Derivation of Damping-reflected Energy Functions in COI Formulation for Direct Analysis of Transient Stability

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Abstract: This paper presents an improved group of energy functions reflecting generator damping effects for multi-machine power systems by using Center of Inertia (COI) formulation as an extension of the previous work. Since rotor angles at the Stable Equilibrium Point (SEP) of post-fault systems are generally calculated in COI, system transient energy can be found without assumption of infinite or slack bus, which is a crucial drawback of the absolute rotor angle frame approach. The developed energy functions have a structure preserving property with which it is very flexible to incorporate various models of power system components, especially various load and generator models. The proposed damping-reflected energy functions are applied to the Potential Energy Boundary Surface (PEBS) method, one of the direct methods. Numerical simulation of WSCC 9-bus shows that conservativeness of the PEBS method can be considerably reduced.

Keywords: damping effect, direct method, energy function, PEBS method, transient stability

1. Introduction

Planning and operation of nonlinear systems such as larger power systems are subjected to the first issue of the stability problem. Many authors have contributed to development of the nonlinear stability theory, yielding the Lyapunov theorem, the Popov theorem and the ultimate confinement theorem for Lure-type nonlinear systems [1-3]. The Lyapunov direct method provides precise and rigorous theoretical backgrounds for stability analysis of nonlinear systems, which enables us to assess the stability directly without solving the differential equations for the entire time period concerned. From the viewpoint of estimate accuracy, incorporating damping effects is a minor but crucial point to ensure a more precise and larger estimate of the domain of attraction, although it is still very difficult to measure and control generator damping coefficients, which will be the focus of future work [4].

There have been many attempts to reflect the damping effects into the Lyapunov function by using the Popov criterion approach [1, 2]. However, there has been no simple way developed to provide a remarkable enlargement of the estimate of attraction region. In most cases, searching for a damping included energy function requires extremely complicated procedures as in the Popov method [2, 3]. The Popov criterion based approaches require us to adjust multiple scalar variables and matrices by trial and error to ensure the largest estimated stability region associated with the given generators' damping [5].

The previous work presented a generalized approach to derive a group of damping reflected energy functions in absolute rotor angle based formulation by considering the first integral of motion for pure reactive multi-machine electric power systems [6]. A new type of energy function was derived where some portion of the damping losses is transformed into the energy function with parameter λ of transform ratio. It has been proven that the time derivative of the damping-reflected energy function well satisfies the semi-negativeness in the normal operating range. For the One-Machine-Infinite-Bus (OMBI) system, it was shown that the damping-reflected energy functions remarkably improve the estimation of domain of attraction compared with that by the conventional energy functions [7].

In the previous work, however, there is room for improvement in that the absolute rotor angle frame is adapted for damping-reflected energy functions of multi-machine systems, which would be a limitation in the first swing transient stability analysis of the direct method for realistic systems. In most physical cases, absolute rotor angles of generators after fault clearing converge to a new operating point rather than the original operating point of the pre-fault system even if governor integral control actions are considered. Moreover, SEP of the post-fault system greatly depends on fault clearing time when it is represented in an absolute value. Hence in order to use energy functions in absolute rotor angle formulation, an infinite bus or slack bus assumption is needed, which is not realistic in transient stability analysis.

This paper presents an improved group of energy

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functions reflecting generator damping effects for multi-machine power systems by using the COI formulation as an extension of the previous work [6]. The developed energy functions guarantee the semi-negativity of time derivative and also have the structure preserving property with which it is very flexible to incorporate various models of power system components, especially various load and generator models. The proposed damping-reflected energy functions are applied to the Potential Energy Boundary Surface (PEBS) method, which is essential in transient stability analysis of the direct method. Numerical simulations with the WSCC 9-bus indicate that the proposed energy functions can be effectively used and conservativeness of the PEBS method can be considerably reduced.

2. Structure-preserving Model of a Power System

Let us consider a power system model to be used throughout the paper. In Fig. 1, a network consists of m generators and n buses connected by transmission lines. There are n ($=N-m$) load buses and the network is augmented with fictitious buses that represent the internal generation voltage E'_{qi}. These are connected to the generator buses in the transmission network via transient reactances. The total number of buses in the augmented network is N ($=n+m$). It is convenient to number these as follows. Each fictitious bus is numbered 1, 2, ..., m and m generator buses are numbered $i+m$ where i is the bus number of the corresponding generator fictitious bus to which it is connected.

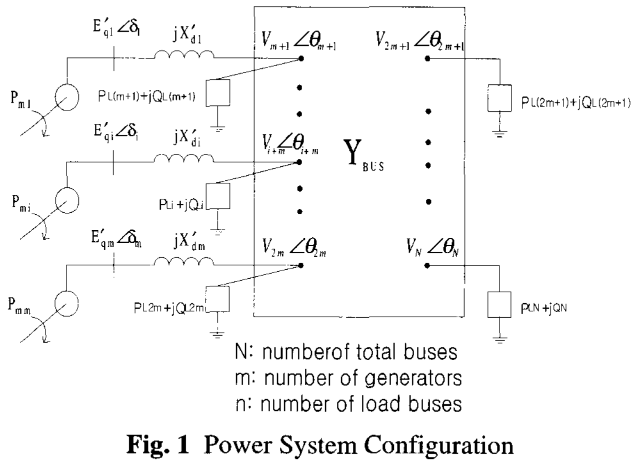

Fig. 1 Power System Configuration

Let the phase angle of i^{th} generator internal voltage and the phase angle of voltage of load bus be δ_i and θ_i respectively. The swing equations representing generator dynamics are given as follows:

$$\dot{\delta}_i = \omega_i \tag{1}$$

$$M_i\dot{\omega}_i + D_i\omega_i = P_{mi} - P_{ei}, \quad i=1, \ldots, m \tag{2}$$

where,

$$P_{ei} = \frac{E'_{qi}V_{i+m}\sin\delta_{Li}}{X'_{di}}, \quad \delta_{Li} = \delta_i - \theta_{i+m}$$

The ω_i is a rotor frequency deviation from a synchronous reference. M_i and D_i represent the generator inertia constant and generator damping constant respectively. For simplicity of analysis, load damping is assumed to be neglected. For the structure preserving models of power systems, Bergen and Hill suggested a structure preserving Lyapunov function based on the Popov stability criterion [8], which was the start of versatile energy functions for stability analysis. Among the various approaches for developing energy functions, Moon et al. systematically derived a structure preserving energy function based on energy conservation law by using a complex line integral [9]. In that approach, energy functions were derived without any assumption, and applicable for any kind of generator model including the exciter/governor control systems. To focus further on derivation of damping-reflected energy functions, we assume that all generators are represented by the classical models, with all generator internal voltages being kept constant and all loads being maintained at constant impedance load. It is also assumed that transfer conductance of the network is neglected. Then, the energy function of multi-machine power systems from conservation law can be written as follows [6, 9]:

$$\begin{aligned} E = &\sum_{i=1}^{m}\frac{1}{2}M_i\omega_i^2 - \frac{1}{2}\sum_{i=m+1}^{N}B_{ii}(V_i^2 - V_{is}^2) + \sum_{i=m+1}^{N}\sum_{j>i}^{N}V_iV_jB_{ij}(\cos\theta_{ij}^s - \cos\theta_{ij}) \\ &- \sum_{i=1}^{m}P_{mi}(\delta_i - \delta_{si}) + \sum_{i=m+1}^{N}{}_c\int_{\theta_{is}}^{\theta_i}G_{ii}V_i^2 d\theta_i \\ &+ \sum_{i=1}^{m}\left(\frac{1}{2}\frac{V_{i+m}^2 - V_{i+m}^{s2}}{X'_{di}} + \frac{E'_{qis}V_{i+m}^s\cos(\delta_{is} - \theta_{i+m}^s) - E'_{qi}V_{i+m}\cos(\delta_i - \theta_{i+m})}{X'_{di}}\right) \\ = &\sum_{i=1}^{m}\int_{t_0}^{t}D_i\omega_i^2 dt + C(\omega_0, \delta_0, E_{q0}) \end{aligned} \tag{3}$$

where $C(\omega_0, \delta_0, E_{q0})$: constant determined by the initial condition In the above energy function (3), path dependent integrals related to the real parts of constant impedance loads can be evaluated by trapezoidal integration or by line integration approximately, which is trivial and has almost no affect on the computation time. The imaginary parts of constant impedance loads are contained in B_{ij} of Y_{BUS}.

The energy functions from the energy conservation law (3) can efficiently be used in transient stability analysis of the direct method. For the above energy function to be applicable to transient stability analysis, it should be assumed that an infinite bus exists or governor integral control actions should be reflected in energy functions. In section 3, improved damping-reflected energy functions will be derived in the Center-Of-Inertial (COI) formulation, which can overcome the limitation of the previous work.

3. Derivation of Damping-reflected Energy Functions for Multi-machine Systems

In this section, improved damping-reflected energy functions are developed in the COI formulation. The angle of the center of inertia (COI) is used as the reference angle, since it represents the "mean motion" of the system and we can calculate system energy without knowing the stable equilibrium points of absolute rotor angles after the fault has cleared. The energy conservation law in the COI is given as follows:

$$\begin{aligned} W = &\sum_{i=1}^{m}\frac{1}{2}M_i\tilde{\omega}_i^2 - \frac{1}{2}\sum_{i=m+1}^{N}B_{ii}(V_i^2 - V_{is}^2) - \sum_{i=m+1}^{N-1}\sum_{j>i}^{N}V_iV_jB_{ij}\cos\tilde{\theta}_{ij} \\ &+\sum_{i=1}^{m}[(V_{i+m}^2 - V_{i+m^s}^2) - 2E_i(V_{i+m}\cos(\tilde{\delta}_i - \tilde{\theta}_{i+m}) \\ &-V_{i+m}^s\cos(\tilde{\delta}_{is} - \tilde{\theta}_{i+m}^s)]/2X_{di}' \\ &-\sum_{i=1}^{m}P_{mi}(\tilde{\delta}_i - \tilde{\delta}_i^s) + \sum_{i=m+1}^{N}\int G_{ii}V_i^2 d\tilde{\theta}_i \\ &+\sum_{i=1}^{m}\int D_i\tilde{\omega}_i^2 dt + \sum_{i=1}^{m}\int D_i(\omega_0 - \omega_s)d\tilde{\delta}_i = 0 \end{aligned} \tag{4}$$

where $\tilde{\delta}_i = \delta_i - \delta_0$, $\tilde{\theta}_i = \theta_i - \delta_0$

$$\begin{aligned} &\tilde{\omega}_i = \omega_i - \omega_0 \\ &\delta_0 = \frac{1}{M_T}\sum_{i=1}^{m}\delta_i M_i, \quad \omega_0 = \frac{1}{M_T}\sum_{i=1}^{m}\omega_i M_i \\ &M_T = \sum_{i=1}^{m}M_i \end{aligned} \tag{5}$$

Energy equation (4) has three path-dependent integral terms, one of which is related to the constant impedance loads. We can evaluate them using a simple trapezoidal integral. Let the potential energy associated with constant impedance loads be $E_l(V,\theta)$ then $E_l(V,\theta)$ can be evaluated using trapezoidal integration as

$$E_l(V,\theta) \triangleq \sum_{i=m+1}^{N}\int G_{ii}V_i^2 d\tilde{\theta} \approx \sum_{i=m+1}^{N} I_i \tag{6}$$

where at the k^{th} step

$$\begin{cases} I_i(k) = I_i(k-1) + \frac{1}{2}G_{ii}(V_i^2(k) + V_i^2(k-1)(\theta_i(k) - \theta_i(k-1)) \\ I_i(0) = 0 \end{cases} \tag{7}$$

In order to reflect the damping effects, it is necessary to change the damping loss term in (5) into an appropriate form of energy integral. This can be done by using integral relationships derived from the system governing equations. The swing equation of the i^{th} generator is given by

$$M_i\Delta\ddot{\delta}_i + D_i\Delta\dot{\delta}_i = P_{mi} - P_{ei} \tag{8}$$

with $\Delta\delta_i = \delta_i - \delta_{is}$, $i = 1,\cdots,m$

δ_{is} : steady state angle of generator i after fault cleared

$$P_{ei} = \frac{E_{qi}'V_{i+m}\sin\delta_{Li}}{X_{di}'}, \quad \delta_{Li} = \delta_i - \theta_{i+m}$$

Using the COI formulation in (5), the swing equation can be rewritten as

$$M_i(\Delta\ddot{\tilde{\delta}}_i + \Delta\ddot{\delta}_0) + D_i(\Delta\dot{\tilde{\delta}}_i + \Delta\dot{\delta}_0) = P_{mi} - P_{ei} \tag{9}$$

where $\Delta\delta_0 = \delta_0 - \delta_{0s} = \frac{1}{M_T}\sum_{i=1}^{m}M_i(\delta_i - \delta_{is})$

Multiplying both sides of (9) by load angle $\Delta\tilde{\delta}_{Li}$ and taking the first integral with respect to time, we obtain

$$\begin{aligned} &\int M_i\Delta\ddot{\tilde{\delta}}_i(\Delta\tilde{\delta}_i - \Delta\tilde{\theta}_i)dt + \int D_i\Delta\dot{\tilde{\delta}}_i(\Delta\tilde{\delta}_i - \Delta\tilde{\theta}_i)dt \\ &= \int(P_{mi} - P_{ei})\Delta\tilde{\delta}_{Li}dt - \int(M_i\Delta\ddot{\delta}_0 + D_i\Delta\dot{\delta}_0)(\Delta\tilde{\delta}_i - \Delta\tilde{\theta}_i)dt \end{aligned} \tag{10}$$

where

$$\begin{aligned} &\tilde{\delta}_{Li} = \tilde{\delta}_i - \tilde{\theta}_{i+m} = \delta_i - \theta_{i+m} = \delta_{Li} \\ &\Delta\tilde{\theta}_i = \tilde{\theta}_i - \tilde{\theta}_{i^s} \\ &\Delta\tilde{\delta}_{Li} = \tilde{\delta}_{Li} - \tilde{\delta}_{Li^s} = \Delta\delta_{Li} \end{aligned} \tag{11}$$

Expanding the left side of (11) and integrating by parts gives

$$\begin{aligned} &M_i\Delta\dot{\tilde{\delta}}_i\Delta\tilde{\delta}_i - M_i\int\Delta\dot{\tilde{\delta}}_i^2 dt + \frac{1}{2}D_i\Delta\tilde{\delta}_i^2 \\ &-\int M_i\Delta\ddot{\tilde{\delta}}_i\Delta\tilde{\theta}_i dt - \int D_i\Delta\dot{\tilde{\delta}}_i\Delta\tilde{\theta}_i dt \\ &= \int(P_{mi} - P_{ei})\Delta\tilde{\delta}_{Li}dt - \int(M_i\Delta\ddot{\delta}_0 + D_i\Delta\dot{\delta}_0)(\Delta\tilde{\delta}_i - \Delta\tilde{\theta}_i)dt \end{aligned} \tag{12}$$

From (12), the damping related term can be written as

$$\int \tilde{\omega}_i^2 dt = -\tilde{\omega}_i \Delta\tilde{\theta}_i +_c \int_{\Delta\theta_{i0}}^{\Delta\theta_i} \tilde{\omega}_i d\Delta\tilde{\theta}_i -_c \int_{\Delta\delta_{i0}}^{\Delta\delta_i} \frac{D_i}{M_i} \Delta\tilde{\theta}_i d\Delta\tilde{\delta}_i + \tilde{\omega}_i \Delta\tilde{\delta}_i + \frac{1}{2}\frac{D_i}{M_i}\Delta\tilde{\delta}_i^2 - \frac{1}{M_i}\int (P_{mi} - P_{ei})\Delta\tilde{\delta}_{Li} dt + \frac{1}{M_i}\int (M_i \Delta\ddot{\delta}_0 + D_i \Delta\dot{\delta}_0)\Delta\tilde{\delta}_{Li} dt \quad (13)$$

The path-dependant integrals in (13) can be approximately evaluated by the straight line integral for the first swing transient stability analysis [10]. From a practical point of view, the evaluation of path-dependent integral terms is trivial and has little affects on computation time.

$$\int \tilde{\omega}_i^2 dt \cong -\tilde{\omega}_i \Delta\tilde{\theta}_i + \frac{1}{2}\Delta\tilde{\theta}_i(\tilde{\omega}_i + \tilde{\omega}_{i0}) - \frac{1}{2}\frac{D_i}{M_i}\Delta\tilde{\theta}_i \Delta\tilde{\delta}_i + \tilde{\omega}_i \Delta\tilde{\delta}_i + \frac{1}{2}\frac{D_i}{M_i}\Delta\tilde{\delta}_i^2 - \frac{1}{M_i}\int (P_{mi} - P_{ei})\Delta\tilde{\delta}_{Li} dt + \sum_{i=1}^{m}\frac{1}{2}\Delta\tilde{\delta}_{Li}\Delta\omega_0 + \sum_{i=1}^{m}\frac{D_i}{M_i}\frac{\Delta\tilde{\delta}_{Li}\Delta\delta_0}{2} \quad (14)$$

The λ portion of the damping loss in the energy conservation law can be transformed into an energy integral by using (14). As a result, we can easily obtain improved damping-reflected energy functions for multi-machine power systems as follows:

$$\begin{aligned} E_\lambda =& \sum_{i=1}^{m}\frac{1}{2}M_i\tilde{\omega}_i^2 - \frac{1}{2}\sum_{i=m+1}^{N} B_{ii}(V_i^2 - V_{is}^2) - \sum_{i=m+1}^{N-1}\sum_{j>i}^{N} V_i V_j B_{ij}\cos\tilde{\theta}_{ij} \\ & - \sum_{i=1}^{m} P_{mi}(\tilde{\delta}_i - \tilde{\delta}_i^s) + \sum_{i=1}^{N}[P_{Li}(\tilde{\theta}_i - \tilde{\theta}_{is}) + Q_{Li}\ln(\frac{V_i}{V_{is}})] \\ & + \sum_{i=1}^{m}[(V_{i+m}^2 - V_{i+m}^{s2}) - 2E_i(V_{i+m}\cos(\delta_i - \theta_{i+m}) - V_{i+m}^s\cos(\delta_{is} - \theta_{i+m}^s))] \quad (15) \\ & /2X_{di}' + \sum_{i=1}^{m}\lambda_i[-\tilde{\omega}_i\Delta\tilde{\theta}_i + \frac{1}{2}\Delta\theta_i(\tilde{\omega}_i + \tilde{\omega}_{i0}) - \frac{1}{2}\frac{D_i}{M_i}\Delta\tilde{\theta}_i\Delta\tilde{\delta}_i \\ & + \frac{1}{2}\frac{D_i}{M_i}\Delta\tilde{\delta}_i^2 + \frac{D_i}{2}\Delta\tilde{\delta}_{Li}\Delta\omega_0 + \frac{D_i^2}{2M_i}\Delta\tilde{\delta}_{Li}\Delta\delta_0] \end{aligned}$$

where $\lambda \in [0,1]$

The proposed energy functions in (15) are usually addressed with the following form:

$$E_\lambda = E_K(\boldsymbol{\omega}) + E_P(\boldsymbol{\delta}, \boldsymbol{\theta}, \mathbf{V}, \boldsymbol{\lambda}) \quad (16)$$

where E_K is the kinetic energy and E_P is the potential energy.

It is interesting to note that damping related energy loss terms are included in the potential energy terms. Damping related energy loss is inevitable if frequency deviation is not zero, i.e. until the system settles down to Stable Equilibrium Point (SEP). Since the total energy of the system once the fault clears remains constant, the change of kinetic energy gives rise to the changes of potential energy and damping loss. With the use of (4), it can be easily proven that the time derivative of the above energy functions satisfies the semi-negativeness as follows [6]:

$$\frac{dE_\lambda}{dt} = \sum_{i=1}^{m}[-(1-\lambda_i)D_i\omega_i^2 + \frac{D_i\lambda_i}{M_i}(P_{mi} - P_{ei})\Delta\tilde{\delta}_{Li}] \le 0 \quad (17)$$

with $0 \le \lambda_i \le 1 \quad (i = 1, \ldots\ldots m)$

Here, it should be noted that $(P_{mi} - P_{ei})\Delta\tilde{\delta}_{Li} \le 0$ for the normal operating range if P_{mi} is kept constant [6]. With the proposed energy functions reflecting damping effects, transient stability analysis is conducted for multi-machine power systems.

4. Numerical Simulations

The proposed energy functions are tested for the WSCC 9-bus system with the transform ratio λ continuously changing. To validate the proposed energy functions, the PEBS method, which is one of the direct methods, was used for estimating critical energy and Critical Clearing Time (CCT) with respect to a disturbance. For simplicity, line conductance is neglected and it is assumed that all generators have identical damping coefficients, which are subject to change in several steps such as 0.01, 0.05, and 0.1[pu/rad]. For a selected damping ratio, the proposed energy function (15) is tested with transform ratio λ changed from 0.0 to 1.0. Each generator may have its own transform ratio λ_i. However, this study adopts the same λ for all generators, for the sake of simplicity. All generators of the test system are assumed to be classical and the constant impedance load model is adapted.

The WSCC 9-bus system data is given in Table 1. It is assumed that a $3-\phi$ fault occurred at Bus 7 and that system topology returned to the original state after fault clearing.

Fig. 2 shows that the energy profile versus time with respect to the fault scenario mentioned above. The energy curves indicate that the proposed energy function sufficiently satisfies the semi-negativeness of the time derivative. We have adopted an approximated integration in (13) in the energy function derivation. It has been demonstrated through the tests that approximation by the straight line integral does not produce as many errors in the estimation of the energy function to change the sign of the time derivative. For weakly damped systems in which all generators have very small damping coefficients, one may observe slight violations in the semi-negativeness of the derivative for large λ near the unity. However, this only slightly degrades the proposed energy function since the damping effects can be neglected in such a weakly damped system seldom encountered in actual practice.

Table 1 Data for the WSCC 9-bus System

(a) Generator/Load (unit: pu; Base 100MVA)

Gen. Bus No.	H (sec)	D [pu/ rad]	P_m [pu]	X'_{di} [pu]	E'_{qi} [pu]
1	23.64	0.01~0.1	0.00	0.0608	1.0566
2	6.4	0.01~0.1	1.63	0.1198	1.0502
3	3.01	0.01~0.1	0.85	0.1813	1.0169
Load Bus No.	P_L	Q_L			
5	1.25	0.5			
6	0.9	0.3			
8	1.00	0.35			

(b) Line Data

Line No.	**Line**	**Line impedance**	**Half line charging**
1	1-4	0.0+j0.0576	j0.000
2	2-7	0.0+j0.0625	j0.000
3	3-9	0.0+j0.0586	j0.000
4	4-5	0.01+j0.085	j0.088
5	4-6	0.017+j0.092	j0.079
6	5-7	0.032+j0.161	j0.153
7	6-9	0.039+j0.170	j0.179
8	7-8	0.0085+j0.072	j0.0745
9	8-9	0.0119+j0.1008	j0.1045

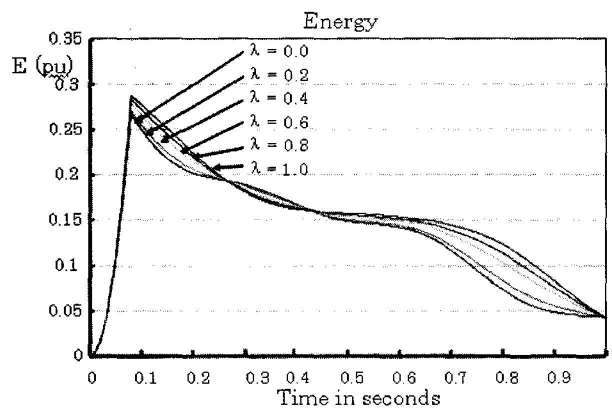

Fig. 2 Energy profiles according to λ variation(D=0.05 [pu/rad])

The PEBS method is applied for estimating the Critical Clearing Time (CCT) of the test systems with respect to the fault using the proposed energy functions. In the PEBS method the fault-on trajectory in machine angle space is traced until it is seen to exit the potential energy boundary surface (PEBS) for the post-fault system. The point (state) where the fault-on trajectory crosses the PEBS is called the exit point and critical energy is calculated at the exit point instead of the CUEP. The time at which the fault system attains critical energy is called the CCT for the fault contingency. The main advantage of the PEBS method is that an estimate of the CCT can be efficiently obtained without calculating the Controlling Unstable Equilibrium Point (CUEP), which is a great computational burden [11]. However, the PEBS method may provide optimistic results when the exit point is not close to the CUEP and system trajectory after fault clearing crosses the PEBS. Due to this limitation, the PEBS method is generally used as a middle step to determine the CUEP or contingency ranking criterion for dynamic security assessment. In this paper, we adopted the PEBS method to show the impact of system damping on system potential energy.

Including damping loss energy in potential energy, we can improve the conservativeness of the PEBS method since damping loss compensates for the critical energy at the exit point $(\delta^*, \theta^*, V^*)$ in which the projected trajectory crosses the PEBS. The critical energy is defined as being the potential energy evaluated at the exit point $E_{CR} = E_P(\delta^*, \theta^*, V^*)$. The exit point is found by tracking the first local maximum of potential energy and sign change of the dot-product between state vector in the projected space and gradient of potential energy. Figure 3 shows an example of estimating CCT and critical energy with respect to a fault.

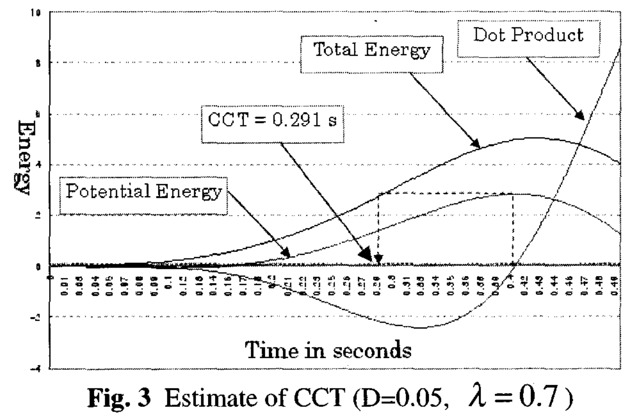

Fig. 3 Estimate of CCT (D=0.05, $\lambda = 0.7$)

The estimated CCTs according to the several damping coefficients and lambda are compared with the results from conventional time simulation. Table 2 shows the estimated CCT from simulation.

The simulation results of Table 1 indicate that as damping transform ratio λ increases, the estimate of the CCT by the PEBS method comes close to the CCT by the conventional time simulation method, which implies that the proposed damping-reflected energy function for multi-machine systems can be efficiently applied to the direct analysis of transient stability. To obtain more accurate estimates of the CCT, the damping related path-dependent terms are integrated by trapezoidal integration. From a computational point of view, the trapezoidal integration of path-dependent terms is very simple and does not considerably give rise to computational burden. However, with the line integral approximation, the authors observed that fairly good estimates can be obtained.

Table 2 Estimate of Critical Clearing Time with variance of λ and Damping Coefficients

	D=0.01[pu/rad] CCT*=0.249(sec)	D=0.05[pu/rad] CCT=0.297(sec)	D=0.1[pu/rad] CCT=0.344(sec)
$\lambda = 0.0$	CCT** = 0.244	0.277	0.327
$\lambda = 0.4$	0.246	0.286	0.335
$\lambda = 0.7$	0.247	0.291	0.340
$\lambda = 1.0$	0.249	0.296	0.343

* Critical Clearing Time by Time Simulation
** Estimate of Critical Clearing Time by the PEBS Method

The proposed energy functions are compared with the previous work in which absolute rotor angle frame is used. In order to apply the previous work to this test case, bus #1 is assumed to be an infinite bus. Table 2 shows that the previous approach gives poor estimates of the CCT in the contingency case. The reason for such poor estimation can be ascribed to the infinite bus assumption, which is the mathematical modeling limitation of the previous approach. However, we conjecture the absolute rotor angle based energy functions can provide superior approximation of the CCT when there exists a generator with very large inertia or a generator that is hardly affected by the disturbance.

Table 3 Comparison of Estimate of CCTs with the Previous Work

	D=0.05[pu/rad] CCT*=0.297(sec)		D=0.1[pu/rad] CCT=0.344(sec)	
	Proposed energy functions (COI)	Previous approach** (absolute angle based)	Proposed energy functions (COI)	Previous approach** (absolute angle based)
$\lambda = 0.0$	0.277	0.218	0.327	0.247
$\lambda = 0.4$	0.286	0.228	0.335	0.263
$\lambda = 0.7$	0.291	0.229	0.340	0.272
$\lambda = 1.0$	0.296	0.238	0.343	0.281

* Critical Clearing Time by Time Simulation
** Generator Bus #1 is assumed to be an Infinite Bus

The simulation results by the PEBS method are in general very good estimates of the critical clearing time. However, there are some cases in which the PEBS method provides non-conservative estimates, that is, the estimated CCT is greater than the real CCT. These cases are such that the system states after the fault has cleared cross the PEBS instead of heading away from the PEBS. Fortunately the fault scenario used in this paper is not such a case. Nonetheless, it can't be assumed that the CCT from the PEBS method is not an optimistic case without further simulation, which is the weakest point of the PEBS method. In spite of this problem, the PEBS method has been applied successfully in contingency analysis combined with the BCU method [12] and other hybrid methods.

5. Conclusions

This paper presents an improved energy function group reflecting the damping effect in multi-machine power systems using the Center of Inertia (COI) formation instead of absolute rotor angle frame. The developed energy functions have a structure preserving property with which it is very flexible to incorporate various models of power system components, especially for various load and generator models. The proposed damping-reflected energy functions are applied to the Potential Energy Boundary Surface (PEBS) method, one of the direct methods. The numerical simulation of the WSCC 9-bus system shows that conservativeness of the PEBS method can be considerably improved and the proposed energy functions can be efficiently used for direct transient stability analysis methods such as the PEBS and BCU methods.

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