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On the Method for Hot-Fire Modeling of High-Frequency Combustion Instability in Liquid Rocket Engines

Chae Hoon Sohn*
Department of Aerospace Engineering, Chosun University, Gwangju 501-759, Korea
Woo-Seok Seol
Rocket Engine Department, Korea Aerospace Research Institute, P.O.Box 113, Yusung, Daejeon 305-600, Korea
Alexander A. Shibanov, Valery P. Pikalov
Research Institute of Chemical Machine Building (NIICHIMMASH), Sergiev Posad, Russia

This study presents the methodological aspects of combustion instability modeling and provides the numerical results of the model (sub-scale) combustion chamber, regarding geometrical dimensions and operating conditions, which are for determining the combustion stability boundaries using the model chamber. An approach to determine the stability limits and acoustic characteristics of injectors is described intensively. Procedures for extrapolation of the model operating parameters to the actual conditions are presented, which allow the hot-fire test data to be presented by parameters of the combustion chamber pressure and mixture (oxidizer/fuel) ratio, which are customary for designers. Tests with the model chamber, based on the suggested scaling method, are far more cost-effective than with the actual (full-scale) chamber and useful for injector screening at the initial stage of the combustor development in a viewpoint of combustion instabilities.

Key Words : Scaling, Combustion Instability Modeling, Stability Boundary, Injector Screening, Model Combustion Chamber

Nomenclature

A_{th} : Nozzle throat area of rocket engine
C : Sound velocity
C^* : Characteristic velocity
f : Frequency
idem : The same as the actual condition
K_m : O/F mixture ratio $(=\dot{m}_o/\dot{m}_f)$
L : Length
M : Mach number
$\dot{m}$: Mass flow rate
n_i : Total number of main injectors
p : Pressure
Q : Volumetric flow rate
q : Dynamic head $(=\rho U^2)$
R : Stability margin
U : Injection velocity
W : Molecular weight

Greek Symbols

α : Excess-oxidizer coefficient
α_{mn} : Eigenvalue variable
Π_1 : Dependent stability parameter
Π_2 : Governing (or independent) stability parameter
ρ : Density
τ : Characteristic time

Subscripts

A : Nominal or design operation regime of actual combustion chamber
a : Actial (full-scale) chamber or condition
ch : Combustion chamber
$chem$: Chemical reaction

* Corresponding Author,
E-mail : chsohn@chosun.ac.kr
TEL : +82-62-230-7123; **FAX** : +82-62-230-7123
Department of Aerospace Engineering, Chosun University, Gwangju 501-759, Korea. (Manuscript **Received** September 1, 2003; **Revised** February 26, 2004)

cyl : Cylindrical part of combustion chamber
(c) : With a closed end
f : Fuel
f_m : Auxiliary injectors for film cooling
i : Per injector orifice
ig : Auxiliary injectors for ignition
L : Longitudinal mode
liq : Liquid
m : Model chamber or condition
mix : Mixing
mL : The m-th longitudinal mode
noz : Nozzle part of combustion chamber
O_2 : Oxygen
o : Oxidizer
(o) : With an open end
tot : Total

1. Introduction

High-frequency combustion instability that results from pressure-wave amplification is often called acoustic instability, which has long gained significant interest in propulsion systems. Under acoustic instability, pressure oscillations are amplified through the in-phase heat addition/extraction from combustion. It may lead to intense pressure fluctuations as well as excessive heat transfer to the combustor wall in combustion systems such as solid and liquid propellant rocket engines, ramjets, turbojet thrust augmentors, utility boilers, and furnaces (Harrje and Reardon, 1972 ; McManus et al., 1993 ; Culick and Yang, 1995 ; Seo, 2003 ; Sohn et al., 2004). Accordingly, it has caused common problems in the course of engine development, i.e., thermal damage on the injector face plate and combustor wall, severe mechanical vibration of rocket body, unpredictable malfunction of engines, etc.

Combustion stabilities as well as engine performance should be checked by engine designers and especially, the impact of candidate injectors on stabilities should be examined for injector screening at the initial stage of the combustor development, and thereby the suitable injector specification will be selected. For the purpose of validation or prediction with elementary hardware component, it is the best and most reliable way to conduct an experimental test using the actual (full-scale) combustion chamber. But, it is a rather exhaustive method in the viewpoints of cost and time. Especially, considering the recent economic constraint applied to the space-technology program, it is required to find cost-effective ways to validate each hardware but without losing essential part of its characteristics. One of them is to use a model (sub-scale) chamber instead of an actual chamber. Thus far, there have been reported lots of sub-scale engine tests in various aspects (Fisher et al., 1995). But, its application to injector screening at the initial developing stage cannot be found.

Regarding injector screening, the present study suggests a scaling method to investigate stability characteristics using a model (sub-scale) chamber equipped with injectors. And one example of scaling practice is proposed here for better understanding of the present method.

2. Basic Consideration on Approximate Modeling of Spontaneous Oscillation

The objective of hot-fire model tests with injectors consists in determination of the operating regimes where acoustic oscillations are excited. A multiple of such regimes form the instability region, which is separated from the steady regimes with the stability boundary. The remoteness of a simulated actual operating regime from the stability boundary reflects the stability margin. The comparison of the stability margins of various injectors will make it possible to choose the most optimized injector configuration in view of high-frequency combustion stability as well as to identify the effects of certain design and performance parameters on the stability.

2.1 General principles and rules of approximate modeling

In hot-fire modeling, there is no other possibility than creating a model to examine only the main feature out of actual physical phenomena. The physical model should not only describe adequately the actual process under study, but also should be far less sophisticated than the real object itself. The comparison of the model test data with the full-scale test data is most convenient to perform, if the scale factor of modeling of key parameters is chosen to be unity.

Based on the above principles, an approach should be used to allow a successful approximate modeling of combustion instability within a combustion chamber. It is associated with the need of invoking physical notion about the complex process under study such as high-frequency combustion instability. Combustion instability is a result of interaction between the acoustic oscillations within a combustion chamber and the phenomena such as atomization, evaporation, mixing, and chemical kinetics (Harrje and Reardon, 1972 ; Sohn, 2002). The accuracy of approximate modeling depends on estimation-correctness of the influence of all these phenomena on combustion stability for a specific combustion chamber and the accuracy of the selected governing phenomenon under model conditions.

According to the similarity theory, the necessary and sufficient conditions for similarity of two objects (phenomena, processes, systems, etc.) are to provide (1) geometrical similarity of the actual object and its model, (2) proportionality of the corresponding parameters involved in the geometry of the spatial area, values of physical constants, initial and boundary conditions including flame condition, and (3) equality of determining criteria for actual object and its model.

Hence, in order to build a model in which the process behavior is similar to that in the actual object, we should select parameters of the model from the theory of similarity and provide equality of determining the similarity criteria, adding the boundary and initial conditions to the determining similarity criteria.

2.2 Major concepts of physical model for the combustion process concerned

The modeling technique developed here is based on the following major concepts of physical model for the combustion process within a combustion chamber.

(1) From all the integrity of in-chamber processes (atomization, evaporation, mixing, and chemical kinetics), propellants mixing is assumed to be the first governing process with respect to susceptibility to acoustic oscillations. This assumption seems to be quite natural, since whatever mixture formation scheme may be used, the possibility of chemical-kinetic interaction of propellants (combustion) is realized only after mixing oxidizer and fuel in the vapor phase.

(2) From the qualitative considerations (Dexter et al., 1995), delay effects in the combustion zone are governed mainly by the mixing time τ_{mix}, of which reciprocal indicates the mixing rate, which is the main limiting phase of the combustion process.

(3) Substitution of the gaseous fuel for the liquid one in the model combustion chamber results in acoustic oscillations generated by heat oscillations, rather than by the mass of material (Natanzon, 1986).

(4) Excitation conditions are assumed to be governed mainly by the processes in the initial section of flames stabilized near the injector exit.

(5) Bi-propellant injectors are assumed to operate independently. Combustion zone under actual operating conditions comprises a large number of separate identical flames established at the injector exit. Main features of the combustion zone pattern are governed by the processes occurring at its initial point with slight interference of flames of adjacent injectors.

(6) High reactivity of the propellants in the combustion process under actual operating conditions extremely reduces the duration of chemical reactions, τ_{chem}. In this regard, the use of actual propellants in modeling is considered to be advantageous, but not strictly required with respect to qualitative comparison with each other for injector screening. But, it is important to ensure selection of such parameters of model gaseous propellants (for example, by varying their temperature) as to make the duration of chemical reactions τ_{chem} as small as possible, in order to satisfy the actual condition, $(\tau_{chem}+\tau_{liq}) \ll \tau_{mix}$, where τ_{liq} denotes the duration of liquid-phase processes such as atomization and

evaporation.

2.3 Geometric similarity

Geometric similarity with injector is provided primarily by using actual injection elements. Many peculiarities in mixing and combustion processes are connected with geometry of a particular injector. Therefore, any attempt to manipulate the geometry of an injector in modeling appears to be unpromising in practice. On the other hand, when selecting the dimensions of the model combustion chamber, a cylindrical chamber or resonator is considered. The resonator specifies a series of acoustic frequencies of longitudinal and transverse (tangential and radial) oscillations interacting with the combustion process.

Since the modeling scale with acoustic modes should be unity for accuracy, the geometry of the model chamber is selected such that natural frequencies of acoustic oscillations of the model and the actual combustion chambers coincide, i.e., $f_{ch}=idem$.

The face plate of the model injector head comprises a blind plate with a hole which accommodates an injector or a series of injectors under study. Thus, the face plate of the model and actual injector head exhibits an acoustically closed end. In order to simulate the conditions of acoustically closed end at the model chamber outlet (as it is the case with the actual combustion chamber), the outlet section of the cylindrical chamber should be closed with a cover fitted with an orifice or a slot of the flow area being equal to actual one.

During hot-fire model tests of injectors, the combustion chamber is normally installed nozzle-upwards on a plate, which is an injector head simulator. In order to facilitate the test operations, the model chamber may be used without a cover at its outlet. In this case, the chamber outlet will serve as an acoustically open end with the range of natural frequencies of longitudinal oscillations within a model combustion chamber. These are actually different from actual ones since calculation is actually made for a tube of length $L_{ch,m}$ with one closed and the other open end at the sound velocity being $C_{ch,m}$. The natural frequency of the m-th longitudinal mode, f_{mL} are calculated by the equation of

$$f_{mL}=\frac{m}{4}\frac{C_{ch,m}}{L_{ch,m}},\ \text{where } m=1,\ 3,\ 5,\ 7,\ \cdots \tag{1}$$

In the meanwhile, calculations of the longitudinal oscillation frequency in the model chamber having a cover are made for a tube with both acoustically closed ends as in the case with the actual combustion chamber by the following equation,

$$f_{mL}=\frac{m}{2}\frac{C_{ch,m}}{L_{ch,m}},\ \text{where } m=1,\ 2,\ 3,\ 4,\ \cdots \tag{2}$$

Condition of the equality of longitudinal oscillation frequency values f_L with the actual values (for example, for the first mode, $f_{1L}=idem$) can be fulfilled by the combustion chamber without the cover if its length is selected 1.5 times larger than that of the chamber with the cover (if sound velocity in both chambers remains equal, i.e., $C_{ch,m}$=const). Thereby, the frequency of the longitudinal oscillations of the second mode in the chamber with an open end, $f_{2L(o)}$ will be numerically equal to $f_{1L(c)}$ in the chamber having an acoustically closed end and to the actual first-mode frequency $f_{1L,a}$. It is demonstrated in Fig. 1.

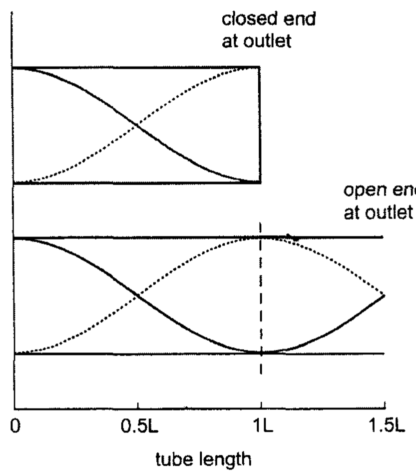

Fig. 1 Acoustic modes at the first longitudinal-mode frequency with closed end, $f_{1L(c)}$ and the second longitudinal-mode frequency with open end $f_{2L(o)}$ at outlet

In this case, however, longitudinal oscillations in the model chamber with $L_{ch(o)}$, equal to $1.5 \times L_{ch(c)}$, may be generated with the first longitudinal-mode frequency, $f_{1L(o)}$, which is less than the actual frequency. Therefore, when comparing the results of model with the actual tests, the effects of $f_{1L(o)}$ generated in the model chamber must be taken out of consideration so as to eliminate the spurious information for the actual combustion chamber.

2.4 Flame-condition similarity

Besides providing geometrical similarity including acoustic boundary conditions throughout the combustion chamber/resonator (acoustically closed ends at the inlet and outlet of the combustion chamber, rigid lateral walls), a similarity for the flame condition applied to combustion zone should also be provided in the modeling. A required condition for similarity of velocity, concentration, temperature fields, etc. in the corresponding cross-sections and points of the initial section of the flame is the similarity of distribution of oxidizer and fuel flows, equal mass exchange rate, and the self-similarity of the physico-chemical transformations. As applied to full-scale injectors employing oxygen and hydrocarbon propellants, these conditions can be satisfied with two following requirements: (1) equality of velocity ratio, $\bar{U}=U_o/U_f=idem$; (2) equality of density ratio, $\bar{\rho}=\rho_o/\rho_f=idem$, where the subscripts o and f denote oxidizer and fuel, respectively. Throughout these requirements, the equality of criterion governing the relation of the dynamic head of oxidizer and fuel streams is ensured automatically, i.e., $q=\bar{\rho}\,\bar{U}^2=idem$.

In case of diffusive combustion of the propellants separately supplied through the injectors, the actual value of the excess-oxidizer coefficient in the model chamber (as one flame condition), i.e., $\alpha=idem$ is preferable, but not strictly required, since chemical reaction in the flame (at its initial point being most sensitive to disturbances) occurs mostly on stoichiometric surfaces ($\alpha=1.0$), and deviation from the strict requirement of $\alpha=idem$, does not noticeably distort the regime-parameters on the stability boundary.

2.5 Criteria for spontaneous oscillation conditions

Qualitative comparison of operating parameters of model and actual combustion chambers on the stability boundary must be performed on the basis of the equality of the dependent stability parameter, $\Pi_1=idem$, with the equality of the selected governing stability parameter (for example, $\Pi_2=idem$) being assured. In graphical demonstration of the stability boundaries, the dependent stability parameter is plotted on the ordinate axis and on the abscissa axis is plotted the governing stability parameter. That is, the plot is made on the coordinates of Π_1 and Π_2 as shown in Fig. 2. The point "A" in this figure denotes a model nominal operating condition of the actual combustion chamber and the shortest distance from this point to the stability boundary in the form of vector R denotes the stability margin. By dimensionless criterion-parameters on coordinates Π_1 and Π_2, stability margin may be presented as follows:

$$R=\left[\left(\frac{\Delta\Pi_1}{\Pi_1}\right)^2+\left(\frac{\Delta\Pi_2}{\Pi_2}\right)^2\right] \tag{3}$$

The best injector configuration in terms of combustion stability features the greatest stability margin R.

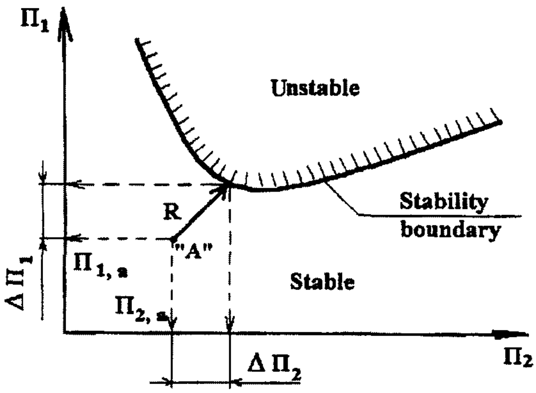

Fig. 2 Stability margin R plotted on the coordinates of criteria parameters, Π_1 and Π_2 for a nominal operating regime "A"

3. Scaling Methods

3.1 Analysis of actual conditions for the combustion process in combustion camber

The full-scale combustion chamber adopted here as a sample, employs a liquid-liquid scheme injector with the propellants of hydrocarbon fuel (kerosene) and cryogenic oxidizer (liquid oxygen). The detailed schematic diagram and basic geometrical dimensions of the combustion chamber can be in Sohn et al.(2003). Major operating parameters for this combustion chamber are listed in Table 1.

3.2 Calculation of injector operating parameters for model conditions

In actual conditions, liquid oxygen and liquid kerosene are supplied through main bi-propellant injectors. Under model conditions at atmospheric pressure, instead of liquid oxygen, actual propellant (oxygen) is used which is fed into injectors in gaseous form at the room temperature and atmospheric pressure (+20℃ and 0.1 MPa). Model oxygen density is $\rho_{o2}=1.288$ kg/m^3.

In a similar way, instead of actual hydrocarbon fuel (liquid kerosene), gaseous hydrocarbon fuel is used for a model combustion chamber which would provide the same as the actual density ratio of oxidizer and fuel. Density ratio in actual condition is $\bar{\rho}=\rho_o/\rho_f=1{,}030/799=1.289$ (Sutton, 1992). Consequently, the density of model gaseous fuel must be $\rho_{f,m}=\rho_{o,m}/\bar{\rho}=1.288/1.289=0.999$ kg/m^3. Application of the vaporized actual propellant (kerosene) as a model fuel is impossible, since kerosene vapor density of $\rho_{vapor,kerosene}=2.95$ kg/m^3 exceeds the required density by a factor of three.

With both model gaseous propellants being fed into the injectors at the same temperature, the density ratio is the same as the ratio of propellant molecular weights, that is, $\bar{\rho}=W_o/W_f$, where W_o and W_f are oxygen and fuel molecular weights, respectively. Combination of two gases may be regarded as gaseous hydrocarbon fuels such as methane and propane whose molecular weights

Table 1 Main parameters of the actual combustion chamber

Parameter	Symbol	Unit	Value
chamber pressure	p_{ch}	MPa	1.4
total mass flow rate of propellants	$\dot{m}_{tot}$	kg/s	58.4
mixture ratio for main injectors	K_m	–	2.39
excess-oxidizer coefficient for main injectors	α	–	0.7
oxygen and kerosene mass flow rates per injector	$\dot{m}_{o,i}$	kg/s	0.189
	$\dot{m}_{f,i}$		0.079
oxygen and kerosene volumetric flow rates per injector	$Q_{o,i}$	l/s	0.183
	$Q_{f,i}$		0.0989
ratio of oxidizer to fuel volumetric flow rate	$\bar{Q}=Q_{o,i}/Q_{fi}$	–	1.85
oxygen density	ρ_o	kg/m^3	1,030
kerosene density	ρ_f	kg/m^3	799
ratio of oxidizer to fuel density	$\bar{\rho}=\rho_o/\rho_f$	–	1.289
velocities of oxidizer and fuel discharged from injector orifices	U_o	m/s	24.13
	U_f	m/s	24.61
ratio of oxidizer to fuel discharge velocity	$\bar{U}=U_o/U_f$	–	0.98
estimated temperature of combustion products	T_{ch}	K	3,388
estimated sound velocity in combustion products	C_{ch}	m/s	1,205

are 16 and 44 kg/kmol, respectively. Molecular weight of the fuel required for injector model tests is $W_{f,m}=W_{O2}/\bar{\rho}=32/1.289=24.8$ kg/kmol. It is evident that the model hydrocarbon fuel with such a molecular weight may be provided by mixing 31.5% methane with 68.5% propane.

With actual injectors used in the model combustion chamber, the mixing process will be governed fully by two parameters: density ratio, $\bar{\rho}=\rho_o/\rho_f$ and velocity ratio, $\bar{U}=U_o/U_f$. For injectors of actual geometry, the parameter $\bar{U}$ is adjusted by the ratio of volumetric flow rates of oxidizer and fuel through injector orifices, $\bar{Q}=Q_{o,i}/Q_{f,i}$. At this point, it is to be noted that one more condition is required, which is $U_o=idem$, to preserve the same characteristic time of disturbances caused by propellant jets in model chamber as in actual chamber (Dexter et al., 1995). Consequently, $Q_{o,i}=idem$, $Q_{f,i}=idem$, and $U_f=idem$ are automatically satisfied.

According to the equality of volumetric flow rates, it becomes $Q_{o,i,m}=0.183$ l/s and $Q_{f,i,m}=0.0989$ l/s from Table 1. Subsequently, propellant mass flow rates in model chamber at the room temperature are

$$\dot{m}_{o,i,m}=\rho_{o,m}\cdot Q_{o,i}=1.288\times 0.183=0.236 \text{ g/s} \quad (4)$$

$$\dot{m}_{f,i,m}=\rho_{f,m}\cdot Q_{f,i}=0.999\times 0.0989=0.0988 \text{ g/s} \quad (5)$$

Mixture ratio in injector flames is $K_{m,m}=\dot{m}_{o,m}/\dot{m}_{f,m}=2.39$, which is the same value as that in Table 1.

3.3 Calculation of model-chamber geometry for hot-fire tests of injectors

Procedures for selecting the geometry of the model combustion chamber acting as an acoustic resonator with a variety of oscillation natural frequencies, f_{ch} consist in choosing such geometric dimensions as to provide the equality of natural frequencies of both model and actual combustion chambers, i.e., to satisfy the condition, $f_{ch}=idem$.

From the condition of the equality of transverse oscillations frequencies, $f_{ch}=idem$ for actual object ("a") and model ("m"), we obtain

$$\alpha_{mn}\frac{C_{ch,a}}{D_{ch,a}}(1-M^2_{ch,a})^{0.5} = \alpha_{mn}\frac{C_{ch,m}}{D_{ch,m}}(1-M^2_{ch,m})^{0.5}, \quad (6)$$

where α_{mn} is the eigenvalue variable depending on each acoustic mode and M is the Mach number (Zucrow and Hoffman, 1977; Natanzon, 1986). From this equation, the diameter of model chamber can be determined as follows:

$$D_{ch,m}=D_{ch,a}\frac{C_{ch,m}}{C_{ch,a}}\left(\frac{1-M^2_{ch,m}}{1-M^2_{ch,a}}\right)^{0.5} \quad (7)$$

If the model combustion chamber with acoustically open end (without a cover) is accepted for operations, then the frequency of its second longitudinal-mode oscillation has to be equal to the frequency of the first longitudinal-mode oscillation in the actual combustion chamber (as described in section 2.3). Accordingly, the length of the model chamber cylindrical portion is obtained from the following equation,

$$\frac{C_{ch,a}}{2L_{eff,a}}(1-M^2_{ch,a})=\frac{3C_{ch,m}}{4L_{ch,m}}(1-M^2_{ch,m}), \quad (8)$$

where L_{eff} is the effective length of the combustion chamber including cylindrical and nozzle parts, and calculated by $L_{cyl}+bL_{noz}$, where the empirical value of b is about 2/3. From this equation, the length of cylindrical model chamber can be determined as follows:

$$L_{ch,m}=1.5L_{eff,a}\frac{C_{ch,m}}{C_{ch,a}}\frac{1-M^2_{ch,m}}{1-M^2_{ch,a}} \quad (9)$$

For the model chamber with acoustically closed end, the coefficient of 1.5 should be deleted from the equation (9).

3.4 Scaling-up (extrapolation) of model-test data to actual operating conditions

A method to extrapolate the model operating conditions to actual ones is based on the postulate that with model propellant parameters chosen, there must be equal volumetric flow rates of oxidizer and fuel per bi-propellant injector installed in the model or actual combustion chambers, i.e., $Q_{o,i}=idem$ and $Q_{f,i}=idem$. Scaling-up procedures are suggested as follows.

(1) With oxidizer (liquid oxygen) density at the injector inlet of actual injector head, i.e., $\rho_{o,a}$ known, we determine the total mass flow rate of oxidizer through the main injectors : $\dot{m}_{o,i,a}=Q_{o,i,m}\cdot\rho_{o,a}\cdot n_i$, where n_i is the total number of main injectors.

(2) With fuel (kerosene) density at the injector inlet of actual injector head known, we determine the total mass flow rate of fuel through the main injectors : $\dot{m}_{f,i,a}=Q_{f,i,m}\cdot\rho_{f,a}\cdot n_i$.

(3) Determine the total mass flow rate of propellants supplied into the chamber : $\dot{m}_{tot}=\dot{m}_{o,tot}+\dot{m}_{f,tot}=\dot{m}_{o,i,a}+\dot{m}_{f,i,a}+\dot{m}_{ig}+\dot{m}_{fm}$, where $\dot{m}_{ig}$ and $\dot{m}_{fm}$ are mass flow rates through auxiliary injectors for ignition and film cooling in the actual chamber, respectively.

(4) Determine the mixture ratio, K_m : $K_m=\dot{m}_{o,tot}/\dot{m}_{f,tot}$.

(5) Determine the combustion chamber pressure : $p_{ch}=\dot{m}_{tot}C^*/A_{th}$, where C^* and A_{th} denote the characteristic velocity (Sutton, 1992) and nozzle throat area, respectively.

To determine the stability margin of the process for the injector of any configuration, it is desirable to use customary coordinates for convenient presentation of stability boundaries, i.e., total mass flow rate, $\dot{m}_{tot}$ vs. mixture ratio, K_m or chamber pressure, p_{ch} vs. mixture ratio, K_m.

When the parameters of Π_1 and Π_2 presented in Fig. 2 are replaced by p_{ch} and K_m, respectively, stability boundary can be plotted in a functional form of $p_{ch}=f(K_m)$. The point "A" denotes the nominal operation regime of actual combustion chamber. The letter R denotes a minimum distance from the nominal regime to the stability boundary. Stability margin is expressed in non-dimensional form,

$$R=(R_{p_{ch}}^2+R_{K_m}^2)^{0.5},\quad \text{where } R_{p_{ch}}=\frac{\Delta P_{ch}}{p_{ch,A}} \text{ and } R_{K_m}=\frac{\Delta K_m}{K_{m,A}} \tag{10}$$

4. Concluding Remarks

The methodological aspects of combustion instability modeling have been presented for the determination of combustion stability boundaries using the model chamber. The results of calculation of model (sub-scale) combustion chamber regarding geometrical dimensions and operating conditions have been provided. An approach for determining stability limits and acoustic characteristics of injectors is described in detail. Procedures for extrapolation of the model operating parameters to the actual conditions are presented. They allow the hot-fire test data to be presented by parameters of the chamber pressure and mixture (oxidizer/fuel) ratio, which are customary for a designer.

Tests with the model chamber are more cost-effective than with the actual (full-scale) chamber and useful for injector screening at the initial stage of combustor development in a viewpoint of high-frequency combustion instabilities. In the present study, only the scaling method has been proposed conceptually, and experimental results using the model chamber, based on the method suggested here, will be introduced as a future work.

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References

Culick, F. E. C. and Yang, V., 1995, in *Liquid Rocket Engine Combustion Instability* (V. Yang, and W. E. Anderson, eds.), *Progress in Astronautics and Aeronautics*, Vol. 169, AIAA, Washington DC, pp. 3~38.

Dexter, C. E., Fisher, M. F., Hulka, J. R., Denisov, K. P., Shibanov, A. A. and Agarkov, A. F., 1995, "Scaling Techniques in Liquid Rocket Engine Combustion Devices Testing," *The Second International Symposium on Liquid Rocket Engines*, Paris.

Fisher, S. C., Dodd, F. E. and Jensen, R. J., 1995, in *Liquid Rocket Engine Combustion Instability* (V. Yang, and W. E. Anderson, eds.), *Progress in Astronautics and Aeronautics*, Vol.

169, AIAA, Washington DC, pp. 545~564.

Harrje, D. J. and Reardon, F. H. (eds.), 1972, *Liquid Propellant Rocket Combustion Instability*, NASA SP-194.

McManus, K. R., Poinsot, T. and Candel, S. M., 1993, "A Review of Active Control of Combustion Instabilities," *Progress in Energy and Combustion Science*, Vol. 19, pp. 1~29.

Natanzon, M. S., 1986, *Combustion Instability* (Translated by F. E. C. Culick in 1996), Mashinostroyeniye, Moscow.

Seo, S., 2003, "Combustion Instability Mechanism of a Lean Premixed Gas Turbine Combustor," *KSME International Journal*, Vol. 17, No. 6, pp. 906~913.

Sohn, C. H., 2002, "Unsteady Analysis of Acoustic Pressure Response in N_2 Diluted H_2 and Air Diffusion Flames," *Combustion and Flame*, Vol. 128, pp. 111~120.

Sohn, C. H., Seol, W. S., Lee, S. Y., Kim, Y. -M. and Lee, D. S., 2003, "Application of Combustion Stabilization Devices to Liquid Rocket Engine," *Journal of The Korean Society for Aeronautical and Space Sciences*, Vol. 31, No. 6, pp. 79~87.

Sohn, C. H., Kim, S. -K. and Kim, Y. -M., 2004, "Effects of Various Baffle Designs on Acoustic Characteristics in Combustion Chamber of Liquid Rocket Engine," *KSME International Journal*, Vol. 18, No. 1, pp. 154~161.

Sutton, G. P., 1992, *Rocket Propulsion Elements*, 6th Ed., John Wiley & Sons, Inc., New York.

Zucrow, M. J. and Hoffman, J. D, 1977, *Gas Dynamics*, Vol. II, John Wiley & Sons, Inc., New York.