International Journal of Reliability and Applications
Vol. 5, No. 2, pp. 59-74, 2004

Robust Reliability Analysis of Vibration Components

Hong-Zhong Huang *
Department of Mechanical Engineering
University of Alberta, Edmonton, Alberta, T6G 2G8, Canada
School of Mechanical Engineering
Dalian University of Technology, Dalian, Liaoning, 116023, P. R. China

Yong-Hua Li
School of Mechanical Engineering
Dalian University of Technology, Dalian, Liaoning, 116023, P. R. China

Ming J. Zuo
Department of Mechanical Engineering
University of Alberta, Edmonton, Alberta, T6G 2G8, Canada

Abstract. There are many uncertain parameters associated with vibration components. Their physical parameters, the machining quality of vibration components, and the applied load acting on them are all uncertain. As a result, the natural frequency and the fatigue limits are also uncertain variables. In this paper, we express these parameters of vibration components and the frequency zone of resonance through interval models; this way, the robust reliability of the vibration components is defined. The robust reliability model measures and assesses the reliability of vibration components. The robust reliability of a cantilever beam is evaluated as an example. The results show that this method is reasonable for robust reliability analysis of vibration components because it does not require a large amount of failure data, it avoids the evaluation of the probability density function, and the computation is simple.

Key Words : *Vibration component, Resonance, Fatigue limits, Interval models, Robust reliability.*

1. INTRODUCTION

* Corresponding Author
E-mail address: hzhhuang@dlut.edu.cn

Vibrational components have two main failure modes, namely, resonance failure and strength failure. As a result, reliability analysis of such components should consider these two aspects of failures. Reported methods of reliability analysis includes the use of probability theory and/or fuzzy sets[1,2]. If these methods are used for strength reliability analysis of vibrational components, a large amount of failure data is needed in order to establish the probability density function. Such data are often not available in reality, especially for large-scale expensive structural systems. Engineers have used data from "similar components" to approximate the behavior of the component under study to alleviate the problem of limited data availability. This leads to questionable reliability estimates, particularly for high reliability levels that are associated with the tails of the density functions.

Many scholars have made attempts to develop new methods that do not require the availability of a large amount of failure data. Ben-Haim[3~6] developed the concept and the theoretical system of robust reliability based on convex models. Convex models are proposed to describe uncertainty when there is limited data on the uncertain factors. A convex model is a convex set of uncertain functions. Each element of the set represents a hypothetical value for the uncertain quantity under investigation. Reliability is then measured by the maximum uncertainty which the system or structure under study can tolerate. A system is reliable if its performance fluctuation is small, its ability to resist interference is strong, or it can tolerate large uncertainties without failure. In other words, a system is reliable if it is robust with respect to load uncertainties. On the other hand, a system is unreliable if it can fail due to even small uncertainties. The robust reliability of a system is then defined as the greatest uncertainty that it can tolerate without failure. The new concept of robust reliability is useful for solving engineering problems that involve severe degrees of uncertainty and limited data availability. With this method, we can determine the limits of uncertain parameters even though we are unable to determine the distributive properties due to data unavailability. In a sense, this method does not have the deficiencies of the traditional probabilistic reliability. It is a rigorous quantitative alternative to the probabilistic theory of reliability when a limited amount of data is available.

The idea of robust reliability has proven to be useful in analysis of mechanical systems subject to severe lack of information[5]. In this paper we will show that the same idea is applicable to reliability analysis of vibrational components. The non-probabilistic robust reliability theory is applied to reliability analysis of vibrational components. Interval convex models are used to describe uncertainties in the parameters and resonance frequency zones of vibrational components. The definition of robust reliability of vibrational components is provided. Finally, this theory is applied to the robust reliability analysis of a cantilever beam and a turbine dynamic blade. The results show that this method provides reasonable measures of the robust reliability of vibrational components. This method avoids the use of probability density functions. Because the requirement for data is lower, fewer test data are needed and the computation is simpler to perform.

2. THE ROBUST RELIABILITY THEORY AND METHOD

Robust reliability is derived from convex models of uncertainties rather than classical probabilistic models of uncertainties. The robust reliability of a system is defined as a measure of its resistance in performance to uncertainties in loads, operating conditions, material properties, and/or geometrical properties. The system is highly reliable if it is able to tolerate a great deal of uncertainty without failure. Conversely, a system is of low reliability if it is vulnerable to uncertainty. In other words, the system is not reliable even if a small degree of uncertainty may entail the possibility of failure. Reliability is in this sense a measure of the robustness of the system to uncertainty. Like probabilistic reliability analysis, robust reliability analysis includes three models: a mechanical model, a failure criterion, and an uncertainty model. By combining these three aspects, we can determine the robust reliability of the system.

Now, we illustrate the method of determining robust reliability. Consider a spring with one degree of freedom driven by an uncertain input, $f(t)$. Suppose that the stiffness and the mass of the spring are constant. The dynamic equation of the spring can be expressed as:

$$m\ddot{x} + kx = f(t) \quad , \quad x(0) = \dot{x}(0) = 0 . \tag{1}$$

The uncertain input, $f(t)$, is described by a uniform-bound convex model centered at the origin

$$F(\alpha) = \{f(t) \ : \ |f(t)| \le \alpha\} \quad , \quad \alpha \ge 0 . \tag{2}$$

The uncertainty parameter of the input is α. The spring's response to input $f(t)$ is given by

$$x(t) = \frac{1}{m\omega} \int_0^t f(t) \sin \omega (t - \tau) d\tau , \tag{3}$$

where $\omega^2 = k/m$.

When $f(t) = \pm\alpha$, the response reaches its maximum, that is

$$\max x(t) = \frac{\alpha}{m\omega} \int_0^t |\sin \omega\tau| d\tau . \tag{4}$$

We assume that there exists a critical value, x_c, such that the spring is considered working if the response does not exceed this critical value for all t such that $t \le T$ (T is a specified time duration), namely

$$|x| \le x_c . \tag{5}$$

According to this scenario, the spring is reliable when the uncertainty parameter, α, satisfies the following expression:

$$\frac{\alpha}{m\omega}\int_0^t |\sin\omega\tau| d\tau \le x_c. \tag{6}$$

If the time period T is very large, it can be expressed as

$$T = \frac{n\pi}{\omega} + \delta \quad , \quad n \ge 1 \quad , \quad \delta < \frac{\pi}{\omega}. \tag{7}$$

Then

$$\int_0^T |\sin\omega\tau| d\tau \approx \frac{2n}{\omega}. \tag{8}$$

Therefore

$$\max x(T) \approx \frac{2n\alpha}{m\omega^2} \approx \frac{2T\alpha}{\pi m\omega}. \tag{9}$$

According to (5) or (6), the uncertainty parameter, α, must satisfy

$$\frac{2T\alpha}{\pi m\omega} < x_c. \tag{10}$$

The greatest value of the uncertainty parameter α which this spring can tolerate and not fail is

$$\hat{\alpha} = \frac{\pi x_c \sqrt{km}}{2T}. \tag{11}$$

The magnitude of the uncertainty parameter ($\hat{\alpha}$) reflects the fluctuating range of the input, $f(t)$, within the time duration of T, that the spring can tolerate. If the input, $f(t)$, is less than $\hat{\alpha}$, this spring is reliable. The larger the value of $\hat{\alpha}$ is, the more reliable this spring is. The value of $\hat{\alpha}$ is the measure of the robust reliability.

2.1 The mathematic base of the robust reliability—Convex models

A convex model of uncertainty is a set of functions or vectors specified by such global characteristics as loads, spectral properties, or functions of bounded energy. In effect, a convex model constrains uncertainty within known bounds. Each function or vector represents a possible realization of the uncertain phenomenon under consideration.

Many different convex models have been proposed[5]. One selects a suitable convex model according to some prior information that characterizes the uncertainty. The following lists some commonly used convex models[5]:

(a) The energy-bound convex model

$$U(\alpha) = \left\{ u(t) : \int_0^L u^T(t) W u(t) dt \leq \alpha^2 \right\}, \tag{12}$$

where W is a positive definite real symmetric matrix that controls the shape of an ellipsoid and α determines the size of the ellipsoid and the degree of uncertainty.

(b) The uniform-bound model

$$G(\alpha) = \{ g(x) : g_1(x) \leq g(x) \leq g_2(x) \}, \tag{13}$$

where $g(x)$ is an unknown function, $g_1(x)$ and $g_2(x)$ are, respectively, the upper and lower bounds of the interval.

(c) The Fourier-bound model

$$C(\alpha) = \left\{ c : (c - \bar{c})^T W (c - \bar{c}) \leq \alpha^2 \right\}, \tag{14}$$

where $\bar{c}$ is a nominal Fourier-coefficient vector and W is a positive definite real symmetric matrix.

(d) The slope bound model

$$L = \left\{ l(t) \quad : \quad l(0) = l_1 \quad , \quad l(T) = l_2 \quad , \quad \frac{dl}{dt} \geq 0 \right\} \tag{15}$$

for use where the time interval is $[0, T]$.

The first advantage of using convex models is that they require less information than the traditional probabilistic models. A convex model is far less information-intensive than a probabilistic model because no information on the frequency of occurrences of possible outcomes is required. The problem of insufficiency of event records is avoided. Of course, this implies that the results of a convex-model analysis are unable to provide a probabilistic distribution description of the uncertain events; nevertheless, useful insight is usually gained through the analysis. This makes the method attractive. The second advantage of convex models is that they are able to provide very reliable analytic results for some important structures. If one uses the traditional probabilistic analysis approach, due to limited availability of statistical data and lack of experience of some designers, a significant amount of conservatism often results from the analysis. However, convex model analysis can lead to highly reasonable design decisions since convex models describe the uncertainty of many events very thoroughly. It is clear that convex models provide an alternative way of analyzing uncertainty when a limited amount of information is available.

2.2 The relation between robust reliability and probabilistic reliability

Both approaches of the probabilistic reliability and the robust reliability deal with the problem of uncertainty. They have direct design implications. The design variables control the performance of the system to be designed. In the probabilistic reliability approach, the design must produce an acceptable level of the probability of failure. In the robust reliability approach, the design decisions must guarantee that the system's performance is within an acceptable range.

The traditional probabilistic reliability approach emphasizes the probability of acceptable behaviors, while the robust reliability approach puts stress on the scope of these acceptable behaviors. Probabilistically, a system is reliable if the probability of unacceptable behavior is sufficiently low. For robust reliability, a system is reliable if the range of performance fluctuations is acceptably small. From the modeling point of view, the convex models are structurally different from the probabilistic models. The probabilistic approach to modeling of uncertainty begins with a space of events and a probability measure on that space. The space is all-inclusive. It includes everything that could occur, and possibly events that will never occur. The probability measure contains all information concerning the relative frequency of different events. The convex model approach to modeling of uncertainty is different. A space of conceivable events is defined as in the probabilistic approach, but no probability measure is defined; instead, sets of allowed events are specified. This removes the need to assume density functions, whose tails may or may not be those of the true density function. The structure of these sets is chosen to reflect available information on what events can and cannot occur. It is remarkable, and of considerable practical significance, that sets whose elements represent spatial or temporal uncertainty are often found to be convex. The probabilistic theory of reliability quantifies this intuition by measuring reliability as the probability that failure will not occur. Probabilistically, a system is reliable if the probability of failure is small. The robust reliability theory quantifies reliability as the amount of uncertainty the system can tolerate before failure. In the convex modeling approach, a system is reliable if large uncertainties are consistent with failure-free operation. These quantifications are different, though both relate satisfactory performance to degree of uncertainty. The probabilistic theory is usually based more on information regarding the likelihood of event-sets while the convex modeling theory is based on information about the clustering of uncertain events. Consequently, the probabilistic reliability is informationally richer if data is available, establishing the degree of likelihood of no-failure, while the convex-modeling reliability only establishes the amount of uncertainty consistent with no-failure. On the other hand, the probabilistic analysis of reliability is vulnerable to substantial error if there are even very small inaccuracies in the description of rare events by the tails of the probability distribution. Another comparison of the probabilistic and the convex-modeling reliabilities concerns normalization. The probability of no-failure is normalized, so that we have a clear intuitive understanding of the meaning of values near zero or near unity, even without reference to any particular system. The uncertainty parameter of a convex model is non-negative and unbounded, but we have no clear intuitive a priori interpretation of any particular numerical value for these quantities.

3. ROBUST RELIABILITY MODELS OF VIBRATIONAL COMPONENTS

Material parameters, geometric parameters, and the machining quality of vibrational components contain uncertainties. As a result, the maximum stress and fatigue limit of vibrational components are uncertain. In addition, variations in vibrational properties of the components such as their natural frequencies, applied vibrational load and its frequency, and vibrational damping properties cause additional uncertainties. If the data on these uncertain parameters is limited, but the variation limits of these uncertain parameters are given or can be determined relatively easily, the non-probabilistic convex models should be adopted to describe these uncertainties for robust reliability analysis of vibrational components. In this study, these uncertain parameters are represented by interval convex models (also called uniform-bound models) for determination of the robust reliability index. The set of uncertain factors, U , which has n elements, is expressed as

$$U = \{u_1, u_2, \Lambda, u_n\}. \tag{16}$$

According to the theory of non-probabilistic convex models proposed by Ben-Haim[3,5], the variation ranges of uncertain but bounded parameters can be described by the interval convex models, $U(\alpha)$, namely

$$U(\alpha) = \{u_j(x) : u_j^l(x) \le u_j(x) \le u_j^u(x)\}, \quad (j = 1,2,\Lambda, n), \tag{17}$$

where $u_j(x)$ $(j = 1,2,\Lambda, n)$ are normal values of the unknown parameters and $u_j^l(x)$ and $u_j^u(x)$ are, respectively, the lower and upper limits of the uncertain parameters that can be tolerated.

Other properties of the interval convex model for uncertain but bounded parameters can be obtained with the following transformations:

$$u_j^c = \frac{u_j^u + u_j^l}{2}, \quad u_j^r = \frac{u_j^u - u_j^l}{2}, \tag{18}$$

$$u_j^u = u_j^c + u_j^r, \quad u_j^l = u_j^c - u_j^r, \tag{19}$$

where u_j^c and u_j^r are the mean and the deviation of the uncertain parameters, respectively. If u_j^r is small, the variation range of the uncertain parameter is small.

As a result, the interval variable of an arbitrary uncertain parameter, u_j, may be expressed as

$$u_j = u_j^c + \alpha_j u_j^r, \quad |\alpha_j| \le 1, \tag{20}$$

where α_j is a standardized interval variable with uncertainty, belonging to the convex zone of $F_\delta = \{\delta : |\delta_i| \le 1, i = 1,2,\Lambda\ ,n\}$. The values of this interval variable will directly influence the magnitude of the parameter u_j.

Geometrically, u_j varies within the hyper-polyhedron R_X of a hyperspace. The bounded convex zone is a two-dimensional rectangular zone with respect to two uncertain parameters. If there are three uncertain parameters, the bounded convex zone is a three-dimensional box. This definition may be expanded to higher dimensions. If there are N uncertain parameters, the bounded convex zone will be an N-dimensional polyhedron. If the uncertain parameters are described through these interval models, the design results of these parameters will be definite forms of interval variables. Thus, the operation rule of interval variables may be used in searching for an optimal design.

The operation of interval variables is linear in nature[7]. Assume that the set of all real value intervals are IR; $X_i \in IR$, $x_i \in X_i$ $(i = 1,2,\Lambda\ ,N)$ are arbitrary uncorrelated interval variables; and k is an arbitrary real number. The mean Y^c and the deviation Y^r of the interval variable $y = \sum_{i=1}^{N} k_i x_i$ are given by:

$$Y^c = \sum_{i-1}^{N} k_i X_i^c \tag{21}$$

$$Y^r = \sum_{i-1}^{N} |k_i| X_i^r \tag{22}$$

where X_i^c and X_i^r are respectively the mean and the deviation of interval variable x_i.

Using the failure criteria of a component, we can express the state function of the component as

$$Y = f(x) = f(x_1, x_2, \Lambda\ , x_n), \tag{23}$$

where the elements of the vector $x = \{x_1, x_2, \Lambda, x_n\}$ represent the types of uncertain factors affecting the performance of this component such as load state, properties of the material, environmental factors, geometric size, and so on. In the conventional reliability theory, this state function expresses the state that the component may fall in. The hyper-surface, $f(x) = 0$, is defined as the failure surface, which separates the basic parameter space of the structure design into two parts, that is, the failure zone $\Omega_f = \{x : f(x) < 0, x \in R\}$ and the safe zone $\Omega_s = \{x : f(x) > 0, x \in R\}$. Variables that are within each of the above-mentioned spaces respectively express either the failure state or the safe state of the component under consideration. If $f(*)$ is a continuous function with respect to x_i $(i = 1,2,\Lambda, n)$, Y is also an interval variable based on the properties of convex sets. According to the non-probabilistic reliability theory of interval analysis[8], the minimum distance from the origin of coordinates to the failure surface is determined by the non-probabilistic reliability index in the expanded space of standardized interval variables. The robust reliability index with respect to the state function of equation (23) may be defined as

$$\eta = \min(\|\alpha\|_\infty) = \min(\max\{|\alpha_1|, |\alpha_2|, \Lambda, |\alpha_n|\}), \tag{24}$$

satisfying the following condition

$$Y = f(x_1, x_2, \Lambda, x_n) = F(\alpha_1, \alpha_2, \Lambda, \alpha_n) = 0, \tag{25}$$

where $\alpha = \{\alpha_1, \alpha_2, \Lambda, \alpha_n\}$ represents the standardized interval vectors corresponding to $x = \{x_1, x_2, \Lambda, x_n\}$, which are described using the convex model.

When $Y = f(x)$ contains linear continuous function of irrelevant variables, x_i $(i = 1,2,\Lambda, n)$, equation (24) is equivalent to

$$\beta = \frac{Y^c}{Y^r}, \tag{26}$$

where Y^c and Y^r are respectively the mean and the deviation of the interval variable Y. From equation (26), we know that if $\beta > 1$, we always have $f(x) > 0$ with respect to $x_i \in X_i^I$ $(i = 1,2,\Lambda, n)$.

4. MEASURING ROBUST RELABILITY OF VIBRATIONAL COMPONENTS

Vibrational components may fail due to resonance of components or due to fatigue stress exceeding the strength of the materials. As a result, the robust reliability analysis of vibrational components should include two parts, namely: (1) robust reliability wherein the vibrational component do not generate resonance; (2) robust reliability of fatigue strength under loads.

4.1 Robust reliability analysis in terms of resonance of vibrational component

Because of the uncertainties of material parameters, geometric parameters and the machining quality of vibrational components, the natural frequency of vibrational components is also uncertain. This uncertainty can be described with an interval model. Likewise, the frequency of applied vibrational load is also uncertain. These are also described using interval models. The so-called robust reliability in terms of resonance indicates that ability of the components in resistance to resonance. Resonance occurs when the natural frequency of the vibrational components is equal to or close to the frequency of the applied vibrational load. As a result, the robust reliability in terms of resonance failure may be measured by the difference between the natural frequency of the vibrational components and the frequency of applied vibrational loads. The mean and the deviation of the natural frequency of vibrational components may be obtained through either an analytic method or the numerical method. The frequency of the applied vibrational load acting on vibrational components can generally be measured by observing the values of frequencies through trials, and then calculating their mean and deviation. Assume that the natural frequency of the vibrational components is f_n, its mean and deviation are respectively f_n^c and f_n^r; the frequency of vibrational load is f_w, its mean and deviation are respectively f_w^c and f_w^r. Then, the difference between the natural frequency of vibrational components and the frequency of vibrational load is $\Delta f = f_n - f_w$, its mean and deviation are respectively Δf^c and Δf^r.

According to the interval operation principle introduced in Section 2, we have

$$f_n = f_n^c + \alpha_n f_n^r$$

$$f_w = f_w^c + \alpha_w f_w^r$$

$$\Delta f = f_n - f_w = \left(f_n^c - f_w^c\right) + \left(\alpha_n f_n^r - \alpha_w f_w^r\right).$$

Thus

$$\Delta f^c = f_n^c - f_w^c \tag{27}$$

$$\Delta f^r = f_n^r + f_w^r. \tag{28}$$

The robust reliability index in terms of resonance of vibrational components is

defined as

$$\beta_1 = \frac{|\Delta f^c|}{\Delta f^r}. \tag{29}$$

When $\beta_1 = 0$, we have $f_n^c = f_w^c$. This means that the natural frequency of the vibrational components equals or approaches the frequency of the applied vibrational loads. In this case, the components surely yield resonance. Therefore, they are certain to fail and are unreliable. When $0 < \beta_1 \le 1$, there is a possibility that the natural frequency of vibrational components equals or approaches the frequency of the applied vibrational load. In other words, the components could possibly fail because of resonance. Under these circumstances, they cannot be considered reliable. When $\beta_1 > 1$, there is little possibility that the natural frequency of the vibrational components will equal or approach the frequency of the vibration load. In other words, the components cannot fail due to resonance. The possibility that the natural frequency of the vibrational components will equal or approach the frequency of the vibrational loads gradually decreases when the frequency difference of β_1 increases. As a result, the components' reliability also gradually increases.

4.2 Robust reliability in terms of fatigue strength of vibrational components

The maximum material stress and fatigue limits of vibrational components are uncertain owing to the uncertain factors of material property parameters, machining quality, and geometrical parameters. The stress amplitudes of vibration components are also uncertain qualities because of the uncertainties of the natural frequency, the applied load and its frequency, and so on. When the vibrational stress exceeds the fatigue strength, the vibration of the component will lead to fatigue failure. The maximum stress and fatigue limits of vibrating components are described using interval models, and they are then used to analyze the robust reliability in terms of fatigue strength of the components. Assume that the maximum stress of the vibrating component is S, its mean and deviation are respectively S^c and S^r; the fatigue limit of the component material is R, its mean and deviation are respectively R^c and R^r. The state function of the component in terms of fatigue failure is defined as

$$M = R - S. \tag{30}$$

According to the interval operation principle introduced in Section 2, we have

$$M^c = R^c - S^c \tag{31}$$

$$M^r = R^r + S^r. \tag{32}$$

From equation (26), the robust reliability may be defined as

$$\beta_2 = \frac{M^c}{M^r}. \tag{33}$$

When $\beta_2 < -1$, namely when the fatigue limit of the component material is lower than its maximum stress, this component is certain to yield to fatigue failure. When $-1 \le \beta_2 \le 1$, namely when the fatigue limit of the component material may be less or greater than its maximum stress, there exists the possibility that fatigue failure will occur. In other words, the reliability of this component cannot be guaranteed. When $\beta_2 > 1$, the component will not yield to fatigue failure. In other words, it is reliable, and its reliability gradually increases with the increase of β_2.

4.3 Robust reliability of the vibrational component

In engineering applications, the failure modes of vibrational components mainly include resonance failure and fatigue failure. So long as one of them yields failure, the component will fail. Thus, the robust reliability index of the component should be the minimum of two individual reliability indexes, namely

$$\beta = \min(\beta_1, \beta_2). \tag{34}$$

5. EXAMPLE

In what follows, two examples are given illustrating the above-mentioned method to analyze their vibrational reliability. The interval convex models are used to describe the uncertain property of the frequency zone of resonance and the vibration parameters.

5.1 Example 1: Vibration analysis of a cantilever beam

A variable cross-section cantilever beam is monitored in an experiment. The mean and the deviation of its natural frequency are respectively 29.7647rad/s and 0.44647rad/s; the mean and the deviation of the frequency of the applied load are respectively 27.318rad/s and 0.8377rad/s. While alternating stress acts on this cantilever beam, the mean and the deviation of its fatigue limit are respectively 156.95MPa and 10.9535MPa; the mean and the deviation of the maximum stress are respectively 96.2778MPa and 6.81815MPa.

According to equations (27) and (28), we have

$$\Delta f^c = f_n^c - f_w^c = 2.4467\text{rad/s},$$
$$\Delta f^r = f_n^r + f_w^r = 1.28417\text{rad/s}.$$

The robust reliability index in terms of resonance of the cantilever beam is

$$\beta_1 = \frac{|\Delta f^c|}{\Delta f^r} = 1.905.$$

According to equations (31) and (32), we have

$$M^c = R^c - S^c = 60.6722\text{MPa},$$
$$M^r = R^r + S^r = 17.77165\text{MPa}.$$

The robust reliability index in terms of fatigue failure of the cantilever beam is

$$\beta_2 = \frac{M^c}{M^r} = 3.414.$$

As a result, the robust reliability index of this cantilever beam is given by

$$\beta = \min(\beta_1, \beta_2) = 1.905.$$

From this analysis, we can conclude that the robust reliability in terms of resonance of this cantilever beam is low. In order to mitigate the vibrational problem of this cantilever beam, its natural frequency may be re-designed and modified. For example, the mean and the deviation of its natural frequency are respectively modified so as to become 30.7111rad/s and 0.42672rad/s, the mean and the deviation of its maximum stress are respectively modified as 102.3038MPa and 8.8249MPa. Through these modifications, the possibility of resonance failure of the cantilever beam is dramatically decreased, as illustrated with the following calculations:

$$\Delta f^c = f_n^c - f_w^c = 3.3931\text{rad/s}$$
$$\Delta f^r = f_n^r + f_w^r = 1.26442\text{rad/s}.$$

The robust reliability index in terms of resonance of the cantilever beam is

$$\beta_1 = \frac{|\Delta f^c|}{\Delta f^r} = 2.6835$$
$$M^c = R^c - S^c = 54.642\text{MPa}$$
$$M^r = R^r + S^r = 19.7784\text{MPa}.$$

The robust reliability index in terms of fatigue failure of the cantilever beam is

$$\beta_2 = \frac{M^c}{M^r} = 2.7627.$$

As a result, the robust reliability index of this cantilever beam becomes

$$\beta = \min(\beta_1, \beta_2) = 2.6835.$$

5.2 Example 2: Vibration analysis of dynamic blade

The above-mentioned method is also applied to the reliability analysis of the dynamic blade of a turbine in an aero-engine. Reliability data are usually difficult to obtain for costly parts or systems such as aero-engines. We can describe the uncertain property of the frequency zone of resonance and the vibrational parameters of the blades using the convex model. The uncertain vibrational parameters of the blades are expressed as interval models. For example, we can obtain test data from a two-level turbine blade through experiments. The mean and the deviation of its natural frequency are respectively 1002rad/s and 68rad/s; the mean and the deviation of the exciting vibrational frequency are respectively 930rad/s and 71rad/s. The mean and the deviation of the maximum vibration stress acting on its section are respectively 224MPa and 7.1MPa; the mean and the deviation of the material fatigue stress are respectively 351MPa and 14.6MPa.

According to equations (26) and (27), we have

$$\begin{aligned} \Delta f^c &= f_n^c - f_w^c = 72\text{rad/s} \\ \Delta f^r &= f_n^r + f_w^r = 139\text{rad/s}. \end{aligned}$$

The robust reliability index in terms of resonance of the blade is

$$\beta_1 = \frac{|\Delta f^c|}{\Delta f^r} = 0.52.$$

According to equations (31) and (32), we have

$$\begin{aligned} M^c &= R^c - S^c = 127\text{MPa} \\ M^r &= R^r + S^r = 21.7\text{MPa}. \end{aligned}$$

The robust reliability index in terms of fatigue failure of the blade is

$$\beta_2 = \frac{M^c}{M^r} = 5.85.$$

As a result, the robust reliability index of this blade is

$$\beta = \min(\beta_1, \beta_2) = 0.52.$$

We now know that the likelihood of this blade's avoiding resonance is very low. The probability of resonance failure occurring is rather high. This blade could encounter the problem of severe vibration. Indeed, this situation actually exists, and in experiments, many two-level blades yield failures due to resonance.

6. CONCLUSIONS

In reliability analysis of vibrational components, we suggest that uncertain parameters should be described using set models when the statistical data are insufficient. These models are also useful when only the limits of uncertain parameters are known while the actual distributions of the variations are unknown. With these models there is no need to derive distribution characteristics and the difficulty of insufficient statistical data is resolved. In this study, the uncertain parameters such as the natural frequency of vibrational components, the frequency of applied vibrational load, the fatigue limits of materials, and the maximum stress applied are described using convex models. This study adopts the robust reliability concept, which makes use of interval arithmetic operations of uncertain parameter values to analyze the reliability of vibrating components. A new non-probabilistic robust reliability measure criterion is proposed. In other words, the non-probabilistic reliability index may be chosen as the shortest distance from the coordinate origin to the failure surface like contiguous equations of probabilistic reliability design. The basic idea is that the degree of safety is ascertained through comparing the transformation range of the component properties with their required transformation range. The advantage of using the interval approach of convex models is that it does not require massive data. This method only needs to know the boundary of uncertain parameters; it does not require the specific distribution information. In addition, its computation is simple. In the study, the proposed method is comparable to conventional probabilistic reliability design; although, unlike conventional probabilistic reliability design, it widely reduces the need for original data and does not involve the concept of probability. We should point out, however, that the correctness and maturity of probabilistic reliability analysis regarding vibrational components are undisputed. The method proposed in this paper does not replace the probabilistic reliability method. In stead, it can only profitably supplement it, perfecting and developing the conventional vibration and fatigue analysis method through providing a new method of studying the reliability design of vibrational components.

The method presented in this paper also provides another choice for measuring structural reliability or conducting vibrational design. We may choose to adopt whichever reliability method that works the best on the basis of how many statistical data points are available to us. In other words, when we have sufficient data to describe the probabilistic characteristic of uncertain parameters, the probability reliability model may be adopted; when we have insufficient data, the non-probabilistic robust reliability model should be chosen; when the above two conditions exist at the same time, we may choose to use the hybrid reliability model. In engineering analysis, the ability of different possible reliability methods allows for mutual compensation for the disadvantages inherent in each individual method.

Finally, we should point out that the present robust reliability analysis is only in its initial phase. For the theory to be mature and useful for real engineering application, further development is needed.

ACKNOWLEDGEMENTS

This research was partially supported by the National Natural Science Foundation of China under contract number 50175010, the Excellent Young Teachers Program of the Ministry of Education under contract number 1766 (China), the National Excellent Doctoral Dissertation Special Foundation under contract number 200232 (China), and the Natural Sciences and Engineering Research Council of Canada.

REFERENCES

Cui, H. and Peng, Z. (1998). "Reliability analysis of vibration structural components", *Journal of Mechanical Strength,* **20**, 312-315.

Wang, Yongfu, Li, Yonghua and Huang, Hong-Zhong. (2002). "Fuzzy reliability analysis of vibration structural components". In: Hong-Zhong Huang eds. Progress in Reliability Engineering (Science Press, Beijing). 187-190.

Ben-Haim, Y. (1994). "A non-probabilistic concept of reliability", *Structural Safety*, **14**, 227-245.

Ben-Haim, Y. (1995). "A non-probabilistic measure of reliability of linear systems based on expansion of convex models", *Structural Safety*, **17**, 91-100.

Ben-Haim, Y. (1996). *Robust reliability in the Mechanical Sciences,* Springer-Verlag, Berlin.

Ben-Haim, Y., Genda, C. and Soong, T. (1996). "Maximum structural response using convex models", *J. of Engineering Mechanics*, **122**, 235-333

Alifeld, G., and Claudio, D. (1998). "The basic properties of interval arithmetic, its software realizations and some applications", *Computers & Structures*, **67**, 3-8.

Guo, S., Lv, Z. and Feng, Y. (2001). "A non-probabilistic model of structural reliability based on interval analysis", *Chinese Journal of Computational Mechanics,* **18**, 56-60.