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Intuitionistic Fuzzy Circles

Mi-Hye Kim

Abstract In this paper, we introduce the intuitionistic fuzzy circle with a pair of degree which are membership and nonmembership functions, and investigate the properties of them. And also we experiment with developing fuzzy geometry by limiting process of the notion of intuitionistic fuzzy circles which restricted the notion of fuzzy spheres in order to apply to 2-D image.

요 약 본 논문에서는 소속과 비소속의 정도를 나타내는 함수를 쌍으로 가진 intuitionistic 퍼지 원을 소개하고, 그들의 성질을 조사해 보았다. 또한, 퍼지 구의 개념을 2차원 영상에 적용하기 위하여 차원을 제한한 intuitionistic 퍼지 원이 극한 프로세스에 의해 퍼지 기하에 기여함을 볼 수 있다.

Keywords : simple closed circles, arc function, intuitionistic fuzzy circle, intuitionstic fuzzy convergent

1. INTRODUCTION

Since Zadeh[1] introduced the concept of fuzzy set in 1965, fuzzy mathematics and its application has been developing rapidly. In fuzzy sets, membership degrees indicate the degree of belongingness of elements to the collection or the degree of satisfaction of elements to the property corresponding to the collection. As a generalization of fuzzy sets, the concepts of intuitionistic fuzzy sets was introduced by Atanassov[2]. Intuitionistic fuzzy sets have a pair of membership degree and nonmembership degree. Rosenfeld [3-6] introduced certain ideas in fuzzy plane geometry, e.g., area, height, width, diameter and perimeter of a fuzzy subset of a plane. The results developed in these papers have applications to pattern recognition. Buckley and Eslami [7] began what will be an extensive development of fuzzy plane geometry. The development used fuzzy real numbers for measures of area, perimeter, and so on while that used real numbers. Cheng[9] proposed fuzzy geometry by limiting process of the notion of fuzzy spheres whose degree of circularity is measured by a fuzzy set, called a circularity function. In [8], we studied fuzzy circle and its application in which we modified the fuzzy sphere in [9] to fuzzy circle in order to detect in 2-D image(a hidden circular shape) as a kind of fuzzy circle application, and using this such theory, and we proposed a circular traffic sign detection process and automatic circular convergence in the problem of drawing a circle by computer. In this paper, we study the expanded version of such circle. For the readers' convenience and completeness, we include most concepts applied in [8].

School of Electrical and Electronics Engineering Chungbuk National University

2. INTUITIONISTIC FUZZY CIRCLES

In this section, we introduce some definitions and basic properties which are necessary to the main arguments in the article.

From now on, $x(t)$ and $y(t)$ are continuous parametric functions on I that define a simple closed curve C on the xy-plane. Let X be a nonempty crisp set. A *fuzzy subset* μ of X is a function from X into the closed unit interval [0,1], that is, $\mu:X\rightarrow[0,1]$. In the literature, a fuzzy set A may also be written as a set of ordered pairs: $A = \{(x, \mu(x)):x \in X\}$, where $\mu_A(x)$ is referred to as the membership function or grade of membership. If the range of the fuzzy set μ contains only two values 0 or 1, then μ is identical to the characteristic function of a subset of X. An intuitionistic fuzzy set A (IFS for short) is an ordered pair $A = \{(x, \mu_A(x), \nu_A(x)):x \in X\}$, where the function $\mu_A:X \rightarrow [0,1]$ and $\nu_A:X \rightarrow [0,1]$ denote the degree of membership and the degree of nonmembership respectively, and $\mu_A(x)+\nu_A(x) \le 1$. For the sake of simplicity, we shall use the symbol $A = \langle \mu_A(x), \nu_A(x) \rangle$. Obviously every fuzzy set μ_A on X is an IFS of the form $A = \langle \mu_A(x), 1-\mu_A(x) \rangle$.

Definition 2.1([2]) Let $A = \langle \mu_A(x), \nu_A(x) \rangle$ and $B = \langle \mu_B(x), \nu_B(x) \rangle$ be IFSs on X. Then

(1) $A \subset B$ iff $\mu_A \le \mu_B$ and $\nu_A \le \nu_B$

(2) A=b iff $A \subset B$ and $B \subset A$

(3) $A^c=(\nu_A, \mu_A)$

(4) $A \cap B = \mu_A \wedge \mu_B, \nu_A \wedge \nu_B$

(5) $A \cup B = \mu_A \vee \mu_B, \nu_A \wedge \nu_B$

Motivated by the concept of equations of motion we can specify curves in the two-dimensional *xy*-plane by using equations, $x = x(t)$, $y = y(t)$ to express the coordinates of a point (x, y) on the curve as functions of auxiliary variable t. This is called parametric equation for the plane, and the variable t is called a parameter. Let C be a simple closed curve consisting of all ordered pair $(x(t), y(t))$ on the *xy*-plane, where $x(t)$ and $y(t)$ are continuous real valued functions defined on a closed Interval $I = [a, b]$. Circles and elliptics are typical examples of simple closed circles.

Definition 2.2 ([9]) (1) A function $\mu_{-c}:(I \rightarrow [0,1])$ is called an *arc function* of C if

① there exits a function $f:R \times R \rightarrow [0,1]$ such that $\mu_{-c}(t) = f(x(t), y(t))$ for all $t \in I$,

② $\mu_{-c}(t) = 1$ for all $t \in I$ when C is a circle.

(2) A function $\nu_{-c}:I \rightarrow [0,1]$ is called an *nonarc function* of C if

① there exits a function $g:R \times R \rightarrow [0,1]$ such that $\nu_{-c}(t) = g(x(t), y(t))$ for all $t \in I$,

② $\nu_{-c}(t) = 1$ for all $t \in I$ when C is a circle.

Clearly, $\mu_{-c}(t)$, $\nu_{-c}(t)$ is a fuzzy subset of I. Intuitively, the arc function $\mu_{-c}(t)$ and the nonarc function $\nu_{-c}(t)$ can be thought of as numerical measures which are degrees of arc and nonarc (or a straight line) for a simple closed curve on the - plane, respectably.

Let C be a simple closed curve on the *xy*-plane defined as above. *An intuitionistic fuzzy circle* on the *xy*-plane is given by

$$\tilde{C} = x(t), y(t), \mu_{-c}(t), \nu_{-c}(t) | t \in I$$

where $0 \le \mu(t) + \nu(t) \le 1$ for all $t \in I$.

Roughly speaking, an intuitionistic fuzzy circle is formed by a simple closed curve C together with an arc functions $\mu_{-c}(t)$, and a nonarc function $\nu_{-c}(t)$ for all $t \in I$. For the sake of simplicity, we shall use the symbol

$\tilde{C} = \langle x(t), y(t), \mu_{-c}(t), \nu_{-c}(t) \rangle$ for an intuitionistic fuzzy circle.

If C_0 is a crisp circle, then the corresponding intuitionistic fuzzy circle must have the maximum degree of arc $\mu_{-C_0}(t) = 1$ and the minimum degree of nonarc $\nu_{-C_0}(t) = 0$ for $t \in I$. So we can write the crisp circle as $\tilde{C}_0 = \langle x(t), y(t), 1, 0 \rangle$.

An arc function $\mu_{\tilde{c}}(t)$, and a nonarc $\nu_{\tilde{c}}(t)$ function can be used as a measure in comparing two intuitionistic fuzzy circles. Let $\tilde{C}_1$ and $\tilde{C}_2$ be two intuitionistic fuzzy circles with arc functions $\mu_{\tilde{c}_1}(t), \mu_{\tilde{c}_2}(t)$ and nonarc functions $\nu_{\tilde{c}_1}(t)$, $\nu_{\tilde{c}_2}(t)$, respectively. Then $\tilde{C}_1$ is called *an intuitionistic fuzzy subcircle* of $\tilde{C}_2$, written as $\tilde{C}_1 < \tilde{C}_2$, if $\mu_{\tilde{C}_1}(t) \le \mu_{\tilde{C}_2}(t)$ and $\nu_{\tilde{C}_1}(t) \le \nu_{\tilde{C}_2}(t)$ for all $t \in I$.

And $\tilde{C}_1$ is said to be *equal* to $\tilde{C}_2$, written as $\tilde{C}_1 \sim \tilde{C}_2$, if $\tilde{C}_1 < \tilde{C}_2$ and $\tilde{C}_2 < \tilde{C}_1$, for all $t \in I$. From the definition of equal relation, we can see that the relation ~ is an equivalence relation.

The union $\{\tilde{C}_n\}$ of a sequence of intuitionistic fuzzy circles, denoted by $\bigcup_{n=1}^{\infty} \tilde{C}_n$, is defined as the intuitionistic fuzzy circle

$$\bigcup_{n=1}^{\infty} \tilde{C}_n = x^*(t), y^*(t), \mu \bigcup \tilde{C}_n(t), \nu \bigcup \tilde{C}_n(t)$$

where $(x^*(t), y^*(t))$ denotes the parametric representation of $\bigcup_{n=1}^{\infty} \tilde{C}_n$. and the arc function $\mu \bigcup \tilde{C}_n(t)$ is given by

$$\mu \bigcup \tilde{C}_n(t) = (\vee \mu_{\tilde{C}_n})(t) = \vee \mu_{\tilde{C}_n}(t),$$

and the nonarc function $\nu_{\bigcup}\tilde{C}_n(t)$ is given by

$$\nu_{\bigcup}\tilde{C}_n(t) = (\ \wedge \nu_{\tilde{C}_n})(t) = \ \wedge \nu_{\tilde{C}_n}(t)\ .$$

we can see that the intersection of the sequence of intuitionistic fuzzy circles $\bigcap_{n=1}^{\infty} \tilde{C}_n$ is similar to the sequence union $\bigcup_{n=1}^{\infty} \tilde{C}_n$ such that

$$\bigcap_{n=1}^{\infty} \tilde{C}_n = \langle (x^*(t), y^*(t)), \mu_{\bigcap}\tilde{C}_n(t), \nu_{\bigcap}\tilde{C}_n(t)\rangle$$

where $(x^*(t), y^*(t))$ denotes the parametric representation of $\bigcap_{n=1}^{\infty} \tilde{C}_n$. and the arc function $\mu_{\bigcap}\tilde{C}_n(t)$ is given by

$$\mu_{\bigcap}\tilde{C}_n(t) = (\ \wedge \mu_{\tilde{C}_n})(t) = \wedge \mu_{\tilde{C}_n}(t)\ ,$$

and the nonarc function $\nu_{\bigcap}\tilde{C}_n(t)$ is given by

$$\nu_{\bigcap}\tilde{C}_n(t) = (\ \vee \nu_{\tilde{C}_n})(t) = \ \vee \nu_{\tilde{C}_n}(t).$$

The complement of $\tilde{C}$, denoted by $\tilde{C}^c$, is defined to be the intuitionistic fuzzy circle

$$\tilde{C}^c = \langle x(t), y(t), 1 - \mu_{\tilde{c}}(t), 1 - \nu_{\tilde{c}}(t)\rangle$$

The following Proposition gives the most elementary properties of the intuitionistic fuzzy circles.

Proposition 2.3 Let $\tilde{C}_1, \tilde{C}_2$ be intuitionistic fuzzy circles, and $\tilde{C}_0$ be formed by a crisp simple closed circle. Then

(1) $\tilde{C}_1 \preccurlyeq \tilde{C}_1 \bigcup \tilde{C}_2,\ \tilde{C}_1 \geq \tilde{C}_1 \bigcap \tilde{C}_2$

(2) $\tilde{C}_1 \preccurlyeq \bigcup_{n=1}^{\infty} \tilde{C}_n,\ \ \tilde{C}_1 \geq \bigcup_{n=1}^{\infty} \tilde{C}_n$

(3) $\tilde{C} \preccurlyeq \tilde{C} \bigcup \tilde{C}^c,\ \tilde{C}^c \preccurlyeq \tilde{C} \bigcup \tilde{C}^c$

(4) $\tilde{C} \geq \tilde{C} \bigcap \tilde{C}^c,\ \tilde{C}^c \geq \tilde{C} \bigcap \tilde{C}^c$

(5) $\tilde{C} \bigcup \tilde{C}^c \neq \tilde{C}_0$

(6) $\tilde{C} \bigcap \tilde{C}^c \neq \varnothing$

Properties (1)-(4) are similar to those of the classical theory, but (5)and (6) are somewhat different from the classical ones. We should note that an intuitionistic fuzzy circle intersection its complement of the intuitionistic fuzzy circle is not emptyset.

In a classical measure theory, there are the monotone convergence theorem, the uniform convergence theorem, and Lebesgue dominated convergence theorem, all of which are well known. For the intuitionistic fuzzy circle sequence, we will show that Convergence Theorem of intuitionistic fuzzy circle sequence for the sequence of monotone arc-nonarc functions.

For $n = 1, 2, \ldots,$ let C_n be a simple closed curve given parametrically in terms of ordered pair $x_n(t), y_n(t)$ for $t \in I$, where $x_n(t), y_n(t)$ are real valued continuous functions on I, and let $\tilde{C}_n$ be the intuitionistic fuzzy circle that is formed by C_n with the arc function $\mu_{\tilde{C}_n}(t)$ and the nonarc function $\nu_{\tilde{C}_n}(t)$, respectably, such that

$$\tilde{C}_n = x_n(t), y_n(t), \mu_{\tilde{C}_n}(t), \nu_{\tilde{C}_n}(t)\ .$$

Since the intuitionistic fuzzy circle has been defined with an arc and a nonarc function, we can introduce a concept of the convergence in the sense of arc function and nonaec function for intuitionistic fuzzy circle.

Definition 2.4 The sequence $\{\tilde{C}_n\}$ of intuitionistic fuzzy circles is said to *converge in the sense of arc-nonarc function* to the intuitionistic fuzzy circle if for all $t \in I$, converges to and converges to at the same time .

The following theorem may be called the "Convergence Theorem of intuitionistic fuzzy circle sequence for the sequence of monotone arc-nonarc functions," or "the Monotone Convergence Theorem".

Theorem 2.5 Let $\tilde{C}_1 \preccurlyeq \tilde{C}_2 \preccurlyeq \cdots \preccurlyeq \tilde{C}_n \preccurlyeq \cdots$ be a nondecreasing sequence of intuitionistic fuzzy circles. Then the sequence $\{\tilde{C}_n\}$ converges in the sense of arc-nonarc to the intuitionistic fuzzy circle $\bigcup_{n=1}^{\infty} \tilde{C}_n$.

Proof. For each $t \in I$, it is easily to see that $\{\mu_{\tilde{C}_n}(t)\}$ is a nondecreasing sequence of real numbers bounded above and $\sup\{\mu_{\tilde{C}_n}(t) \mid n = 1, 2, \ldots\} = \bigvee_{n=1}^{\infty}$.

Thus, it follows from Theorem 2.5 of [5] that $\lim_{n \to \infty} \mu_{\tilde{C}_n}(t) = \bigvee_{n=1}^{\infty} \mu_{\tilde{C}_n}(t)$ for all $t \in I$.

Since the arc function of $\bigcup_{n=1}^{\infty} \tilde{C}_n$ is given by $\bigvee_{n=1}^{\infty} \mu_{\tilde{C}_n}(t)$, the nondecreasing sequence of intuitionistic fuzzy circles converges in the sense of arc to the intuitionistic fuzzy circle .

It remains to prove that the sequence converges in the sense of nonarc to the intuitionistic fuzzy circle $\bigcup_{n=1}^{\infty} \tilde{C}_n$.

But the remaining part is similar to the above. So we can get the conclusion. . °·

By using Theorem 2.5, we immediately obtain the following corollary.

Corollary 2.6 Let $\tilde{C}_1 \succcurlyeq \tilde{C}_2 \succcurlyeq \cdots \succcurlyeq \tilde{C}_n \succcurlyeq \cdots$ be a nonincreasing sequence of intuitionistic fuzzy circles. Then $\{C_n\}$ converges in the sense of arc-nonarc to the intuitionistic fuzzy circle $\bigcap_{n=1}^{\infty} \tilde{C}_n$, that is, $\lim_{n \to \infty} \tilde{C}_n = \bigcap_{n=1}^{\infty} \tilde{C}_n$ in the sense of arc-nonarc function.

3. GEOMETRIC INTERPRETATION OF THE INTUITIONISTIC FUZZY CIRCLES

In this section, we will interpretate the intuitionistic fuzzy circles geometrically and consider the problem of drawing a circle by computer as the arc function gets closer and closer to 1 from intuitionistic fuzzy circle.

In [8], we introduced an example of arc function and in this section we consider that example with nonarc function.

Let $C = \{(x(t), y(t)) \mid t \in I\}$ be a simple closed curve on xy-plane. Then we can take Γ the smallest closed rectangle including. Let be the intersect point of two diagonals of Γ.

Define the function $\mu_{-C}(t)$ as following

$$\mu_{-C}(t) = \begin{cases} \dfrac{Min\{|P - P_0| : P \in C\}}{Max\{|P - P_0| : P \in C\}}, & P_0 \in \mathrm{Int}(C) \\ 0, & otherwise \end{cases}$$

where int(C) is the set of all interior point of C. It is easily to see that for $P_0 \in \mathrm{Int}(C)$,

$$0 \le \frac{Min\{|P - P_0| : P \in C\}}{Max\{|P - P_0| : P \in C\}} \le \frac{Max\{|P - P_0| : P \in C\}}{Max\{|P - P_0| : P \in C\}} = 1$$

So, we can see that $\mu_{\tilde{C}}(t)$ is an arc function of C. And we take the nonarc function $\nu_{\tilde{C}}(t) = 1 - \mu_{\tilde{C}}(t)$ for all $t \in I$ for the convention.

Hence we called the intuitionistic fuzzy circle $\tilde{C} = \langle (x(t), y(t)), \mu_{\tilde{C}}(t), \nu_{\tilde{C}}(t) \rangle$ $\mu_{\tilde{C}}(t) - \nu_{\tilde{C}}(t)$ intuitionistic fuzzy circle. From now on, we can see this from the following examples and consider the geometric significance of the intuitionistic fuzzy circle.

If C_0 is a crisp circle, then the parametric equations are given by

$x(t) = R\cos t,\ y(t) = R\sin t$,

where $t \in [0, 2\pi]$, R is a real constant.

Since C_0 is a crisp circle , we know that for all $t \in [0, 2\pi]$,

$$Min\{|P(t) - P_0| : P_0 \in C\} = Max\{|P(t) - P_0| : P_0 \in C\}$$

Hence the calculated arc function is $\mu_{-C}(t) = 1$, nonarc function is $\nu_{\tilde{C}}(t) = 0$ for all $t \in [0, 2\pi]$. Therefore the intuitionistic fuzzy circle $\tilde{C}$ is given by

$$C = \langle (R\cos t, R\sin t), 1, 0 \rangle.$$

Similarly, we can consider in case that C is an elliptic. Let $a > b$ and for $t \in [0, 2\pi]$. Then the parametric equations of an elliptic curve are given by $x(t) = a\cos t,\ y(t) = b\sin t$.

So the arc function is $\mu_{\tilde{C}}(t) = \dfrac{b}{a}$ and the nonarc

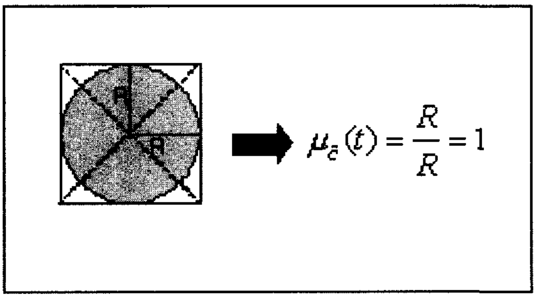

Fig. 1. is a circle

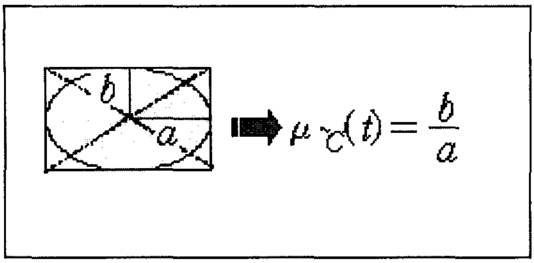

Fig. 2. is an elliptic

function is $\nu_{\tilde{c}}(t) = 1 - \frac{b}{a}$

Thus the intuitionistic fuzzy circle of an elliptic is expressed by $\tilde{C} = \langle (acost, bsint), \frac{b}{a}, 1 - \frac{b}{a} \rangle$.

If C is an arbitrary simple closed curve, the intuitionistic fuzzy circle equation as follows.

$$\tilde{C} = \langle x(t), y(t), \mu_{\tilde{c}}(t), \nu_{\tilde{c}}(t) \rangle$$

where $\mu_{\tilde{c}}(t)$ and $\nu_{\tilde{c}}(t)$ are defined same way to the above.

In the problem of drawing a circle by computer, we first consider a sequence of intuitionistic fuzzy circles with their arc functions in a non-decreasing order / their nonarc functions in a non-increasing order.

Initially, we begin with a given intuitionistic fuzzy circle. At those points near which the curve changes sharply, we may improve their degree of arc by adjusting the arc function step by step.

As the arc function gets closer and closer to 1and the nonarc function gets closer and closer to 0, we would expect that the intuitionistic fuzzy circle gets closer and closer to a crisp circle.

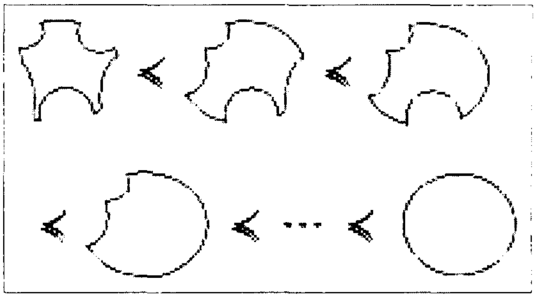

Fig. 3. Drawing a circle by computer

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