

시간지연 퍼지 시스템의 지연 종속 H_∞ 필터 설계

Delay-dependent H_∞ Filter Design for Delayed Fuzzy Dynamic Systems

이 갑 래*
(Kap Rai Lee)

Abstract : This paper presents a delay dependent fuzzy H_∞ filter design method for delayed fuzzy dynamic systems. Using delay-dependent Lyapunov function, the global exponential stability and H_∞ performance problem are discussed. A sufficient condition for the existence of fuzzy filter is presented in terms of linear matrix inequalities(LMIs). The filter design utilize the concept of parallel distributed compensation. And the filter gains can also be directly obtained from the LMI solutions. A simulation example is given to illustrate the design procedures and performance of the proposed methods.

Keywords : delay dependent fuzzy filter, H_∞ performance, linear matrix inequalities

I. Introduction

During the last decades, considerable attention has been devoted to the problem of stability analysis and controller design for the fuzzy dynamic systems. Tanaka et al. [1][2] presented stability analysis for a class of fuzzy dynamic systems. Ma et al. [3] presented the analysis and design of the fuzzy controller and fuzzy observer on the basis of T-S fuzzy model using separation property. Han[4] and Lee et al.[5] presented the design method of fuzzy H_∞ controller to satisfy an H_∞ norm bound constraint on disturbance attenuation.

Since time delay is frequently a source of instability and encountered in various engineering systems, the H_∞ control problem for delayed systems has received considerable attention over the last few decades. In fuzzy control systems, Cao et al.[7] presented stability analysis and synthesis of delayed fuzzy dynamic systems. Lee et al. [8] presented output feedback fuzzy H_∞ control of delayed fuzzy dynamic systems. Lee et al.[9],[10] also presented H_∞ and mixed H_2/H_∞ filtering of delayed fuzzy dynamic system.

However these methods are delay independent stabilization methods. The delay-independent stabilization provides a controller which can stabilize a system irrespective of the size of the delay. On the other hand, the delay-dependent stabilization is concerned with the size of the delay and considered less conservative than the delay-independent case[11][12]. In this paper, we extended

the delay-dependent method into the stabilization for time-delay fuzzy systems.

This method not only guarantees a stability, but also an induced L_2 norm bound constraint on disturbance attenuation. A sufficient condition for the existence of fuzzy filter with H_∞ constraint is then presented in terms of linear matrix inequalities(LMIs).

II. Problem formulation

The continuous fuzzy dynamic model, proposed by Takagi and Sugeno, is described by fuzzy IF-THEN rules which represented local linear input-output relations of nonlinear system. Consider a nonlinear system with time-varying delayed states that can be described by the following T-S fuzzy model with time-varying delayed states:

Plant Rule i :

IF $z_1(t)$ is M_{i1} and ... and $z_g(t)$ is M_{ig}

THEN $\dot{x}(t) = A_i x(t) + A_{di} x(t-\tau) + B_i w(t)$

$y(t) = C_i x(t) + D_i w(t), \quad i=1,2,\dots,r$

$e(t) = C_{zi} x(t), \quad i=1,2,\dots,r \quad (1)$

$x(t) = \phi(t), \quad t \in [-d(0), 0]$

where M_{ij} is the fuzzy set, $x(t) \in R^n$ is the state vector, $\phi(t) \in R^n$ is the continuous unknown initial value function, $w(t) \in R^p \in L_2(0, T)$ is the square-integrable noise signal, $y(t) \in R^m$ is the measurement, $e(t) \in R^q$ is the signal to be estimated, r is the number of IF-THEN rules, $z_1 \sim z_g$ are some measurable system variables, i.e., the premise variables, and all matrices are constant matrices

* 책임저자(Corresponding Author)

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이갑래 : 평택대학교 정보과학부(krlee@ptuniv.ac.kr)

with appropriate dimensions, τ is the delay of the systems with following assumptions:

$$0 \leq \tau \leq \bar{\tau} \tag{2}$$

Given a pair of $(x(t), y(t))$ by using a center average defuzzifier, product inference, and singleton fuzzifier, the dynamic fuzzy model (1) can be expressed by the following global model:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r h_i(z(t)) \{A_i x(t) + A_{di} x(t-\tau) + B_i w(t)\} \\ y(t) &= \sum_{i=1}^r h_i(z(t)) \{C_i x(t) + D_i w(t)\} \\ e(t) &= \sum_{i=1}^r h_i(t) C_{zi} x(t) \\ x(t) &= \phi(t), \quad t \in [-\tau, 0] \end{aligned} \tag{3}$$

where

$$\begin{aligned} w_i(z(t)) &= \prod_{j=1}^n M_{ij}(z_j(t)) \\ h_i(z(t)) &= w_i(z(t)) / \sum_{i=1}^r w_i(z(t)) \\ z(t) &= [z_1(t) \ z_2(t) \ \dots \ z_n(t)]^T \end{aligned}$$

where $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij} .

$$\begin{aligned} w_i(z(t)) &\geq 0, \quad i = 1, 2, \dots, r \\ \sum_{i=1}^r w_i(z(t)) &> 0 \end{aligned} \tag{4}$$

for all t . Then we can obtain the following conditions:

$$\begin{aligned} h_i(z(t)) &\geq 0, \quad i = 1, 2, \dots, r \\ \sum_{i=1}^r h_i(z(t)) &= 1 \end{aligned} \tag{5}$$

for all t . As a fuzzy H_∞ filter of the fuzzy system (1), we consider the following structure:

Filtering Rule i :

$$\begin{aligned} \text{IF } z_1(t) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_n(t) \text{ is } M_{in} \\ \text{THEN } \hat{x}(t) &= F_i \hat{x}(t) + G_i y(t), \quad i = 1, 2, \dots, r \\ \hat{e}(t) &= C_{zi} \hat{x}(t) \\ \hat{x}(t) &= 0, \quad t \leq 0 \end{aligned} \tag{6}$$

where the matrix F_i and G_i are to be determined. The final output of this fuzzy filter is

$$\hat{x}(t) = \sum_{i=1}^r h_i(z(t)) \{F_i \hat{x}(t) + G_i y(t)\}, \quad \hat{x}(0) = 0 \tag{7}$$

From (3) and (7), we obtain the following estimation error system

$$\begin{aligned} \dot{\zeta}(t) &= \mathcal{A}(z)\zeta(t) + \mathcal{A}_1(z)\zeta(t-d(t)) + \mathcal{B}(z)w(t) \\ \zeta(t) &= [\phi(t)^T \ \phi(t)^T]^T, \quad t \in [-\tau, 0] \\ \hat{e}(t) &= \mathcal{C}(z)\zeta(t) \end{aligned} \tag{8}$$

with the following notations

$$\begin{aligned} \mathcal{A}(z) &= \sum_{i=1}^r \sum_{j=1}^r h_i(z(t)) h_j(z(t)) \mathcal{A}_{ij} \\ \mathcal{C}(z) &= \sum_{i=1}^r h_i(z(t)) \mathcal{C}_i \\ \mathcal{A}_1(z) &= \sum_{i=1}^r h_i(z(t)) \mathcal{A}_{1i} \\ \mathcal{B}(z) &= \sum_{i=1}^r \sum_{j=1}^r h_i(z(t)) h_j(z(t)) \mathcal{B}_{ij} \end{aligned} \tag{9}$$

where $\zeta(t) = [(x(t) - \hat{x}(t))^T \ x^T(t)]^T$, $\hat{e}(t) = e(t) - \hat{e}(t)$,

$$\begin{aligned} \mathcal{A}_{ij} &= \begin{bmatrix} F_i & A_i - G_i C_j - F_i \\ 0 & A_i \end{bmatrix}, \quad \mathcal{A}_{1i} = \begin{bmatrix} 0 & A_{di}^T \\ 0 & A_{di}^T \end{bmatrix}, \\ \mathcal{B}_{ij} &= [(B_i - G_i D_j)^T \ B_i^T]^T, \quad \mathcal{C}_i = [C_{zi} \ 0]. \end{aligned} \tag{10}$$

Since H_∞ filtering is filter design to efficiently eliminate the effect of $w(t)$, for given γ we define H_∞ filtering performance of the system (10) as the quantity

$$\begin{aligned} J_\infty &= \int_0^T \|\tilde{e}(t)\|^2 dt \leq \gamma^2 \left[\int_0^T \|w(t)\|^2 dt \right. \\ &\quad \left. + x^T(0) Q_0 x(0) + \int_{-\tau}^0 x^T(\tau) Q_1 x(\tau) d\tau \right. \\ &\quad \left. + \int_{-\tau}^0 \int_{\theta}^0 \dot{x}^T(s) Q_2 \dot{x}(s) ds d\theta \right] \end{aligned} \tag{11}$$

for all $T > 0$ and all $w \in L_2[0 \ T]$, where $\|\cdot\|$ denotes the Euclidean norm. The weighting matrix $Q_i, i = 0, 1, 2$, in (11) are measure of the initial state uncertainty at $t \leq 0$ relative to the uncertainty in $w(t)$. A large value of Q_i indicates that the state at $t \leq 0$ is very close to zero.

This paper addresses designing fuzzy filter (7) for the system (3) such that the estimation error system is globally exponentially stable and achieves H_∞ performance (11).

III. Fuzzy H_∞ Filter Design

Lemma 1[12]: Assume that $a(\cdot) \in R^{n_a}, b(\cdot) \in R^{n_b}$ and $N(\cdot) \in R^{n_a \times n_b}$ are defined on the interval \mathcal{Q} . Then, for any matrices $X \in R^{n_a \times n_a}, Y \in R^{n_a \times n_b}$ and $Z \in R^{n_b \times n_b}$, the following holds

$$\begin{aligned} &-2 \int_{\mathcal{Q}} a^T(a) N b(a) da \\ &\leq \int_{\mathcal{Q}} \begin{bmatrix} a(a) \\ b(a) \end{bmatrix} \begin{bmatrix} X & Y - N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a(a) \\ b(a) \end{bmatrix} da \end{aligned} \tag{12}$$

where

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0$$

Lemma 2 : Consider the system (3) with assumption (2), and let $\gamma > 0$ be a given scalar. If there exist matrices $P > 0, S > 0, R > 0, \bar{R} > 0, R$ and positive scalar α satisfying the following inequalities

$$\Omega_{ii} < 0, \quad i = 1, 2, \dots, r \tag{13}$$

$$\Omega_{ij} + \Omega_{ji} < 0, \quad i < j < r, \tag{14}$$

$$\begin{bmatrix} R & R \\ R^T & S \end{bmatrix} \geq 0 \tag{15}$$

$$[I_n \ I_n]P[I_n \ I_n]^T - \gamma^2 Q_0 < 0$$

$$[I_n \ I_n]R[I_n \ I_n]^T - \gamma^2 Q_1 < 0 \tag{16}$$

$$[I_n \ I_n]S[I_n \ I_n]^T - \gamma^2 Q_2 < 0$$

then the corresponding estimation error system is globally exponentially stable and achieves H_∞ filtering performance (11) for all $w \in L_2[0, T]$. In here

$$\Omega_{ij} = \begin{bmatrix} (1,1) + \bar{C}_i^T \bar{C}_i + \bar{S}_a & P \bar{A}_{ii} - R & P \bar{B}_{ij} & \bar{A}_{ij}^T \\ * & -R & 0 & \bar{A}_{ii}^T \\ * & * & -\gamma^2 I & \bar{B}_{ij}^T \\ * & * & * & -(\bar{\tau}S)^{-1} \end{bmatrix} \tag{17}$$

$$\bar{S}_a = \begin{bmatrix} 0 & 0 \\ 0 & \alpha I \end{bmatrix} \tag{18}$$

where * represents the elements below the main diagonal of a symmetric matrix and

$$(1, 1) = \bar{A}^T P + P \bar{A}_{ij} + \bar{\tau}R + 2R + R. \tag{19}$$

Proof : Consider the following Lyapunov functional as

$$V(t) = V_1(t) + V_2(t) + V_3(t). \tag{20}$$

In here

$$V_1(t) = \zeta^T(t) P \zeta(t) \tag{21}$$

$$V_2(t) = \int_{-\tau}^0 \int_{t+\theta}^t \zeta^T(s) S \zeta(s) ds d\theta \tag{22}$$

$$V_3(t) = \int_{t-\tau}^t \zeta^T(s) R \zeta(s) ds \tag{23}$$

where $P > 0, S > 0$ and $R > 0$. Then there exist positive a scalar δ_1 and δ_2 such that

$$\delta_1 \|\zeta(t)\|^2 \leq V(\zeta, t) \leq \delta_2 \sup_{\theta} \|\zeta(t + \theta)\|^2. \tag{24}$$

If there exists scalar $\alpha > 0$ such that $V(\zeta, t) \leq -$

$\alpha \|\zeta\|$, then the unforced system of (8) is globally exponentially stable[13].

Consider $V(\zeta, t) \leq -\alpha \|\zeta\|$ and the following condition

$$J_a(t) := V(\zeta, t) + \tilde{\zeta}^T(t) \tilde{\zeta}(t) - \gamma^2 w^T(t) w(t) \leq 0. \tag{25}$$

By considering the following condition

$$\zeta(t - \tau) = \zeta(t) - \int_{t-\tau}^t \zeta(s) ds \tag{26}$$

the estimation error system can be written as

$$\dot{\zeta}(t) = [\bar{A}(z) + \bar{A}_1(z)] \zeta(t). \tag{27}$$

$$- \bar{A}_1(z) \int_{t-\tau}^t \zeta(s) ds + B(z)w(t)$$

The derivative of $V_1(t)$ satisfies the relation

$$\begin{aligned} \dot{V}_1(t) &= \zeta^T \{ [\bar{A}(z) + \bar{A}_1(z)]^T P + P [\bar{A}(z) + \bar{A}_1(z)] \} \zeta(t) \\ &\quad - 2\zeta^T(t) P \bar{A}_1(z) \int_{t-\tau}^t \zeta(s) ds + 2w^T(t) B^T(z) P \zeta(t). \end{aligned} \tag{28}$$

Applying lemma 1 will supply

$$\begin{aligned} \dot{V}_1(t) &\leq \zeta^T \{ [\bar{A}(z) + \bar{A}_1(z)]^T P + P [\bar{A}(z) + \bar{A}_1(z)] \} \zeta(t) \\ &\quad + \int_{t-\tau}^t \left[\begin{array}{c} \zeta(t) \\ \zeta(s) \end{array} \right]^T \left[\begin{array}{cc} R & R - P \bar{A}_1 \\ * & S \end{array} \right] \left[\begin{array}{c} \zeta(t) \\ \zeta(s) \end{array} \right] ds + 2w^T(t) B^T(z) P \zeta(t) \\ &\leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \zeta^T \{ \bar{A}_{ij}^T P + P \bar{A}_{ij} + \bar{\tau}R + 2R \} \zeta(t) \\ &\quad + \sum_{i=1}^r 2h_i \zeta^T(t) (P \bar{A}_{ii} - R) \zeta(t - \tau) + \int_{t-\tau}^t \zeta^T(s) S \zeta(s) ds \\ &\quad + \sum_{i=1}^r \sum_{j=1}^r 2\zeta^T(t) P \bar{B}_{ij}(z) w(t) \end{aligned} \tag{29}$$

By considering the following condition

$$\sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r \sum_{l=1}^r h_i h_j h_k h_l X_{ij}^T S X_{kl} \leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j X_{ij}^T S X_{ij} \tag{30}$$

we have the derivative of V_2 as

$$\begin{aligned} \dot{V}_2(t) &\leq \tau [\bar{A}(z) \zeta(t) + \bar{A}_1(z) \zeta(t - \tau) \\ &\quad + B(z)w(t)]^T S [\bar{A}(z) \zeta(t) + \bar{A}_1(z) \zeta(t - \tau) + B(z)w(t)] \\ &\quad - \int_{t-\tau}^t \zeta^T(s) S \zeta(s) ds \\ &\leq \bar{\tau} \sum_{i=1}^r \sum_{j=1}^r h_i h_j [\bar{A}_{ij} \zeta(t) + \bar{A}_{ii} \zeta(t - \tau) \\ &\quad + B_{ij} w(t)]^T S [\bar{A}_{ij} \zeta(t) + \bar{A}_{ii} \zeta(t - \tau) + B_{ij} w(t)] \\ &\quad - \int_{t-\tau}^t \zeta^T(s) S \zeta(s) ds. \end{aligned} \tag{31}$$

Since V_3 yields the relation

$$\dot{V}_3(t) = \zeta^T(t) R \zeta(t) - \zeta^T(t - \tau) R \zeta(t - \tau) \tag{32}$$

J_a can be presented as follows

$$\begin{aligned}
 J_a \leq & \sum_{i=1}^r \sum_{j=1}^r h_i h_j \begin{bmatrix} \zeta(t) \\ \zeta(t-d) \\ w(t) \end{bmatrix} \begin{bmatrix} (1,1) + \bar{\tau} \mathcal{A}_{ij}^T S \mathcal{A}_{ij} & P \mathcal{A}_{1i} - R' + \bar{\tau} \mathcal{A}_{ij}^T S \mathcal{A}_{1i} & P \mathcal{B}_{ij} + \bar{\tau} \mathcal{A}_{ij}^T S \mathcal{B}_{ij} \\ * & -R + \bar{\tau} \mathcal{A}_{1i} S \mathcal{A}_{1i} & \bar{\tau} \mathcal{A}_{1i} S \mathcal{B}_{ij} \\ * & * & -\gamma^2 I + \bar{\tau} \mathcal{B}_{ij}^T S \mathcal{B}_{ij} \end{bmatrix} \begin{bmatrix} \zeta(t) \\ \zeta(t-d) \\ w(t) \end{bmatrix} \\
 & + \sum_{i=1}^r \sum_{j=1}^r h_i h_j (\mathcal{C}_i \zeta(t))^T \mathcal{C}_j \zeta(t). \tag{33}
 \end{aligned}$$

By applying the Schur complement[14] and considering the following condition

$$\begin{aligned}
 & \sum_{i=1}^r \sum_{j=1}^r h_i h_j (\mathcal{C}_i \zeta(t))^T \mathcal{C}_j \zeta(t) \leq \\
 & \sum_{i=1}^r h_i \zeta^T(t) \mathcal{C}_i^T \mathcal{C}_i \zeta(t) \tag{34}
 \end{aligned}$$

J_a can be presented as follows

$$J_a \leq \sum_{i=1}^r h_i(z(t)) h_i(z(t)) \Omega_{ii} + \sum_{i < j}^r h_i(z(t)) h_j(z(t)) \{\Omega_{ij} + \Omega_{ji}\}. \tag{35}$$

Thus if (13) and (14) are satisfied, then $\nabla_a(\zeta, t) \leq d\|x\|^2$ and (25) is satisfied. From (25)

$$\begin{aligned}
 & \int_0^T \|\tilde{e}(t)\|^2 dt - \gamma^2 \int_0^T \|w(t)\|^2 dt \\
 & \leq \zeta^T(0) P \zeta(0) + \int_{-\tau}^0 \zeta^T(\tau) \mathcal{R} \zeta(\tau) d\tau \\
 & + \int_{-\tau}^0 \int_{\theta}^0 \zeta^T(s) S \zeta(s) ds d\theta \tag{36}
 \end{aligned}$$

because $V(\zeta, T) > 0$. It follows from initial condition (8) that

$$\begin{aligned}
 & \int_0^T \|\tilde{e}(t)\|^2 dt - \gamma^2 \left[\int_0^T \|w(t)\|^2 dt + x^T(0) Q_0 x(0) \right. \\
 & + \int_{-\tau}^0 x^T(\tau) Q_1 x(\tau) d\tau + \int_{-\tau}^0 \int_{\theta}^0 \dot{x}^T(s) Q_2 \dot{x}(s) ds d\theta \left. \right] \\
 & \leq \zeta^T(0) P \zeta(0) - \gamma^2 x^T(0) Q_0 x(0) \\
 & + \int_{-\tau}^0 \zeta^T(\tau) \mathcal{R} \zeta(\tau) d\tau - \gamma^2 \int_{-\tau}^0 x^T(\tau) Q_1 x(\tau) d\tau \\
 & + \int_{-\tau}^0 \int_{\theta}^0 \zeta^T(s) S \zeta(s) ds d\theta - \gamma^2 \int_{-\tau}^0 \int_{\theta}^0 \dot{x}^T(s) Q_2 \dot{x}(s) ds d\theta \\
 & \leq x_0^T \{ [I_n \ I_n] P [I_n \ I_n]^T - \gamma^2 Q_0 \} x_0 \\
 & + \int_{-\tau}^0 x^T(\tau) \{ [I_n \ I_n] \mathcal{R} [I_n \ I_n]^T - \gamma^2 Q_1 \} x(\tau) d\tau \\
 & + \int_{-\tau}^0 \int_{\theta}^0 \dot{x}^T(s) \{ [I_n \ I_n] S [I_n \ I_n]^T - \gamma^2 Q_2 \} \dot{x}(s) ds d\theta. \tag{37}
 \end{aligned}$$

Thus if (13)-(16) are satisfied, H_∞ control performance (11) is satisfied. ■

The next Theorem 1 presents a solution to the H_∞ filtering problem for the delayed fuzzy model in terms of LMIs from lemma 2.

Theorem 1 : Consider the system (3), and let $\gamma > 0$ be a given scalar. If there exist common positive matrices $X > 0, Z > 0, S_{11} > 0, S_{22} > 0, \mathcal{R}_{11} > 0, \mathcal{R}_{22} > 0, R_{11} > 0, R_{22} > 0$, and matrices $R'_{11}, R'_{22}, W_i, Y_i, i=1,2,\dots,r$, and positive scalar α satisfying the following LMIs:

$$\Phi_{ii} < 0, \quad i=1,2,\dots,r, \tag{38}$$

$$\Phi_{ij} + \Phi_{ji} < 0, \quad i < j < r, \quad i=1,2,\dots,r \tag{39}$$

$$\begin{bmatrix} R_{11} & 0 & R'_{11} & 0 \\ * & R_{22} & 0 & R'_{22} \\ * & * & S_{11} & 0 \\ * & * & * & S_{22} \end{bmatrix} \geq 0 \tag{40}$$

$$X + Z - \gamma^2 Q_0 < 0$$

$$\mathcal{R}_{11} + \mathcal{R}_{22} - \gamma^2 Q_1 < 0 \tag{41}$$

$$S_{11} + S_{22} - \gamma^2 Q_2 < 0$$

then there exists a fuzzy filter (7) such that the estimation error system (8) achieves the H_∞ constraint (11). In here

$$\Phi_{ij} = \begin{bmatrix} (1,1) + C_i^T C_{ai} & (1,2) - R'_{11} & X A_{di} & X B_i - Y_i D_i & W_i^T & 0 \\ * & \Delta_i & 0 & Z A_{di} - R'_{22} & Z B_i & (1,2)^T & A_i^T Z^T \\ * & * & -\mathcal{R}_{11} & 0 & 0 & 0 & 0 \\ * & * & 0 & -\mathcal{R}_{22} & 0 & A_i^T X & A_i^T Z \\ * & * & * & * & -\gamma^2 I & B_i^T X - D_i^T Y_i^T & B_i^T Z \\ * & * & * & * & * & -2X + \bar{\tau} S_{11} & 0 \\ * & * & * & * & * & 0 & -2Z + \bar{\tau} S_{22} \end{bmatrix} \tag{42}$$

$$\Delta_i = Z A_i + A_i^T Z + \bar{\tau} R_{22} + R'_{22} + R_{22}^T + \mathcal{R}_{22} + \alpha I \tag{43}$$

where

$$(1,1) = W_i + W_i^T + \bar{\tau} R_{11} + R'_{11} + R_{11}^T + \mathcal{R}_{11} \tag{44}$$

$$(1,2) = X A_i - Y_i C_j - W_i$$

Furthermore, filter gains are given by

$$F_i = X^{-1} W_i, \quad G_i = X^{-1} Y_i. \tag{45}$$

Proof : Let

$$R = \begin{bmatrix} R_{11} & 0 \\ 0 & R_{22} \end{bmatrix}, \quad R' = \begin{bmatrix} R'_{11} & 0 \\ 0 & R'_{22} \end{bmatrix}, \quad \mathcal{R} = \begin{bmatrix} \mathcal{R}_{11} & 0 \\ 0 & \mathcal{R}_{22} \end{bmatrix}$$

$$, \quad S = \begin{bmatrix} S_{11} & 0 \\ 0 & S_{22} \end{bmatrix}, \quad P = \begin{bmatrix} X & 0 \\ 0 & Z \end{bmatrix} \quad (46)$$

where X and Z are symmetric positive definite matrices to be found. Now pre and postmultiply $\text{diag}\{I \ I \ I \ P\}$ to (17) and apply

$$-P(\bar{\tau}S)^{-1}P < -P - P + \bar{\tau}S. \quad (47)$$

Next, let $W_i = XF_i$ and $Y_i = XG_i$, $i = 1, 2, \dots, r$, we get (38) and (42) from lemma 2. ■

It has been seen that the filter design problem of the fuzzy system (3) can be transformed into a linear algebra problem. This set of LMIs constitutes a finite-dimensional convex feasibility problem. There are several efficient algorithms to solve the above convex LMIs problem [14]-[16].

IV. Design example

We will design a fuzzy filter for the following nonlinear system;

$$\dot{x}_1(t) = -5.125x_1(t) - 0.5x_1(t-d) - 2x_2(t) - 6.7x_2^3(t) - 0.2x_2(t-d) - 0.67x_2^3(t-d) + w(t)$$

$$\dot{x}_2(t) = x_1(t) \quad (48)$$

$$y(t) = x_2(t) + 0.1w(t)$$

$$e(t) = 0.01x_1(t)$$

where time delay is

$$d = 1.5 \quad (49)$$

$x_1(t)$ is estimated using a fuzzy filter and assume that $x_2(t)$ is observable. It is also assumed that

$$x_1(t) \in [-1.5 \ 1.5], \quad x_2(t) \in [-1.5 \ 1.5]. \quad (50)$$

Then, the nonlinear term can be represented as

$$-6.7x_2^3(t) = M_{11} \cdot 0 \cdot x_2(t) - (1 - M_{11}) \cdot 15.075 x_2(t). \quad (51)$$

By solving the equation, M_{11} is obtained as follows:

$$M_{11}(x_2(t)) = 1 - \frac{x_2(t)^2}{2.25}$$

$$M_{12}(x_2(t)) = \frac{x_2(t)^2}{2.25}. \quad (52)$$

M_{11} and M_{12} can be interpreted as membership functions of fuzzy set. By using these fuzzy sets, the nonlinear system can be presented by the following T-S fuzzy model

Plant Rule 1: IF $x_2(t)$ is M_{11} THEN}

$$\dot{x}(t) = A_1x(t) + A_{d1}x(t-d(t)) + B_1w(t)$$

$$y(t) = C_1x(t) + D_1w(t) \quad (53)$$

$$e(t) = C_{z1}x(t)$$

Plant Rule 2: IF $x_2(t)$ is M_{12} THEN}

$$\dot{x}(t) = A_2x(t) + A_{d2}x(t-d(t)) + B_2w(t)$$

$$y(t) = C_2x(t) + D_2w(t) \quad (54)$$

$$e(t) = C_{z2}x(t)$$

where $x(t) = [x_1(t) \ x_2(t)]^T$,

$$A_1 = \begin{bmatrix} -5.125 & -2 \\ 1 & 0 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -0.5 & -0.2 \\ 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -5.125 & -17.075 \\ 1 & 0 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -0.5 & -1.71 \\ 0 & 0 \end{bmatrix} \quad (55)$$

$$B_1 = B_2 = [1 \ 0]^T, \quad C_1 = C_2 = [0 \ 1],$$

$$D_1 = D_2 = 0.1,$$

$$C_{z1} = C_{z2} = [0.01 \ 0].$$

Let $\gamma = 3$, $\bar{\tau} = 1.5$, $Q_0 = Q_1 = Q_2 = \text{diag}(0.03, 0.03)$, then filter gains obtained from Theorem 1 are

$$F_1 = \begin{bmatrix} -5.0082 & 0.3292 \\ 0.7196 & -1.3822 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} -4.2479 & 2.81112 \\ 0.5813 & -2.0899 \end{bmatrix} \quad (56)$$

$$G_1 = \begin{bmatrix} -3.4168 \\ 1.6855 \end{bmatrix}, \quad G_2 = \begin{bmatrix} -19.3688 \\ 1.2523 \end{bmatrix}$$

The simulation result of nonlinear system with time delay is shown in Fig. 1.

For these simulation, the noise signal $w(t)$ is

$$w(t) = 0.1 \cdot \cos(\pi t) \quad (57)$$

and the initial value of the state is assumed by

$$[x_1^T(t) \ x_2^T(t)]^T = [1 \ -1]^T, \quad t \leq 0. \quad (58)$$

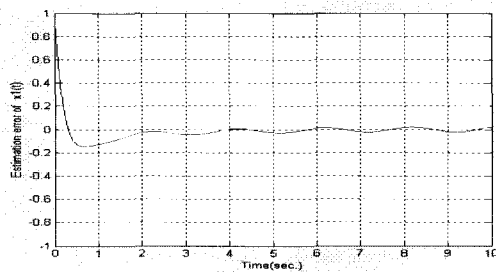
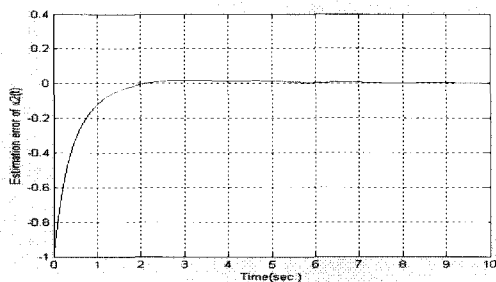
(a) $x_1(t) - \hat{x}_1(t)$ (b) $x_2(t) - \hat{x}_2(t)$

그림 1. 시간지연 비선형시스템에 대한 시뮬레이션 결과.

Fig. 1. The simulation result of nonlinear system with time delays.

The designed fuzzy filter estimate the states of the nonlinear system without the steady state errors and attains disturbance attenuation effect. The value of γ in the delay independent fuzzy H_∞ filter design method[9] is $\gamma = 10$. Thus we can see that delay dependent fuzzy H_∞ filter design method is less conservative compared to delay independent fuzzy H_∞ filter design method[9].

V. Conclusion

In this paper, we have developed delay dependent fuzzy H_∞ filter design method for delayed fuzzy dynamic systems. Using delay-dependent Lyapunov function, we have obtained sufficient conditions for the existence of fuzzy filters such that the estimation error system is globally exponentially stable and achieves H_∞ performance simultaneously. The filter design has utilized the concept of parallel distributed compensation and the filter gains can also be directly obtained from the LMI solutions. One example was provided to explain the ideas involved in the presented approaches. From the example, we could see that delay dependent fuzzy H_∞ filter design method is less conservative compared to delay independent fuzzy H_∞ filter design method.

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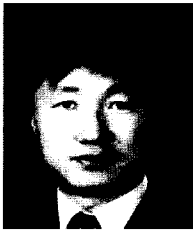
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이 감 래

1964년 11월 22일생. 1987년 경북대학교 전자공학과(공학사). 1990년 동대학원 전자공학과(공학석사), 1999년 동대학원 전자공학과(공학박사). 1990~1995년 국방과학연구소 연구원. 1997년~2001년 두원공과대학 조교수. 2001년~

현재 평택대학교 정보과학부(정보통신 전공) 조교수. 관심분야는 지능제어, 필드버스 네트워크, 임베디드 시스템 등임.