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An algorithm to find all solutions of blind deconvolution

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Abstract

This paper shows that blind deconvolution has only finite solutions when an original image and a point spread function are nonzero over a restricted domain, in other words, an observed image has a compact support. The key of the proof is to use z-transformations and factorizations of polynomials. Then, we propose an algorithm to find all finite solutions under the boundary condition. Finally, we confirm that we can extract all sets of an original image and a point spread function from a degraded image by using our algorithm in numerical examples.

Keywords : Image Restoration, Digital Signal Processing, Blind Deconvolution

I. Introduction

Generally, observed images are degraded and represented by a convolution of two signals. Here, one is an original image and the other is a Point Spread Function (PSF) that causes a degradation of an original image. The problem to estimate both an original image and a PSF from only one degraded image is called Blind Deconvolution (BD)[1].

Ayers and Dainty have proposed Fourier iterative method that is one of methods to solve this problem [2]. This iterative method is refining the approximation by using several restrictions for images and Fourier transformations and tries to minimize an evaluation function that measures the error between the estimated image and the original image. Iterative methods can easily handle evaluation functions and add various restrictions. Therefore, after then, many iterative methods for BD have been proposed. On the other hand, Lane and Bates proposed zero-sheet method[3]. To solve BD, in this method, z-transformations and factorizations of polynomials are used. Both methods assume that BD has a unique solution.

However, BD does not always have a unique solution if there are no special restrictions for signals. In the worst case, BD has infinite solutions and become an indeterminate problem. Nevertheless, few papers treat this determining problem.

In this paper, we prove that BD has only finite solutions if an original image and a PSF take zero except a restricted domain. Then, we propose an algorithm to find all finite solutions under this boundary condition. Finally, we confirm in numerical examples that we can extract all sets of an original image and a PSF from a degraded image by using our algorithm. In this paper, we treat only digital images.

II. Blind Deconvolution

Let $g(x, y)$ be a degraded observed image, $f(x, y)$ be an original image and $h(x, y)$ be an unknown PSF. Then we get a relation among them as follows:

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$$g(x, y) = h(x, y) * f(x, y) \quad (1)$$

where $*$ denotes the convolution operator. A PSF $h(x, y)$ also satisfies the next condition:

$$\sum h(x, y) = 1. \quad (2)$$

In general, the set of an original image and a PSF to satisfy equations (1) and (2) for a given $g(x, y)$ is infinite and makes a space of solutions. Therefore, some restrictions for original images and PSFs are necessary to solve BD.

Ⅲ. Solution set of BD

As seen in the paper of Lane and Bates[4], an explosion of studies for BD assume the boundedness of original images and PSFs. That is to say, every signal takes zero except a restricted domain. When an image satisfies this condition, we say that the image has a compact support. Especially, in this paper, the minimum rectangle that includes an image is called minimum support. The condition of this compactness is satisfied when images have a unique background color, for examples, astronomical photographs, micrographs or sign boards of photographs. In last case, we need to extract the part image from photographs. In this paper, we need this boundary condition for every image.

At first, we treat BD for one dimensional signals.

Theorem 1 : Let one dimensional signal $g(x)$ takes non-negative in a finite region and takes zero outside of the region. The signal is represented by a convolution of two signals such as

$$g(x) = h(x) * f(x)$$

where one dimensional signals $h(x)$ and $f(x)$ take non-negative in finite intervals and take zero outside of the intervals like $g(x)$. Moreover, $h(x)$ satisfies

$$\sum h(x) = p$$

for a positive constant p. Then, the sets of $f(x)$ and $h(x)$ to satisfy above conditions is finite.

- ***Proof*** : Let M be a length of the minimum support of $g(x)$ and $K(K=1, 2, \cdots, M)$ be a length of the minimum support of $h(x)$. Then, the length of the minimum support of $f(x)$ is $M-K+1$. By a z-transformation of $g(x)$, we can get

$$G(z) = \sum_{i=1}^{M} g(i) z^{i-1},$$

where $G(z)$ is a polynomial of degree $M-1$. By using the fundamental theorem of algebra, there exist $M-1$ complex roots $\alpha_1, \cdots \alpha_{M-1}$ and we have a factorization of $G(z)$ as follows:

$$G(z) = g(M) \prod_{i=1}^{M-1} (z - \alpha_i)$$

Let $H(z)$ be a polynomial product of $K-1$ factors from $z-\alpha_1, z-\alpha_2, \cdots, z-\alpha_{M-1}$ and $F(z)$ be a product of left factors of $G(z)$. The combination of selections is given by ${}_{M-1}C_{K-1}$. Hence, the combination of factors is less than it because each coefficient of expanded polynomials $H(z)$ and $F(z)$ must take non-negative. Let find all selections to satisfy the condition and make inverse z-transformations of two polynomials. Then, we normalize $h(x)$ and $f(x)$ to satisfy $\sum h(x) = p$ and get the set of all solutions for $h(x)$ and $f(x)$. The set is less than $\sum_{k=1}^{M} {}_{M-1}C_{K-1} = 2^{M-1}$.

- ***Example 1*** : When an observed signal is given by

$$g(x) = (15 \quad 68 \quad 100 \quad 48),$$

it holds $M=4$ and we get the factorization

$$G(z) = 48z^3 + 100z^2 + 68z + 15$$
$$48\left(z + \frac{1}{2}\right)\left(z + \frac{3}{4}\right)\left(z + \frac{5}{6}\right)$$

Here, we suppose $p = \sum h(x) = 1$.

(1) In the case of $K=1$, since $H(z)=1$ holds, we can get

$$h(x)=(1),\ f(x)=(15\quad 68\quad 100\quad 48)$$

Hence, the degraded image is equal to the original image.

(2) In the case of $K=2$,

(a) if we select $H(z)=z+\frac{1}{2}$, we can get

$$h(x)=\left(\frac{2}{3}\ \frac{1}{3}\right),\ f(x)=(45\quad 114\quad 72).$$

(b) if we select $H(z)=z+\frac{3}{4}$, we can get

$$h(x)=\left(\frac{4}{7}\ \frac{3}{7}\right),\ f(x)=(35\quad 112\quad 84).$$

(c) if we select $H(z)=z+\frac{5}{6}$, we can get

$$h(x)=\left(\frac{6}{11}\ \frac{5}{11}\right),\ f(x)=(33\quad 110\quad 88).$$

(3) In the case of $K=3$,

(a) if we select $H(z)=\left(z+\frac{3}{4}\right)\left(z+\frac{5}{6}\right)$, we can get

$$h(x)=\left(\frac{24}{77}\ \frac{38}{77}\ \frac{15}{77}\right),\ f(x)=(77\quad 154).$$

(b) if we select $H(z)=\left(z+\frac{1}{2}\right)\left(z+\frac{5}{6}\right)$, we can get

$$h(x)=\left(\frac{12}{33}\ \frac{16}{33}\ \frac{5}{33}\right),\ f(x)=(99\quad 132).$$

(c) if we select $H(z)=\left(z+\frac{1}{2}\right)\left(z+\frac{3}{4}\right)$, we can get

$$h(x)=\left(\frac{8}{22}\ \frac{11}{22}\ \frac{3}{22}\right),\ f(x)=(110\quad 132).$$

(4) In the case of $K=4$,

from $H(z)=\left(z+\frac{1}{2}\right)\left(z+\frac{3}{4}\right)\left(z+\frac{5}{6}\right)$, we can get

$$h(x)=\left(\frac{48}{231}\ \frac{100}{231}\ \frac{68}{231}\ \frac{15}{231}\right),\ f(x)=(231).$$

Therefore, there exist $8(=2^{4-1})$ solutions for a given image $g(x)$.

The same property of theorem 1 is easily extended to two dimensional BD problem.

Theorem 2 **:** If a degraded image has a compact support, the set of $h(x,y)$ and $f(x,y)$ satisfying equations (1) and (2) is finite.

- ***Proof*** **:** Let $M\times N$ be a size of the minimum support of a degraded image. Then $g(x,y)$ takes zero outside of the support. Moreover, we can assume that the support is located at from $(1,1)$ to (M,N), if necessary relocated by using a coordinate transformation. When we assume the size of the minimum support of a PSF $h(x,y)$ is $K\times L$ $(1\le K\le M, 1\le L\le N)$, the size of an original image $f(x,y)$ become $(M-K+1)\times(N-L+1)$. By using z-transformation of $g(x,y)$, we obtain

$$G(z,w)=\sum_{i=1}^{M}\sum_{j=1}^{N}g(i,j)z^{i-1}w^{j-1}$$

Then, by the uniqueness of the factorization of a polynomial $G(z,w)$ to irreducible monic factor (See theorem 4.9 in [5]), we can get

$$G(z,w)=g(M,N)\prod\varphi_i(z,w),$$

where each irreducible factor $\varphi_i(z,w)$ is monic. Let $H(z,w)$ be a product of several irreducible factors $\varphi_i(z,w)$ with degree $K-1$ for z and degree $L-1$ for w. Then, let $F(z,w)$ be a product of left $\varphi_i(z,w)$ of $G(z,w)$. The combination of factors is finite. Moreover, taking account to the restrictions of images, each coefficient of expanded polynomials $H(z,w)$ and $F(z,w)$ must be non-negative numbers. The combination is less than it. By using inverse z-transformations of two polynomials and normalizing in order to satisfy $\sum h(x,y)=1$, we obtain all solutions of $h(x,y)$ and $f(x,y)$.

- ***Example 2*** **:** When an observed image is given by

$$g(x,y)=\begin{pmatrix}6 & 10 & 4\\ 21 & 35 & 14\\ 9 & 15 & 6\end{pmatrix},$$

we can get a factorization

$$G(z,w) = 6 + 10z + 4z^2 + 21w + 35zw + 14z^2w + 9w^2 + 15zw^2 + 6z^2w^2 = (z+1)(2z+3)(w+2)(3w+1)$$

Here, we assume the size of a PSF is 2×2. Then we get four solutions in the following.

(1) If we select $H(z,w) = (z+1)(w+2)$, we get

$$h(x,y) = \frac{1}{6}\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix},\ f(x,y) = 6\begin{pmatrix} 3 & 2 \\ 9 & 6 \end{pmatrix}.$$

(2) If we select $H(z,w) = (z+1)(3w+1)$, we get

$$h(x,y) = \frac{1}{9}\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix},\ f(x,y) = 9\begin{pmatrix} 6 & 4 \\ 3 & 2 \end{pmatrix}.$$

(3) If we select $H(z,w) = (2z+3)(w+2)$, we get

$$h(x,y) = \frac{1}{15}\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix},\ f(x,y) = 15\begin{pmatrix} 1 & 1 \\ 3 & 3 \end{pmatrix}.$$

(4) If we select $H(z,w) = (2z+3)(3w+1)$, we get

$$h(x,y) = \frac{1}{21}\begin{pmatrix} 6 & 9 \\ 2 & 3 \end{pmatrix},\ f(x,y) = 21\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}.$$

Ⅳ. Algorithm for two dimensional BD

We showed that BD has only finite solutions. However, there are two critical problems to solve BD using theorem 2. First is how we transform a polynomial $G(z,w)$ with two variables to a product of irreducible factors. Second is that it takes too much time to factor polynomials with two variables when a given degraded image is large. In the case of a polynomial with only one variable, it is easy to factor a polynomial to a product of polynomials of degree one by using Durand-Kerner-Aberth method[6] (DKA method). This means to find all roots of a polynomial in complex number. Therefore, we propose an algorithm to find all irreducible factors of polynomials with two variables. It is extension of the method of factorizations of polynomials with one variable.

Algorithm for two dimensional BD

(ⅰ) Let $M\times N$ be the size of the minimum support of a degraded image $g(x,y)$. Let put

$$g(y) = \sum_{x=1}^{M} g(x,y)$$

for $y = 1,2,\cdots,N$. Then, it holds $g(y) \neq 0$ for $y = 1,2,\cdots,N$.

(ⅱ) Let $G(z)$ be a z-transformation of $g(y)$. Then, the polynomial $G(z)$ is factored to the product of N-1 factors with one degree by using DKA method.

(ⅲ) When we suppose the size of a PSF to be $K\times L$, $H(z)$ is a product of L factors of $G(z)$. Here, we must select L factors to satisfy that each coefficient of the expansion of polynomial $H(z)$ become a non-negative number. Then, from the coefficients of the expansion of $H(z)$, we can obtain sums of each line of $h(x,y)$:

$$p_i = \sum_{x=1}^{K} h(x,i)$$

for $i = 1,2,\cdots,L$.

(ⅳ) Let put $p = p_1$ in theorem 1. Solve one dimensional BD problem of the convolution of $h(x,1)$ and $f(x,N)$.

(ⅴ) To estimate a PSF $h(x,y)$, for each $k = 2,3,\cdots,L$, we solve a system of $M+1$ linear equations:

$$\sum_{i=1}^{K} h(i,k) = p_k$$

$$\sum_{i=1}^{K}\sum_{j=1}^{L} h(i,j) f(i+l-K, N-1+1-k+j) = g(l, N+1-k),$$

for $1 \le l \le M$. Then we obtain every value of a PSF $h(x,y)$. If the obtained PSF has a negative element, return to step (ⅲ) and try again with another selection of factors.

(vi) To estimate an original image $f(x, y)$, we solve a system of $(M-K+1)\times(N-L+1)$ linear equations selected from $M\times N$ linear equations of the convolution $h(x, y) * f(x, y) = g(x, y)$. That is

$$\sum_{i=1}^{K}\sum_{j=1}^{L} h(i, j) f(i-[K/2]+k-1, j-[L/2]+l-1)$$
$$= g(k+[(K-1)/2], l+[(K-1)/2])$$

for $1\le k\le N-K+1$ and $1\le l\le N-L+1$. Here, $[x]$ is Gauss' symbol. Then, we can obtain all values of the original image $f(x, y)$.

(vii) We verify that the convolution of obtained $h(x, y)$ and $f(x, y)$ is equal to $g(x, y)$. If it does not hold, return to step iii) and try again with another selection of factors.

- ***Remark 1*** : In step (v), we cannot always obtain a unique solution of the systems of linear equations. For Example

$$\begin{pmatrix} 0.5 & 0 \\ h_{12} & h_{22} \end{pmatrix} * \begin{pmatrix} f_{11} & f_{21} \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Since it holds $0f_{11}+0h_{22}=0$, we can only three linear equations for four unknown variables:

$$h_{12}+h_{22}=0.5$$
$$0.5f_{11}+2h_{22}=2$$
$$0.5f_{21}+2h_{12}=0.$$

This is a rare case but we should attention to some special cases. For example, let add one easy equation such as $f_{11}\cdot h_{22}=1$.

- ***Remark 2*** : If there exists an inverse filter of the estimated PSF in step (v), we can use it instead of step (vi) to obtain an original image.

- ***Example 3*** : When an observed image is given by

$$g(x, y) = \begin{pmatrix} 3 & 8 & 13 & 6 \\ 13 & 25 & 31 & 12 \\ 25 & 43 & 49 & 18 \\ 7 & 8 & 9 & 0 \end{pmatrix}$$

we get

$$g(1) = 3+8+13+6 = 30$$
$$g(2) = 13+25+31+12 = 81$$
$$g(3) = 25+43+49+18 = 135$$
$$g(4) = 7+8+9+0 = 24.$$

Then we obtain only one factorization

$$G(z) = 30+81z+135z^2+24z^3$$
$$= 3(5+z)(2+5z+8z^2)$$

to satisfy each coefficient of factors becomes non-negative. Here, when we suppose the size of an original image $f(x, y)$ to be 3×3, the size of a PSF $h(x, y)$ becomes 2×2. Therefore, it holds $H(z)=5+z$. From the coefficients, we obtain

$$p_1 = \frac{1}{1+5} = \frac{1}{6}$$
$$p_2 = \frac{5}{1+5} = \frac{5}{6}.$$

Next, we solve the convolution $h(x, 1)*f(x, 3)=g(x, 4)$. Since the factorization must satisfy that each coefficient of factors of $G(z, 4)$ is non-negative, we obtain only one factorization

$$G(z,4) = 7+8z+9z^2+0z^3$$
$$= (1+0z)(7+8z+9z^2).$$

Here, from $p_1=\frac{1}{6}$, we have

$$h(x, 1) = \begin{pmatrix} 0 & \frac{1}{6} \end{pmatrix} \text{ and } f(x, 3) = 6(7 \quad 8 \quad 9).$$

Then we solve a system of five linear equations:

$$h(1,2)+h(2,2) = p_2$$
$$\sum_{i=2}^{2}\sum_{j=1}^{2} h(i, j) f(i-1, j+1) = g(1,3)$$
$$\sum_{i=1}^{2}\sum_{j=1}^{2} h(i, j) f(i, j+1) = g(2,3)$$

$$\sum_{i=1}^{2}\sum_{j=1}^{2} h(i,j)f(i+1,j+1) = g(3,3)$$

$$\sum_{i=1}^{1}\sum_{j=1}^{2} h(i,j)f(i+2,j+1) = g(4,3),$$

we get

$$h(x,2) = \begin{pmatrix} \frac{1}{3} & \frac{1}{2} \end{pmatrix} \text{ and } f(x,2) = 6\begin{pmatrix} 4 & 5 & 6 \end{pmatrix}.$$

Therefore we obtain a PSF

$$h(x,y) = \frac{1}{6}\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}.$$

Finally, we solve a system of 3×3 linear equations obtained from the convolution

$$h(x,y) * f(x,y) = g(x,y).$$

Then, we can obtain only one original image

$$f(x,y) = 6\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

V. Experimental Results

1. Problem Setting

Fig. 1 is a standard image of SIDBA 79[7] (named GIRL) with 256 gray level, sampled as 256×256 pixels.

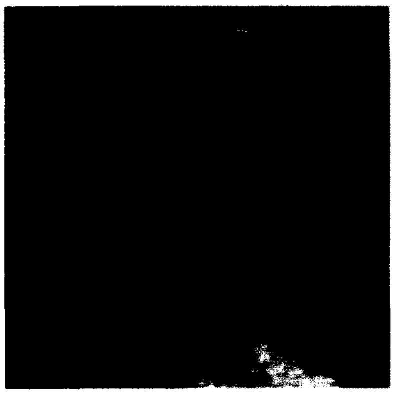

Fig. 1. Test Image.

Next, we define PSFs used in this section. Let $n(x,y)$ be a Gaussian Distribution as follows:

$$n(x,y) = \frac{1}{2\pi\tau^2}\exp\left(-\frac{x^2+y^2}{2\tau^2}\right), \quad (3)$$

where τ is a standard derivation and means blurring degree. We sample equation (3) to arrive at a discrete approximation:

$$\tilde{n}(i,j) = \int_{i-1/2}^{i+1/2}\int_{j-1/2}^{j+1/2} n(x,y)\,dxdy.$$

Then, we generate a PSF $h(i,j)$ with $K \times L$ size by normalizing and shifting $\tilde{n}(i,j)$ as follows:

$$h(i,j) = \frac{\tilde{n}(i-1-[K/2], j-1-[L/2])}{\sum_{1\le i\le K}\sum_{1\le j\le L}\tilde{n}(i-1-[K/2], j-1-[L/2])},$$

where $i = 1,2,\cdots,K$ and $j = 1,2,\cdots,L$. Here, [x] is Gauss' symbol. For example, let put $\tau = 0.3$ and $K = L = 3$. Then we obtain the PSF $h(i,j)$:

$$h(i,j) = \begin{pmatrix} 0.002285 & 0.043234 & 0.002285 \\ 0.043234 & 0.817919 & 0.043234 \\ 0.002285 & 0.043234 & 0.002285 \end{pmatrix}.$$

We used a personal computer with 2GHz CPU, 512MB memories and double floating system with 64bits in calculation.

2. Degraded images with no noise

To analyze the influence of the size of Images on our proposed method, four kind of original images were prepared. These are the original size GIRL (256×256) half size (128×128), quarter size (64×64) and 1/8 size (32×32) of it. Also, we prepared four kind of PSFs with $\tau = 0.3, 0.5, 0.7, 0.9$ and $K = L = 3$. Then, we made 16 degraded images, that is, four original images times four PSFs. Table 1 shows the number of all PSFs obtained in step (v) by using our proposed method. In the case of the original size (256×256), we cannot obtain all roots of polynomials $G(z)=0$ in step (ii). The reason seems to be the influence of overflow errors. In other cases, each true PSF is included in estimated PSFs obtained in step (v). It seems that the number of PSFs obtained in step (v) tends to become large when blurring degree is high. In step (vii), PSFs reduced only one true PSF in every case. They took less than some minutes from step (i) to step (v). However, in step (vi), if the size of images becomes twice, the time for solving a system of linear equations expands 16 times.

Table 1. Number of PSFs estimated in step (v)

	0.3	0.5	0.7	0.9
1/8	2	2	2	3
1/4	2	3	3	5
1/2	1	1	1	10
Original size	NG	NG	NG	NG

In any cases except original size, PSFs and original images are obtained with sufficiently accuracies (less than 10^{-8}).

3. Degraded images with pulse noise

Next, we treated degraded images with pulse noise. Let an original image to be 1/4 size of GIRL and a PSF has $\tau=0.5$ and $K=L=3$. At first, we made a blurred image by using the original image and the PSF. Then we added pulse noise with ± 0.5 amplitude. Pulse noise was added to the blurred image with 0%, 5% and 10% pixels selected from the image at random, respectively. Then we restored original images from these three degraded images by using our proposal method. We used SN rate to evaluate the difference of two images. SN rate of two images is defined by

$$SNR\left(f_1(x,y), f_2(x,y)\right) = 10\log_{10}\frac{255^2}{MSE},$$

where $$MSE = \frac{1}{MN}\sum_{M,N}\left\{f_1(x,y) - f_2(x,y)\right\}^2 .$$

Table 2 shows SN rates between the original image and blurred images with noise or between the original image and restored images. When pulse noise did not add to the blurred image, we could restore the original image almost completely. When additive noise is equal to 5%, SNR extremely improved from 28.6dB to 60.5dB. Fig. 2 shows original image with 1/4 size of GIRL. Fig. 3 is a degraded image added 5% pulse

Table 2. SN rates for the original image

SNR (dB)	0%	5%	10%
Degraded Image	28.6	28.6	28.5
Restored Image	289.7	60.5	NG

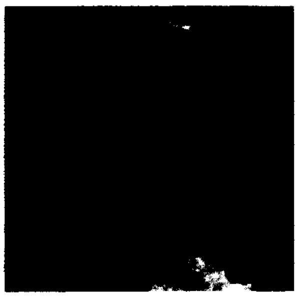

Fig. 2. Original image

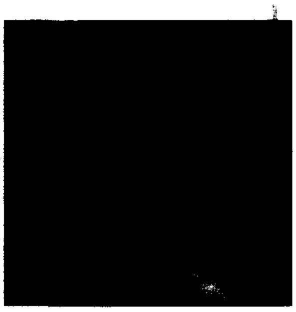

Fig. 3. Degraded image

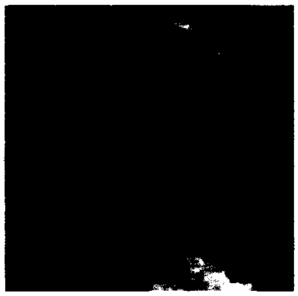

Fig. 4. Restored image

noise to the blurred image. Fig.4 is a restored image of it by using our method. However, when 10% noise was added to the blurred image, we could not obtain any PSF in step (v). Consequently, our proposal method is sensitive to noise. Therefore, some efforts to reduce noise are demanded before using our proposed method.

4. PSF with large size

When we try to estimate PSFs, we should suppose that PSFs have a small size. A PSF itself may be the convolution of two signals. Especially, it sometimes holds when variables of PSFs can be a separated such as $f(x,y)=f_1(x)f_2(y)$. In this case, we can use our proposal method iteratively. For example, the Gaussian PSF with $\tau=0.5$ and $K=L=5$ can be deconvoluted as follows:

$$h(x,y)=\begin{pmatrix} 0.0000 & 0.0002 & 0.0009 & 0.0002 & 0.0000 \\ 0.0002 & 0.0247 & 0.1074 & 0.0247 & 0.0002 \\ 0.0009 & 0.1074 & 0.4660 & 0.1074 & 0.0009 \\ 0.0002 & 0.0247 & 0.1074 & 0.0247 & 0.0002 \\ 0.0000 & 0.0002 & 0.0009 & 0.0002 & 0.0000 \end{pmatrix}$$

$$=\begin{pmatrix} 0.000 & 0.008 & 0.000 \\ 0.008 & 0.9652 & 0.008 \\ 0.000 & 0.008 & 0.000 \end{pmatrix} * \begin{pmatrix} 0.023 & 0.106 & 0.023 \\ 0.106 & 0.479 & 0.106 \\ 0.023 & 0.106 & 0.023 \end{pmatrix}$$

.

In this case, if we suppose PSF has a 3x3 size, we obtain at lest two solutions and we can refine a degraded image twice by using our method.

VI. Conclusions

This paper has proved that blind deconvolution has only finite solutions if an observed image has a compact support. Then, we proposed an algorithm to find all finite solutions under the condition. Finally, we confirm that we can extract an original image and estimate a point spread function from a degraded image with no noise in a numerical experiment by using our algorithm. However, our proposal method is sensitive to noise. Therefore, some efforts to reduce noise are demanded before using our proposed method.

When images are large, we cannot find all roots by using DKA method. This is future work.

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