{k_x^i}{k_z^i}\sin^{-1} k_x^i - 1\right)\right]$$

where

$$f_1' = \frac{2k_x^i \eta^2}{1-\eta^2 k_z^{i2}}\left(g_1 k_x^i k_z^{i2} + \frac{2ik_z^i}{\eta}\sin^{-1} k_x^i\right)$$

I_2 is more complicated, and using L'Hospital rule, the final result is given by

$$I_2 = \frac{\eta}{1-\eta^2 k_z^{i2}}\left[I_4 - i\pi e^{ik_0 k_x^i w}\left\{k_x^i(2\eta - \frac{1}{k_z^i}) - ik_0 k_z^i w(k_z^i \eta - 1)\right\}\right.$$
$$- e^{ik_0 w}\left\{e^{-i\pi/4} - \left(\sqrt{2} - \frac{1}{\sqrt{1+k_x^i}}\right)\cdot\sqrt{\frac{\pi}{k_0 w}} + 2\pi i\left(\frac{k_x^i}{k_z^i}\, w(\sqrt{k_0 w}\, s_2^i)\right.\right.$$
$$\left.\left.\left. + e^{j\pi/4}\sqrt{k_0 w}\sqrt{1+k_x^i}\left(-\sqrt{k_0 w}\, s_2^i w(\sqrt{k_0 w}\, s_2^i) + \frac{1}{\sqrt{\pi}}\right)\right)\right\}\right]$$
$$- \frac{2\eta^3 k_x^i e^{ik_0 w}}{(1-\eta^2 k_z^{i2})^2}\left[\eta k_z^{i2}\left\{T^i\sqrt{\frac{\pi}{k_0 w}} + i\pi b^i w(\sqrt{k_0 w}\, s_1)\right\} + P_i' - (\sqrt{2}(1+k_x^i)\right.$$
$$\left. - 2e^{-j\pi/4}\sqrt{1+k_x^i}\sqrt{\frac{\pi}{k_0 w}} + i2\pi k_z^i w(\sqrt{k_0 w}\, s_2^i)\right]$$
$$- \frac{2\eta^2 k_x^i e^{ik_0 w}}{1-\eta^2 k_z^{i2}}\left[T^i\sqrt{\frac{\pi}{k_0 w}} + i\pi b^i w(\sqrt{k_0 w}\, s_1)\right]$$

where $P_i' = i\pi e^{ik_0 k_x^i w}(\eta k_z^i - 1)$. For the case of $k_x^i = k_x^s$ and $|k_x^i| = 1$(forward scattering when edge-on incidence), the above expression contains several divergent terms such as $\frac{1}{\sqrt{1+k_x^i}}$, but these terms cancel each other. Therefore by rearranging the divergent terms and using the first-order asymptotic expansion of $w(\cdot)$ function[6], (1) is modified slightly into

$$I \sim -\frac{I_1}{2} + \frac{I_2'}{4}e^{-ik_0 k_x^i w} + \frac{I_3'}{4}e^{ik_0 k_x^i w} + 4\eta\left[e^{i\pi/4}\sqrt{\frac{\pi k_0 w}{2}} - i\right] \tag{4}$$

In I_1, f_1 is simplified again into $f_1 = -2\eta^2(g_1 - \frac{i}{\eta})$. I_2' and I_3' are given by

$$I_2' \text{ or } I_3' = I_4 - i2\pi\eta e^{ik_0 k_x^i w} - \pi H_0^{(1)}(k_0 w)$$
$$+ (1 \pm k_x^i)e^{ik_0 w}\left[\frac{1}{2\sqrt{2}} + \sqrt{\pi k_0 w}\,(1+i)\right]$$

Here "+" is chosen for I_2' and " − " for I_3'.

Ⅲ. Numerical Results

The first example is a calculation of scatterings in forward and back directions from a thin dielectric strip with thickness 0.025 λ_0 and ε_r = 4+i0.4. Fig. 2 and 3 show comparisons of normalized radar echo width of the strip as a function of w/λ_0, which are calculated by the proposed solution given in Section 2 and MoM. Excellent agreement is observed for the two cases. In Fig. 3, contributions of single and multiply diffracted fields are also plotted. For even very small strips, the new formulation generates very accurate results. For the

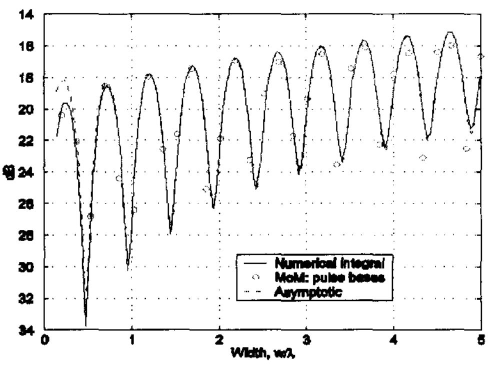
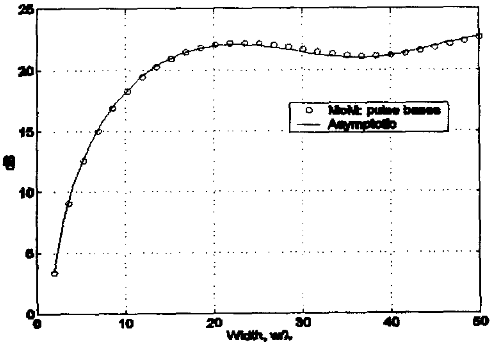
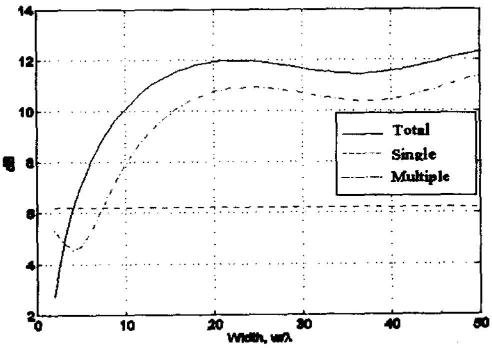


Fig. 2. Backscattering by a thin strip for edge-on incidence as a function of width. The dielectric constant and thickness of the strip are *4 + i0.4* and 0.025 λ_0, respectively.



(a) Comparison of results calculated by the new formulation and MoM



(b) Single diffraction and multiple diffraction contributions

Fig. 3. Forward scattering by a thin strip for edge-on incidence as a function of width. The dielectric constant and thickness of the strip are $4+i0.4$ and $0.025\ \lambda_0$, respectively.

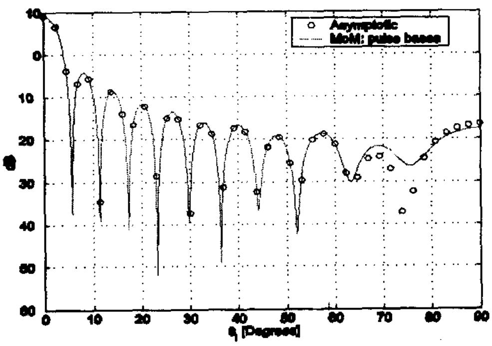


Fig. 4. Backscattering as a function of incidence angles by a thin dielectric strip, whose width is $5\lambda_0$, dielectric constant is $4+i0.4$, and thickness $0.025\ \lambda_0$.

rest of calculations presented here the width of the strip is fixed to be $5\lambda_0$. Fig. 4 is a plot of backscattering as a function of incidence angles. Except at some angles around θ_i=75°, the new uniform solution provides very accurate results. Fig. 5 and 6 are plots of bistatic scattering by the strip with the same dielectric constant and thickness as the previous case, for θ_i=30° and θ_i=90° (edge-on incidence), respectively. As shown in these figures, the asymptotic solution produces very accurate results, but some discrepancy is observed at angles around 75° for edge-on incidence. The final example is an investigation of effect of dielectric constant. Fig. 7 shows a comparison of echo width as a function of the real part of dielectric constant with a fixed imaginary part of 0.4. Fig. 8 is the echo width as a function of the imaginary part of the dielectric

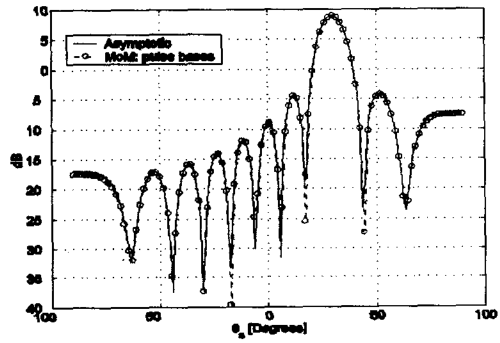


Fig. 5. Bistatic scattering as a function of observation angles for θ_i=30° by a thin dielectric strip, whose width is $5\lambda_0$, dielectric constant is $4+i0.4$, and thickness $0.025\ \lambda_0$.

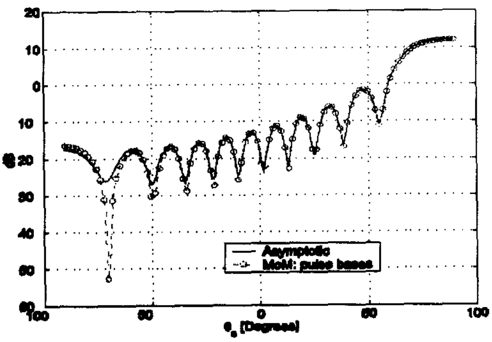


Fig. 6. Bistatic scattering as a function of observation angles for θ_i=90° by a thin dielectric strip, whose width is $5\lambda_0$, dielectric constant is $4+i0.4$, and thickness $0.025\ \lambda_0$.

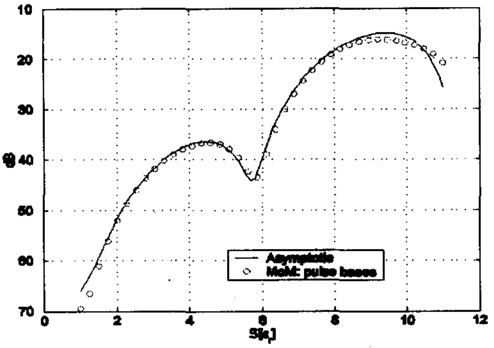


Fig. 7. Forward scattering by a thin dielectric strip as a function of real part of dielectric constant with a fixed imaginary part of 0.4. Wave is incident at 30°, and the strip's width, and thickness are $5\lambda_0$, $0.025\lambda_0$, respectively.

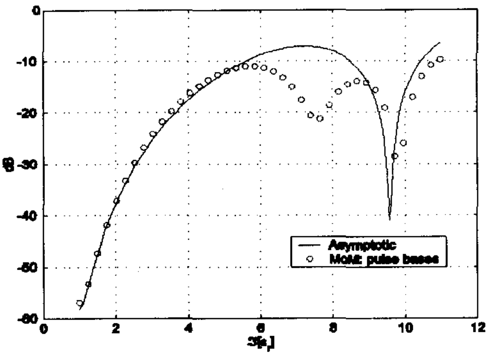


Fig. 9. Backscattering by a thin dielectric strip as a function of real part of dielectric constant with a fixed imaginary part of 0.4. Wave is incident at 90°, and the width, and thickness of the strip are $5\lambda_0$, $0.025\lambda_0$, respectively.

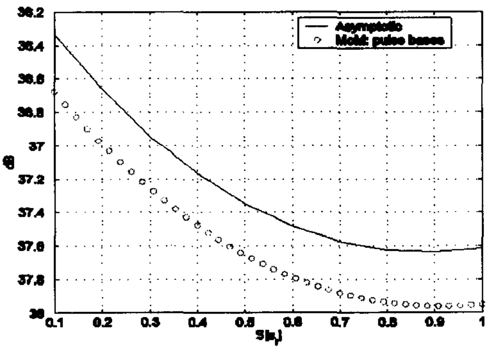


Fig. 8. Forward scattering by a thin dielectric strip as a function of imaginary part of dielectric constant with a fixed real part of 20. Wave is incident at 30°, and the strip's width, and thickness are $5\lambda_0$, $0.025\lambda_0$, respectively.

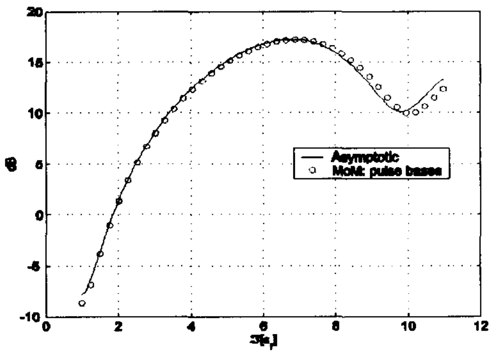


Fig. 10. Forward scattering by a thin dielectric strip as a function of real part of dielectric constant with a fixed imaginary part of 0.4. Wave is incident at 90°, and the width, and thickness of the strip are $5\lambda_0$, $0.025\lambda_0$, respectively.

constant with a fixed real part of 20. For these simulations the incidence angle(θ_i) is fixed to be 30° and the observation point(θ_s)(forward direction). The asymptotic solution is again in excellent agreement with MoM. For the next two figures, edge-on incidence is considered, and the echo width is plotted in backscattering and forward scattering directions as a function of real part of the dielectric constant with a fixed imaginary part of 0.4. Fig. 9 shows some discrepancy, but Fig. 10 shows excellent agreement. Since a pulse basis MoM is used, at high dielectric constants some error is observed in Fig. 9 and 10, and hence the error can be reduced with increasing number of cells. From the examinations, it is observed that when an incident wave or/and observation point are at near-grazing angles and the observation point is in the backward direction, there are some discrepancies between the exact and approximate results. The error may be caused by the fact that the original formulation misses some surface wave components. However the accuracy of the formulation is overall very high, and hence this new uniform solution can be used for any incidence and observation angles without a big error.

Ⅳ. Conclusions

In this paper, a new uniform formulation of scattering by a very thin homogeneous dielectric strip for a TM wave incidence is obtained. It contains only elementary functions such as trigonometric functions and a transcendental function known as GTD transition function. The solution is verified by comparisons with MoM in various situations. It is observed that the new solution is very accurate, but sometimes the accuracy of the formulation is degenerated when the incidence angle or/and the observation point are at near-grazing angles. Since the original formulation is approximate, the error may be caused by missing some surface components. Hence it is also observed that the degeneration of the accuracy is dependant of dielectric constant, width, and thickness of the strip.

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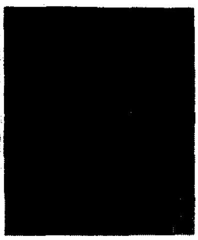

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