

Automatic NC-Data Generation Method for 5-axis Cutting of Turbine-Blades by Finding Safe Heel-Angles and Adaptive Path-Intervals

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In this paper, an efficient method for generating 5-axis cutting data for a turbine blade is presented. The interference elimination of 5-axis cutting currently is very complicated, and it takes up a lot of time. The proposed method can generate an interference-free tool path, within an allowance range. Generating the cutting data just point to the cutting process and using it to obtain NC data by calculating the feed rate, allows us to maintain the proper feed rate of the 5-axis machine. This paper includes the algorithms for: (1) CL data generation by detecting an interference-free heel angle, (2) finding the optimal tool path interval considering the cusp-height, (3) finding the adaptive feed rate values for each cutter path, and (4) the inverse kinematics depending on the structure of the 5-axis machine, for generating the NC data.

Key Words: 5-axis Cutting, Turbine Blade, Heel Angle, Interference Avoidance, Cusp-height, Tool Path Interval

1. Introduction

Five-axis cutting issued for manufacturing the shapes that can't be done by 3-axis cutting. It is successfully used in the production of turbine blades, dies or master models for automobile tires, screws for ships, etc. It offers many advantages over 3-axis cutting, such as better tool accessibility and improved surface finish, but the

process for generating the NC data is much more complicated and the manufacturing costs are higher than those of 3-axis cutting (Li's, 1994; Park's, 1995; Lee's 1996).

A turbine blade is a part that is mounted on aircraft or on the turbines of ship's engines. Fig. 1 shows an example of a turbine blade. By using a 3-axis milling machine, we cannot make a blade without changing set-up posture. Though 4-axis machining can make the blade without changing the posture by using a ball end-mill, only 5-axis machining guarantee the efficiency of using a flat end-mill without changing the posture.

In order to generate proper 5-axis NC codes for cutting a turbine blade, we have to determine

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Revised January 5, 2004)



Fig. 1 Example model of a turbine blade

the tool axis vector as well as the feed rates and to check the interferences of the tool-paths. This paper proposes to create a method for generating 5-axis NC-data, allowing us to solve these problems using the polyhedral model of a turbine blade.

A lot of research have been conducted regarding 5-axis cutting and the generation (Li's, 1994 ; Park's, 1995 ; Lee's, 1996 ; Jung's, 1998 ; Lee's, 1998) and verification of generated NC data (Huang's 1995). In regards to the 5-axis tool path, research is being conducted on the interference-free tool paths and the regulation of material-removal-rate. However, a lot of computation time is required to verify the interference path (Park's, 1995). A parametric surface was generally used (Li's, 1994 ; Jung's, 1998, Lee's, 1998 ; Choi's, 1993) and, recently, polyhedron models have been introduced to generate the NC data (Li's, 1994 ; Park's, 1995). To generate the cutting data for the NC cutting of blade shapes, the wasted part of the blade is measured and approximated into a curve with the help of the related library (Fleisig's, 1997). In particular 4-axis cutting, the method of producing tool orientation has already eliminated the interference of a ball end-mill (Suh's, 1992) If the blade surface is machined by a flat end-mill, 5-axis control is necessary and the performance could be improved by regulating the heel angle.

Various methods have been used to generate 5-axis NC-data by using parametric surface models, Z-map models or polyhedron models. The alternative processes are proposed for making a propeller by using above models (Park's, 1995). It also presents a method for improving the speed by using the definition of a Bucket-map, which

means the elimination of interference in a polyhedral model.

We establish the CC (Cutter Contact Point) by using a polyhedral model of the blade surface. A CC-parametric path can't be generated because most of the blades can't be modeled using only one surface. Therefore, we generate a CC-Cartesian path by using a polyhedron model of the blade. The resultant 5-axis tool path is generated so as to machine the cross section of a blade simultaneously, the tool path intervals keep the cusp-height within allowance and calculate the minimum intervals between two adjacent paths, in the shortest time (Lee's, 2001 ; Cho's, 1995 ; Lee's, 1996 ; Kim's, 1994 ; Lee's, 2001).

In 5-axis cutting, the rigid linear and rotating motions occur simultaneously but most 5-axis machines are controlled by regular feed rates under any kind of motion. Therefore, we need a procedure to calculate the feed rates along the tool path, taking into considering the tool-axis and kinematical structures of 5-axis machine. Research for high accuracy 5-axis cutting has been conducted, taking into consideration the material-removal-rate, and the cusp-height, and presented examples of various sculptured surfaces.

2. Introduction of a System

The system for the 5-axis cutting of a turbine blade is illustrated in Fig. 2. Data regarding the blade surface are needed in order to generate the CC (Cutter Contact) data and tool orientation. The surfaces of a blade can be represented by using triangular mesh. We use the approximated surface for the computational efficiency. The input of the proposed system is polyhedral surface in STL format. The program generates the position of a cutting cross-section, which satisfies the required cusp-height in the blade surface. The intersection curve between the generated section and polyhedral model is the location of the tool CC point. The normal vector of the triangle in the polyhedron is kept as an intersection curve vector and is used to modify the tool point (CL point). Though the tool heel angle is known at the very

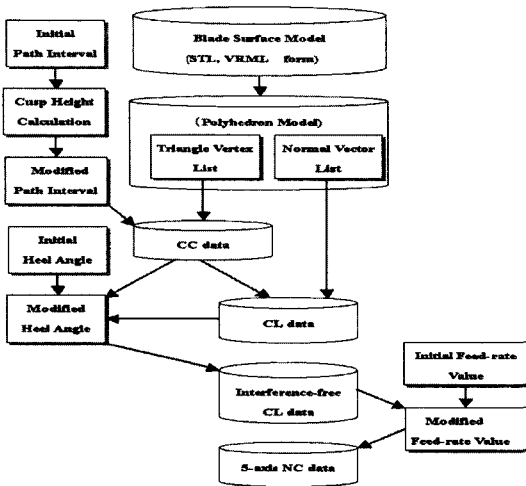


Fig. 2 Practical steps in calculating 5-axis NC-data for cutting turbine blades

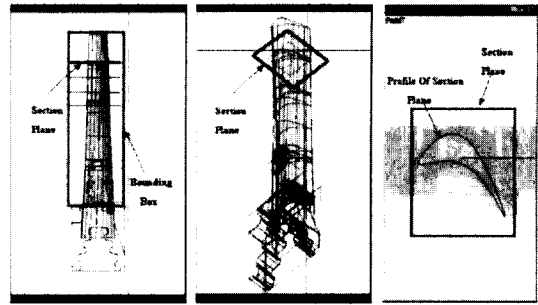
beginning, it becomes an interference-free heel angle through the process of offered interference checking. In fact, the motion rate of the axis feed rate value and the tool's tip is different in 5-axis cutting. So in real cutting, the feed rate value should be modified according to the tool tip cutting rate. By using these data, we can make 5-axis NC data.

3. The Generation of Interference-Free CL-data and the Determination of Tool Path Intervals

3.1 CC data generation

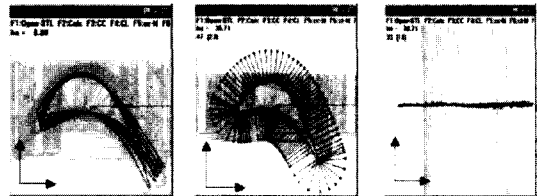
Though there are many methods for generating a 5-axis cutter path, we use CC data which is the procedure for generating the intersection curve between a cross section and the polyhedron. As shown in Fig. 3(a), (c) assumed that the calculated tool path intervals of the section planes are the boundaries for the minimum and maximum blade length in the bounding box. Fig. 3(b), (c) illustrates the generation of the section planes and intersecting lines of the blade surface.

If all the triangles and intersection line segments are calculated, it is possible to connect the intersection lines by comparing the starting and ending points. The intersection lines can be generated by moving the location of the section



(a) Side view (b) Isometric view (c) Top view

Fig. 3 Example of a blade profile intersection of blade surface and section plane



(a) Section profiles of a blade (b) Top view (c) Side view

Fig. 4 Blade profile and their normal vectors

plane, and the algorithm for the calculation of the section intervals is discussed in section 3.5. As shown in Fig. 4(a), the resultant intersection curves of the cross-sections are the profile of its section, Fig. 4(b) and Fig. 4(c) represent the normal vectors along the curve.

3.2 Orientation of tool

The tool orientation will be determined by using the profile of a blade cross section and normal vector at every point. If the direction of the profile of the normal vector and the tool axis vector are identical as in Fig. 5(a), it's possible to use 5-axis cutting. However, in the case of the cutting region including tool bottom, it would be impossible to use 5-axis cutting with a flat end-mill, as it does not have a cutter in the bottom center. As shown in Fig. 5(b), if cutting with the heel of tool, the tool axis vectors will be identical but the CC point and CL points won't be. If the tool radius of the flat end-mill is r , the normal vector of the surface is \mathbf{n} , the tool forward vector is \mathbf{f} , and the CC point of the profile is \mathbf{C} , the CL point orientation \mathbf{L} , can be

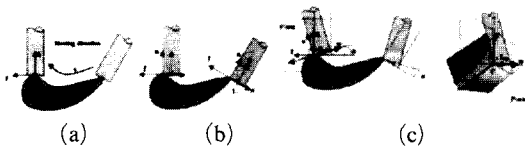


Fig. 5 Cutter location for 5-axis flat-end cutter
 (a) Heel angle=0 (b) Heel angle=0
 (c) Heel angle>0

calculated as follows :

$$\mathbf{L} = \mathbf{C} + r\mathbf{w} \quad (1)$$

where, $\mathbf{w} = -\mathbf{f}$.

In fact, the heel angle should be used to improve quality of cutting-surface and eliminate the interference of tool. As in Fig. 5(c), normal vector of cross section is the axis of rotation and the profile rotate degree of heel angle around it. If so, we can get new vector as tool axis vector and CL point orientation \mathbf{L}_θ is like Eq. (2), if the heel angle value is available.

$$\mathbf{L}_\theta = \mathbf{C} + r\mathbf{w} \quad (2)$$

where, $\mathbf{w} = \mathbf{n} \times (\mathbf{n} \times \mathbf{u}) / |\mathbf{n} \times \mathbf{u}|$
 $\mathbf{u} = \mathbf{n} \cdot \cos \theta + \mathbf{f} \cdot \sin \theta$
 $\mathbf{v} = (\mathbf{n} \times \mathbf{u}) / |\mathbf{n} \times \mathbf{u}|$
 $\mathbf{w} = \mathbf{n} \times \mathbf{v}$

3.3 Interference occurrence

In 5-axis cutting, the flat end-mill can reduce the cusp-height and improve cutting by using a varied heel angle in comparison to ball end-mill. It is better to maintain the initial heel angle if there is no interference.

At the cross section profile of the blade shape, there is a concave section which requires the elimination of interference. This is due to the cross section profile of the blade length direction being a smooth curve. As curvature is relatively minimal, there is no need to eliminate the interference with the length direction of the blade. If a cutter path is generated according to a cross section profile, interference can only be eliminated by modifying the heel angle. In fact, interference checking is conducted as many times as the number of cross section profiles, and sometimes it generates a heel angle fixed to one profile. If this occurs, the whole surface can be machined

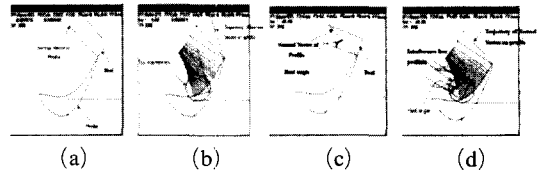


Fig. 6 Example of interference-free tool-path in 5-axis machining of a turbine blade (heel angle=detected)

with a similar heel angle.

If a flat end-mill is used to machine blade the surface, interference will occur when the direction of the surface normal vector and the tool axis vectors are identical (angle=0). The tool orientation, when the normal vector of the profile and the tool axis vectors are identical, is shown in Fig. 6(a). When the value of the heel angle is zero, the tool interference in the concave parts is like that shown in Fig. 6(b). Tool interference can be avoided by changing the tool orientation or when, the blade length direction has less curvature. The profile of the cross section should be used as a tool path. Any interference between the profile of the cross section of the blade and the tool should be eliminated. By modifying the heel angle it was not only possible to vary the tool orientation but also to increase the possibility for a heel angle not to interfere with the whole profile. As a result, the heel angles should be determined differently at the interference locations.

Results show that it was possible to generate heel angles that perfectly eliminate interferences at every profile. The method for eliminating interference was through process of calculating appropriate heel angle at every CC point. When the appropriate heel angle is determined, an interference-free tool path was generated as in Fig. 6 (c) and Fig. 6(d). Fig. 6(c) shows that the direction of the tool axis incline is at the same degree as that of the heel angle. The tool path machined by the modified heel angle is shown in Fig. 6(d). When heel angle=0, there is no interference in the concave part.

3.4 Determination of heel angle

The range of the interference free heel angle

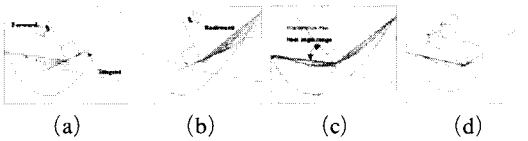


Fig. 7 Auto-detection of the interference-free heel angle of the cutter. (a) Forward detection (b) Backward detection (c) Feasible range of heel angle (d) Auto detection feasible cutter

can be determined by calculating the minimum and maximum in the range of the variable tool orientation. The profile of the cross section is the trajectory of the CC-points. Fixing one of the CC-points enables us to find the valid range of the heel angle. If the range of the interference free heel angle has been calculated, only the heel angle with a certain standard in the range of the profile interval needs to be selected. For example, the heel angle selected, should be close to the initial heel angle, or nearly equal to the medium value within the range.

Every point in the point sequence of a profile is an interference free heel angle, and the proper heel angle must be selected within the range. There are three proposed methods for selecting. The first method is to make the fixed proper heel angle application. The second method is the modification of a heel angle applied to a path only where interference exists. The third, proposed method to select a proper angle within the range of interference-free heel angles, in order to minimize the change of the heel angle at each continuous point.

It is assumed that the cutting tool method in a counter-clockwise direction along the profile curve as shown in Fig. 7. In reality, the profile must be kept in an orderly sequence of points. The direction of movement was classified as a forward or backward direction. Fig. 7(a) shows a semi-straight line, which started at the initial cutting position and finished forward of the cutting position. The tangent vector of the profile must be maintained, on the semi-straight line. The same must be done in a reverse direction. The range of the heel angle that is interference-free is determined by putting the two direction vectors

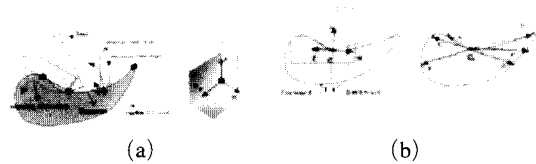


Fig. 8 Calculation of the feasible range of the heel angle using a directional line from the CC point to another point on the profile

on the positions.

When the range of the heel angle is the same as that of Fig. 8(a) the forward vector and the backward vector, which determines the range, will be the same as that of Fig. 8(b). It can be calculated easily by using the vector operation for each profile point sequence. This is done after calculating the outer product vector \mathbf{v} of the tool moving direction vector, and the tool axis vector. The procedure for determining the direction vectors of the tool, so as to ensure a safe and collision free heel angle, is shown in Appendix B.

3.5 Determination of tool path interval

After being machined by a flat end mill in 5-axis NC cutting, an uncut amount is left between a set of adjacent cutter paths. The height, from the top of the uncut volume to the surface, is called the cusp-height. In NC cutting, the tool path interval is generally pre-determined in order to get the required surface roughness, and consider the cutting precision and the cutting efficiency. This method is however, not applicable to 5-axis NC cutting as the tool axis vector changes according to the tool position, while cutting along the tool path, significantly influencing the cusp-height. As a result, the tool path interval, if decided at the initial stage, can not maintain quantitative cusp-height at each position.

If the tool path interval is shortened in order to eliminate the error, in the cusp-height, after actual cutting, it will repeatedly cut at an unnecessary location. It has become inefficient as it simply increases the quantity of CC and CL data.

This paper suggests the method for determining tool path interval based on the calculable cusp-height on the common plane. The common plane is a plane formed by the vector connecting two

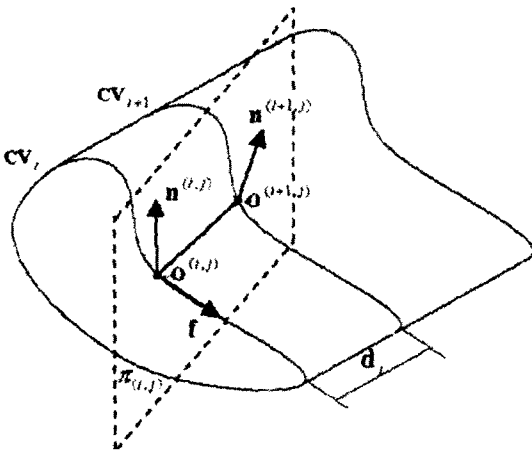


Fig. 9 Generation of the tool path interval of a blade

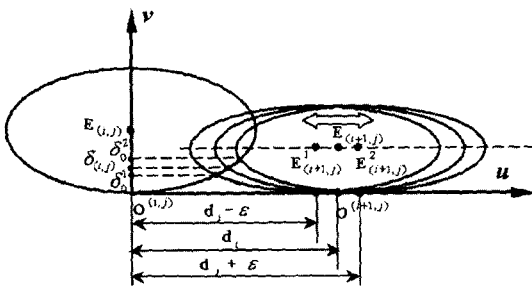


Fig. 10 Calculation of the tool path interval

CC-points between the present and the next tool paths, and the summation vector of the normal vector (Fig. 9). The cusp-height on a common plane is calculated between two adjacent tool paths. This method allows us to determine the optimal tool path, for cutting in the shortest time by keeping the cusp-height in an acceptable error range in 5-axis NC cutting of a turbine blade.

After defining the common plane, which is, d_i , away with the two profile and including corresponded two points $O^{(i,j)}$, and, $O^{(i+1,j)}$ of connecting tool path. We can derive two ellipses which are intersection curves between the common plane and the tool bodies. $O^{(i,j)}$ and $O^{(i+1,j)}$ are the CC-points. As shown in Fig. 10, the cusp-height will be calculated geometrically. $\delta_{(i,j)}$ must be calculated at every points along the curve. If is maximal value of $\delta_{(i,j)}$, $\delta_{(i,j)max}$ must be smaller the allowance. Because $\delta_{(i,j)}$ varies with d_i proportionally, we can choose proper

path interval within allowance by changing d_i .

4. Generation of 5-axis Cutting Data of the Blade

4.1 NC data generation

After the cutting location and the direction of the tool axis vectors have been established, the NC data of appropriate kinematics must be calculated. For the purpose of this research the 5-axis machine tool for the cutting blade had three axes of linear translating (x, y, z), two axis for the rotating (A, B) as shown in Fig. 11.

Due to the tool interference elimination process, tool axis vector u is calculated using the generated heel angle giving the cutting location (P). It is possible to generate because of tool interference elimination, and cutting location P are given, NC data generate NC data that complies with the structure of 5-axis tool kinematics by applying inverse kinematics. Consider the coordinate of the CL point that is calculated in Eq. (4). The machine tool tip point $P(p_x, p_y, p_z)$, utilizes the applicable tool axis vector $u(u_x, u_y, u_z)$. This can be then generate $T(x, y, z)$ as a value of the X, Y and Z-axis, and the tilting angle a, b as a value of the A, B axis. This can order the NC machine tool according to inverse kinematics by calculation that agree with to the kinematics structure of the 5-axis machine tool.

Figure 11 illustrates the 5-axis machine tool used to generate the cutting data for this research. The related inverse kinematics expressions were calculated as shown in Eq. (3). Eq. (4) shows the related inverse kinematics expressed as a matrix.

$$T = P \text{ Rot } X(a) \text{ Trans}(d, 0, l \cdot u_x - l) \quad (3)$$

$$\begin{pmatrix} x & y & z & 1 \end{pmatrix} = \begin{pmatrix} p_x & p_y & p_z & 1 \end{pmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos a & \sin a & 0 \\ 0 & -\sin a & \cos a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ d & 0 & l \cdot (u_x - 1) \end{bmatrix} \quad (4)$$

where $b = \text{atan } 2(n_x, d)$ (here, $d = \sqrt{u_y^2 + u_z^2}$)
 $a = \text{atan } 2(u_z, u_y)$

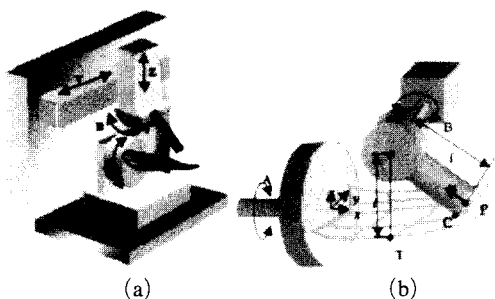


Fig. 11 Schematic of the 5-axis milling machine for machining a turbine. (a) Coordinate system of the Machine (b) Geometry for inverse-kinematics

- l : the distance from the axis of rotation to the tool tip
- $P(p_x, p_y, p_z)$: the tool tip of the considered heel angle
- $u(ux, uy, uz)$: the vector (unit vector) of the considered heel angle

4.2 Modification of the feed-rate

The 5-axis machine tool used in this research had a FANUC 15 M as the NC controller. When the feed rate of each axes is calculated, it is important to calculate the feed rate over the given feed rate. An 1 mm distance is corresponding to an 1 degree angle without considering if it is a linear or a rotary motion (FANUC's, 1997). In the case of 5-axis cutting, where there is both a linear and rotary motion, the feed rate for each NC block must be calculated separately in order to get the required feed rate at the tool tip (Choi's, 1998 ; Lee's, 1996).

In the case of calculating the movement between two points, the values of the X, Y, Z, A and B-axis of the NC data are x, y, z, a and b respectively. If the feed rate is F_0 , the distance L_0 is calculated as shown in Eq. (5). The feed-time t_0 calculated as shown in Eq. (6). However, the value for each coordinate is simply a relative value.

When the real distance, L , is moving, the CNC controller determines the feed rate of each axis according to the time t_0 calculated according to L_0 . Time, t , when it is moving according to L with the given feed rate F_0 , is the same as shown

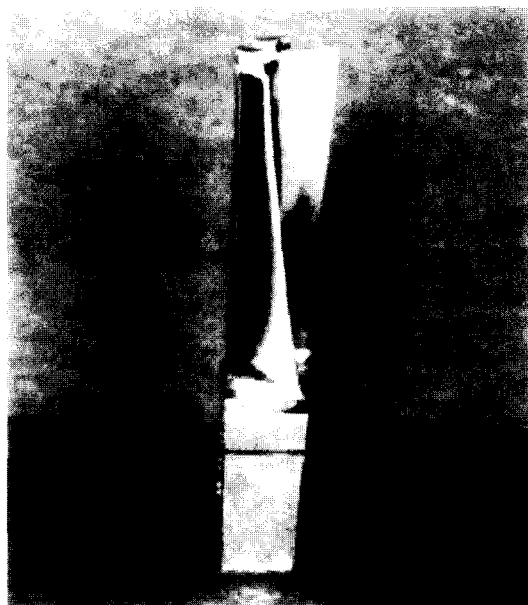


Fig. 12 Photograph of a machined turbine blade by proposed method

in Eq. (7).

In this case, if we want to keep the feed rate in the actual moving distance, L , a modified feed rate, F needs be applied to the relevant NC block (the feed rate is revised by time t , which is corresponds to L). The new feed rate can be calculated as shown by Eq. (8), with a feed-rate of F_0 , the rate of computed t and t_0 after working out Eq. (7) and Eq. (8). And Fig. 12 shows a sample that is machined by using result of the proposed algorithm.

$$L_0 = \sqrt{x^2 + y^2 + z^2 + a^2 + b^2} \tag{5}$$

$$t_0 = L \frac{L_0}{F_0} \tag{6}$$

$$t = \frac{L}{F} \tag{7}$$

$$F = F_0 \frac{t_0}{t} \tag{8}$$

5. Conclusion

This paper investigates a method of optimizing 5-axis NC data for cutting turbine blades. The NC data has interference-free heel angles and has an optimized tool path interval considering the

cuspl-height. The cross sectional curves are used to determine CC-points and the tool orientation. They are derived from a polyhedral model.

We propose a proper algorithm to generate a interference-free CL data by using heel angle. An efficient method is presented to make NC data according to CL data and the structure of the 5-axis machine. These are written in C-language. We tested these by machining several samples and the result is satisfactory for industrial use.

Appendix

A. Finding the intersection curve of a section plane and the mesh of the blade

Step 1. Define the section plane in the form of an implicit equation.

E.g.) The expression of the plane

$$f(x, y, z) = ax + by + cz + d = 0 \quad (9)$$

(where, the normal vector of the plane = (a, b, c))

Step 2. Calculate the expression that corresponds to the edge according to the $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, $P_3(x_3, y_3, z_3)$ of three vertexes of triangle.

E.g.) The expression of the edge line

$$R(t) = P_1 + t(P_2 - P_1) \quad (10)$$

(where, $t = [0, 1]$)

Step 3. Calculate the intersection lines of the section plane and triangle (three line segments, the plane that intersects at the two points of intersection, and the two points of intersection, form the intersection line segment).

E.g.) In order to calculate a point of intersection between a straight line and a plane, use Eq. (9) and Eq. (10) simultaneously to calculate t^* , which exists between 0 and 1. In this case, the intersection point is $R(t^*)$.

Step 4. Preserve the two points of the intersection line segment and the normal vector $\mathbf{n}(nx, ny, nz)$ of the triangle in the arrangement.

Step 5. Repeat steps 2, 3, and 4 for all the triangles.

Step 6. Align the segment, which is preserved in the arrangement, in linking form.

B. Procedure for finding the safe heel-angle to get the collision-free tool vector

The direction vector can be calculated by using each position of the CC-point in profile. As shown in Fig. 10, it is assumed that \mathbf{f} is a tool direction vector and \mathbf{n} is a surface normal vector, \mathbf{u} is an axis vector of a tilting heel angle degree, and define the point C_i is defined (i is the sequence number of C). The method of calculating the range from the initial angle to the final angle is as follows:

$$\mathbf{f} = C_{i+1} - C_i = C_{ahead(i)} - C_i$$

$$\mathbf{w} = -\mathbf{f}$$

$$\mathbf{v} = \mathbf{n} \times \mathbf{f}$$

$k = ahead(i)$ /* tool moving direction, that is, $k = i + 1$ */

$$\mathbf{a} = C_k - C_i$$

while ($k \neq back(i)$) {

$$k = ahead(k)$$

$$\mathbf{c} = C_k - C_i$$

if $(\mathbf{c} \cdot \mathbf{f} < 0)$ then break; /* forward, if not, stop */

if $(\mathbf{a} \times \mathbf{c}) \cdot \mathbf{v} < 0$ then $\mathbf{a} = \mathbf{c}$

$k = back(i)$ /* tool moving direction, that is, $k = i - 1$ */

$$\mathbf{b} = C_{i-1} - C_i$$

while ($k \neq ahead(i)$) {

$$k = back(k)$$

$$\mathbf{c} = C_k - C_i$$

if $(\mathbf{c} \cdot \mathbf{w} < 0)$ then break; /* backward, if not, stop */

if $(\mathbf{b} \times \mathbf{c}) \cdot \mathbf{v} > 0$ then $\mathbf{b} = \mathbf{c}$

}

$$\mathbf{t} = \mathbf{a} \times \mathbf{v}$$

if $(\mathbf{b} \times \mathbf{w}) \cdot \mathbf{v} < 0$

heel angle_{min} = 0 /* obtuse angle management */

else

$$\text{heel angle}_{\min} = \mathbf{a} \cos(\mathbf{w} \cdot \mathbf{b} / (|\mathbf{w}| |\mathbf{b}|))$$

$$\text{heel angle}_{\max} = \mathbf{a} \cos(\mathbf{t} \cdot \mathbf{b} / (|\mathbf{t}| |\mathbf{b}|))$$

References

Cho, H. D. and Yang, M. Y., 1995, "A Study on the Five-axis End Milling for Sculptur-

ed Surfaces," *KSME Journal*, Vol. 9, No. 4, pp. 428~437.

Choi, B. K. and Jerard, R. B., 1998, "Sculptured Surface Cutting," Kluwer Academic Publishers.

Choi, B. K., Park, J. W. and Jun, C. S., 1993, "Cutter-Location Data Optimization in 5-axis Surface Cutting," *Computer-Aided Design*, Vol. 25, No. 6 June, pp. 377~386.

FANUC Ltd., 1997, "FANUC Series 15MA Operators Manual (programming) for Cutting," FANUC.

Fleisig, R. V. and Spence, A. D., 1997, "Integrated Digitizing, Path Planning and Five-axis Cutting for the Refurbishment of Compressor and Turbine Blade," *Manufacturing Science and Technology, MED-Vol. 6-1 Volume 1*.

Huang, Y. H. and Oliver, J. H., 1995, "Integrated Simulation, Error Assessment, and Tool Path Correction for Five-axis NC Milling," *Journal of Manufacturing Systems*, Volume 14 Number 5.

Jung, J. Y., 1998, "Parametric Shape Design and CNC Tool Path Generation of a Propeller Blade," *Journal of the KSPE*, Vol. 15, No. 8, pp. 46~59.

Kim, B. H. and Chu, C. N., 1994, "A Study Quantitative Analysis of Surface Roughness for Precision Cutting of Sculptured Surface," *KSME Journal*, Vol. 18, No. 6, pp. 1483~1495.

Lee, C. S., 1996, "CAD/CAM ," Turbo-Tech (KOREAN).

Lee, C. S., 1996, "Surface Modeling and 5-axis NC Cutting of Automobile Tire Model," *IE-Interface*, Vol. 9, No. 2, pp. 129~141.

Lee, G. Y., Nam, W. W. and S. J. Lee, 2001, "The Tool path Generation for Free-Formed Surface with the Flat End-mill," *Journal of KSPE*, Vol. 18, No. 4, pp. 104~111.

Lee, S. G. and Yang, S. H., 2001, "CNC Tool Path Planning for Free-Form Sculptured Surface with a New Tool Path Interval Algorithm," *Journal of KSPE*, Vol. 18, pp. 43~49.

Lee, Y. S., 1998, "Non-Isoparametric Tool Path Planning by Cutting Strip Evaluation for 5-axis Sculptured Surface Cutting," *Computer-Aided Design*, Vol. 30, No. 7, pp. 559~570.

Lee, Y. S. and Chang, T. C., 1996, "Machined Surface Error Analysis for 5-axis Cutting," *INT. J. PROD. RES.*, Vol. 34, No 1, pp. 111~135.

Li, Susan X. and Jerad, R. B., 1994, "5-axis Cutting of Sculptured Surfaces with a Flat-end Cutter," *Computer-Aided Design*, Volume 26 Number 3, pp. 165~178.

Park, Z. H., 1995, "Development of A Five-axis CAM System for Free-Surface Grinding," Doctor thesis, Mechanical Engineering Dept., KAIST.

Suh, S. H. and Lee, K. S., 1992, "Avoiding Tool Inference in 4-axis NC Cutting of Rotationally Free Surfaces," *IEEE Trans on Robotics & Automation*, 8 (6).