Fibers and Polymers 2004, Vol.5, No.2, 139-144

Orthotropic Theory for the Prediction of Mechanical Performance in Thermally Point-bonded Nonwovens

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(Received January 12, 2004; Revised April 3, 2004; Accepted April 10, 2004)

Abstract: The orthotropic theory is applied for the nonwoven fabrics that have a preferred orientation direction, the case if the structure is not isotropic. The polynomial regression analysis is employed to allow the attainment of more statistically meaningful information. A functional form based on the transformation rule is developed for the orthotropic approach. The predictions thus obtained are seen to be in excellent agreements with experimental data and the resulting compliances exhibit meaningful relationships for the processing conditions. The compatibility of the compliances from tensile and shear analyses has been explored prior to a practical application of the four compliances defining the in-plane strain-stress field.

Keywords: Nonwovens, Orthotropic theory, Compatibility

Introduction

Point-bonded fiberwebs have become the cornerstone of thin ("two-dimensional") nonwoven fabrics, for a large number of applications, over the past twenty some odd years. Apart from any economies in processing, they recommend themselves by their high strength coupled with high degree of flexibility. In the absence of reliable and theoretical models, the industry has developed proprietary empirical rules to design these fabrics for specific end uses. Whether the empirical rules are optimal or not, is an open question. Over the last several decades, a number of approaches have been developed in attempts to derive basic properties for thermally point-bonded nonwovens [1-6] and to provide quantitative measures of the nonwoven structures [7,8].

One of the outstanding applications of theoretical modeling is the orthotropic theory to the nonwovens reported by Backer and Petterson who obtained the constants from the principal-direction tensile tests and simultaneous measurements of the materials Poisson's ratio [9,10]. The validity of the conventional theory of orthotropic materials should be evaluated as it pertains to nonwovens.

In this study, we employed a different approach from Backer and Petterson [9,10] to allow the attainment of more meaningful compliance constants of nonwovens without the need for the ambiguous measurement of Poisson's ratio. The angular mechanical performance predicted through the polynomial regression analysis based on the transformation rule had been compared with the experimental data by the application of simple stress fields, (i) uniaxial stress and (ii) shear stress, for three series of oriented nonwoven samples.

The primary objective in this study has been to apply orthotropic symmetry theory to help predict the behavior of a nonwoven fabric at different angles. This is accomplished through the polynomial regression analysis to characterize the constants of nonwoven fabrics and to investigate the relationships between predicted constants and processing conditions.

Experimental

Materials

In the experimental part of this series of papers [11,12], three series of oriented nonwoven fabrics were produced at different bonding temperature (varied from 140 °C to 180 °C in 10 °C increments), pressure (from 30 psi to 50 psi in 5 psi increments) and percent bond area (bond area; 15 %, 40 %, and 100 %) of the calendar rolls. The 100 % bond area refers to an "area-bonded" fabric, obtained by calendering with smooth rolls. Respectively, the other processing conditions were kept constants. The precursor fabric was a unidirectional, two-ply, carded fabric of poly(propylene) fibers, with a staple length of 1.5 inch (5.08 cm) produced by sequentially combining the webs from two cards. The final fabrics have a weight of ~20 g/yd^2 (21.9 g/m^2).

Mechanical Properties

Each nonwoven sample for tensile test was measured 15 × 2.5 cm. The samples were tested on an Instron tensile testing machine at an extension rate of 100 %/min. The clamps used were 5 cm wide and 2.54 cm high. The gage length used was 10 cm. Testing was carried out on samples cut at ten-degree azimuthal intervals. The data represent the averages and the standard deviations obtained from five measurements in each case. The maximum stress, the elongation at maximum stress and the secant modulus at 10 % elongation were obtained from the load-elongation data.

Shear was measured on a Kawabata Evaluation System (KES) shear tester, with 20 × 20 cm samples. These tests were run on strips cut at 30° increments in the azimuthal angle. The KES system is normally programmed to deform the fabric to a shear angle of 8° and report a shear modulus that is

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*Corresponding author: hanseongkim@pusan.ac.kr calculated automatically between 0.5° and 2.5° shear angle. This produces significant errors when dealing with many nonwovens, due to significant fabric buckling and yield at angles above 0.5°. The maximum shear angle in our study was therefore limited to ±0.5°, and the shear modulus was calculated from the linear-response region occurring between 0° and ±0.5°.

Results and Discussion

Compliance from Tensile

The tensile moduli of nonwovens can be predicted by the orthotropic symmetry approach using the in-plane strain-stress relations with four compliances, S_{11}, S_{22}, S_{12}, and S_{44} [9,10,13].

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{44} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{1}$$

A polynomial regression analysis method was employed to allow the attainment of more statistically meaningful constants. The polynomial regression analysis was processed by using the functional form of equation (2) developed for planar orthotropic approach and the engineering definition [14] as follows:

$$\frac{1}{E_x(\theta)} = S_{11}(\theta) = S_{11}\cos^4\theta + S_{22}\sin^4\theta$$

$$+2S_{12}\cos^2\theta\sin^2 + 4S_{44}\cos^2\theta\sin^2\theta$$

$$= cons1 + cons2\sin^2\theta + cons3\sin^4\theta$$

where

$$\begin{aligned} cons1 &= S_{11} \\ cons2 &= -2S_{11} + 2S_{12} + S_{44} \\ cons3 &= S_{11} + S_{22} - 2S_{12} - S_{44} \end{aligned} \tag{2}$$

The secant tensile moduli $E_x(\theta)$ from uni-axial tensile measurements at ten-degree azimuthal intervals and those direction angles were used for input parameters in the polynomial regression analysis.

The computed predictions are reported and compared with the experimental data for the nonwoven samples produced at three different processing conditions as shown in Figure 1. The predicted results are shown as solid lines and are seen to be in excellent agreements with experimental data shown as marks in the bonding temperature series (Figure 1(a)). The same excellent level of agreement is seen for the pressure and bonded area series (Figures 1(b) and (c)). The excellent agreement between this approach and experimental data suggests that the orthotropic theory is desirable to predict angular mechanical performance of nonwovens produced at the various processing conditions.

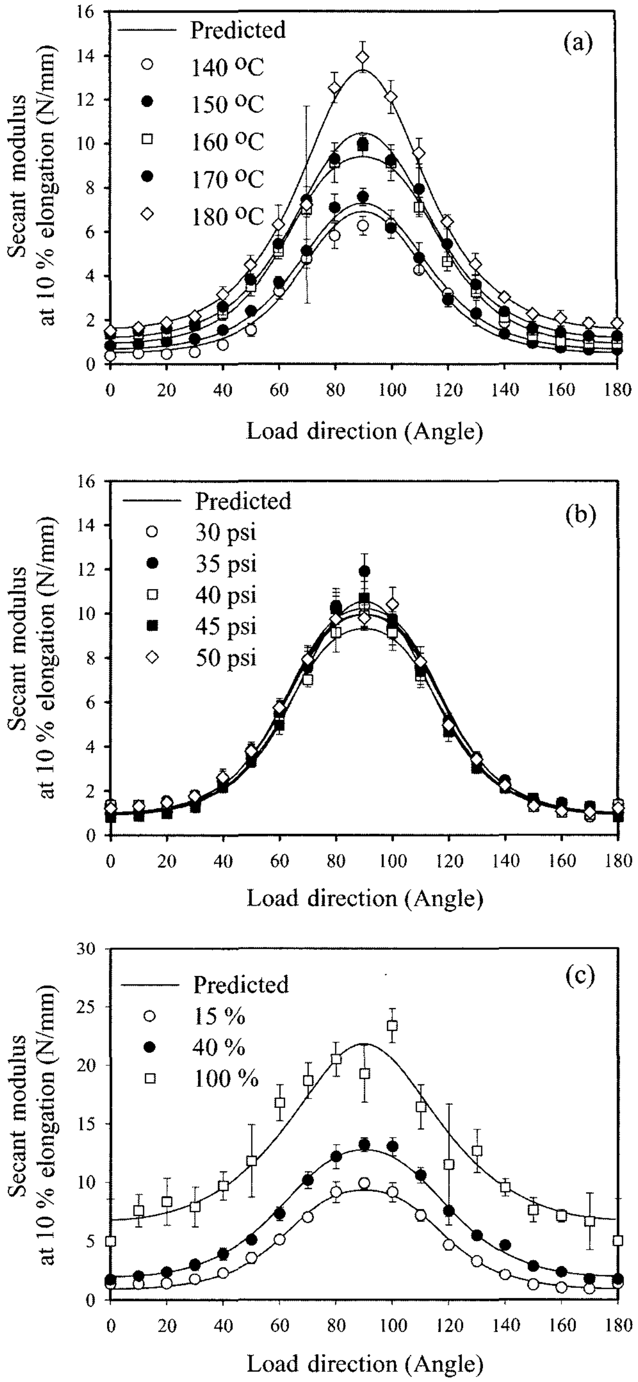

Figure 1. (a) Comparison of experimental and predicted tensile moduli for the nonwovens produced in temperature series, (b) Comparison of experimental and predicted tensile moduli for the nonwovens produced in pressure series, and (c) Comparison of experimental and predicted tensile moduli for the nonwovens produced in bond area series.

After determining the three polynomial regression constants

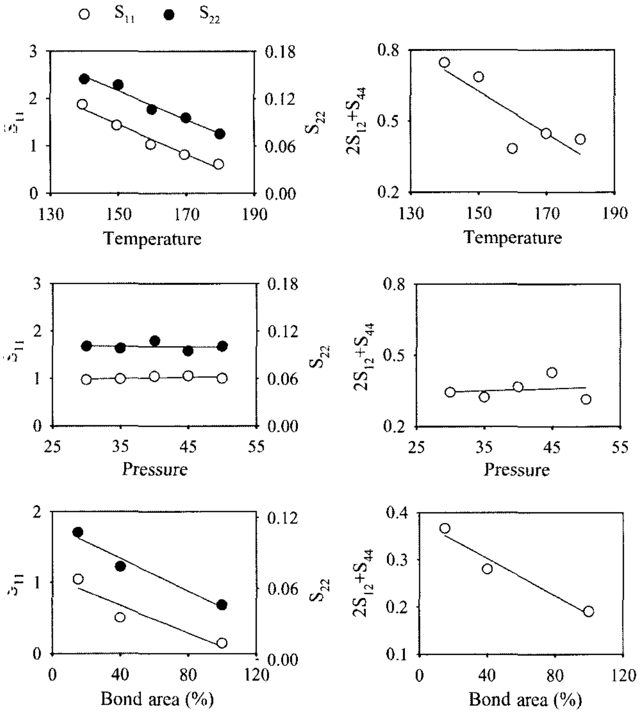

Figure 2. Two compliance constants and one combination computed from the Polynomial regression analysis for the nonwovens produced in three different processing parameters (top: temperature, middle: pressure, bottom: bond area series).

by applying the polynomial regression analysis using the functional form equation (2), two of these, S_{11} and S_{22}, and a combination of the other two, $2S_{12}+S_{44}$, are calculated by the following relations.

$$\begin{aligned} S_{11} &= cons1 \\ S_{22} &= cons1 + cons2 + cons3 \\ 2S_{12}+S_{44} &= cons2 + 2cons1 \end{aligned} \tag{3}$$

With the two constants and one combination of the other two, the tensile modulus at any azimuthal intervals can be successfully predicted and also these parameters provide useful information for investigating the effects of the different processing conditions. The two constants and one combination are respectively reported in the Figure 2 with three different processing conditions. The constants in temperature series well explain the meaningful performance changes of nonwoven samples due to the modulus changes of bond site with increasing bonding temperature. Note that in the pressure series, there are no significant changes observed in the typical window of our experimental between 30 to 50 psi in 5-psi increments. This tendency is also successfully predicted. The changes of compliance due to restriction of regional or entire movebility of the constitute fibers in a fibrous assembly with increasing bond area are well explained in bond area series.

Constants from Tensile and Shear

To complete the in-plane strain-stress relations, equation (1), the combination of $2S_{12}+S_{44}$ should be separated. The separation is accomplished by performing shear measurements. The statistically meaningful S_{44} is computed by the orthotropic approach with the experimental shear moduli $G_{xy}(\theta)$ at different shear directions and the direction angles. The polynomial regression analysis is performed for the functional form of equation (4) as follows:

$$\begin{aligned} \frac{1}{G_{xy}(\theta)} &= S_{44}(\theta) = 2(2S_{11}+2S_{22}-4S_{12}-S_{44})\sin^2\theta\cos^2\theta \\ &\quad + S_{44}(\sin^4\theta+\cos^4\theta) \\ &= cons1 + cons2\sin^2\theta + cons3\sin^4 \end{aligned} \tag{4}$$

where

$$\begin{aligned} cons1 &= S_{44} \\ cons2 &= 4S_{11} + 4S_{12} - 8S_{12} - 4S_{44} \\ cons3 &= -4S_{11} - 4S_{12} + 8S_{12} + 4S_{44} \end{aligned}$$

When the orthotropic theory is applied for the shear measurements of the oriented nonwoven fabrics, two maximum values of the shear moduli is expected between ±90° to the main orientation direction from equation (4). However just one maximum value was observed in the actual experimental data (refer to Figure 3). Therefore the predicted values in Figure 3 ware obtained by applying the polynomial approach separately in the respective ranges of 0 to $\pi/2$ and $\pi/2$ to π.

If the oriented nonwoven samples, which have the main Orientation Distribution Function (ODF) [15] at one direction, are subjected to shear deformations at positive and negative angles, tension will be dominant for positive angle and compression for negative angle to the preferred direction of

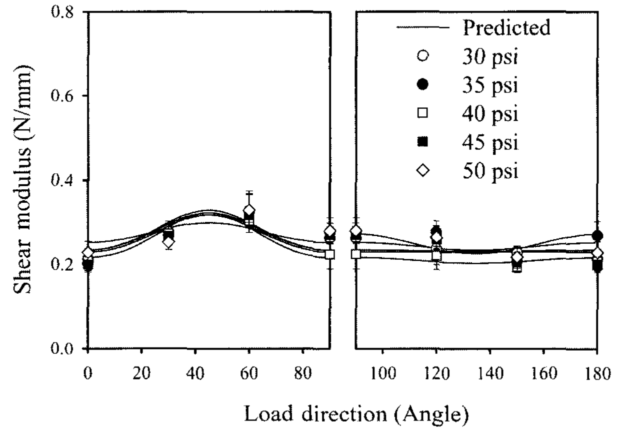

Figure 3. Comparison of experimental and predicted values for shear moduli obtained at ±0.5° of shear deformation angle for the nonwovens produced in pressure series.

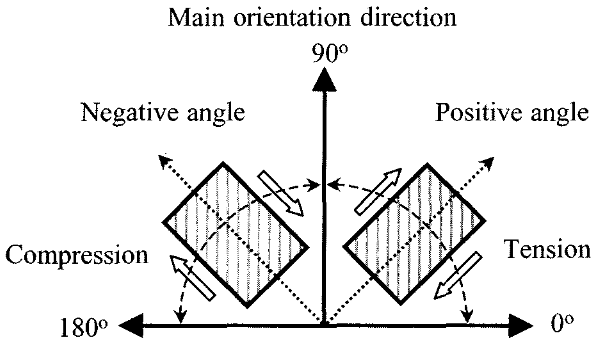

Figure 4. Schematic diagram of shear deformation for the negative and positive angle direction.

constituent fibers (refer to Figure 4). The in-plan compression force at the negative angle resulting from shear deformation easily causes the fiber bending, buckling or the fabric buckling even though the applied shear deformation is small enough. Thus the shear behavior depends not only on the amount of fiber distribution but also on the direction of applied deformation. Note here that the shear behavior in the 0 to +0.5 ° direction is different from that measured in the 0 to –0.5 ° range. This is often likely to be the case if the structure is not isotropic. Consider an anisotropic nonwoevns with the machine direction as the preferred direction of its fiber orientation distribution, and two sample strips from it that have been cut along the directions +θ and −θ, respectively (refer Figure 4). If sample is tested for shear in the 0 to +0.5 ° range, a larger fraction of the constituent fibers will be under tension when compared to the same sample tested in the 0 to –0.5 ° range. Therefore, the shear modulus obtained from the 0 to +0.5 ° shear test would be higher than that from the 0 to –0.5 ° test. The situation will be exactly reversed at the other side. For this reason, the mean of the two has been considered as the appropriate measure. The mean shear modulus is not the statistical mean value only for obtaining a representative value but the mechanical mean including some fraction of extension and compression together for the constituent fibers. The mean values meet the condition for the symmetry of equation (4).

The shear properties were successfully characterized in the three cases (Figures 3 and 5). The predicted results are shown as solid lines and are seen to be in excellent agreements with experimental data shown as marks. Figure 5 shows the

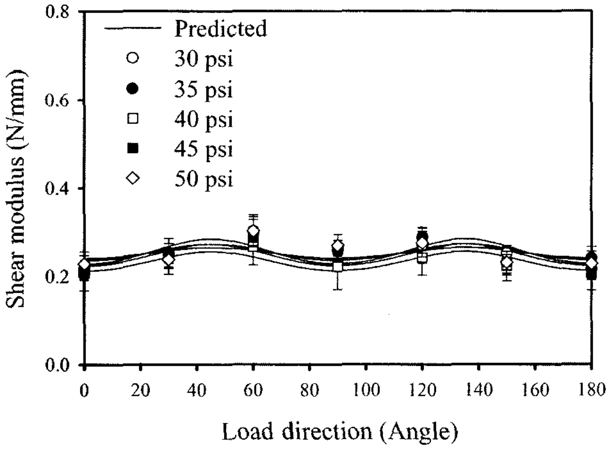

Figure 5. Comparison of experimental and predicted values for mean shear moduli for the nonwovens produced in pressure series.

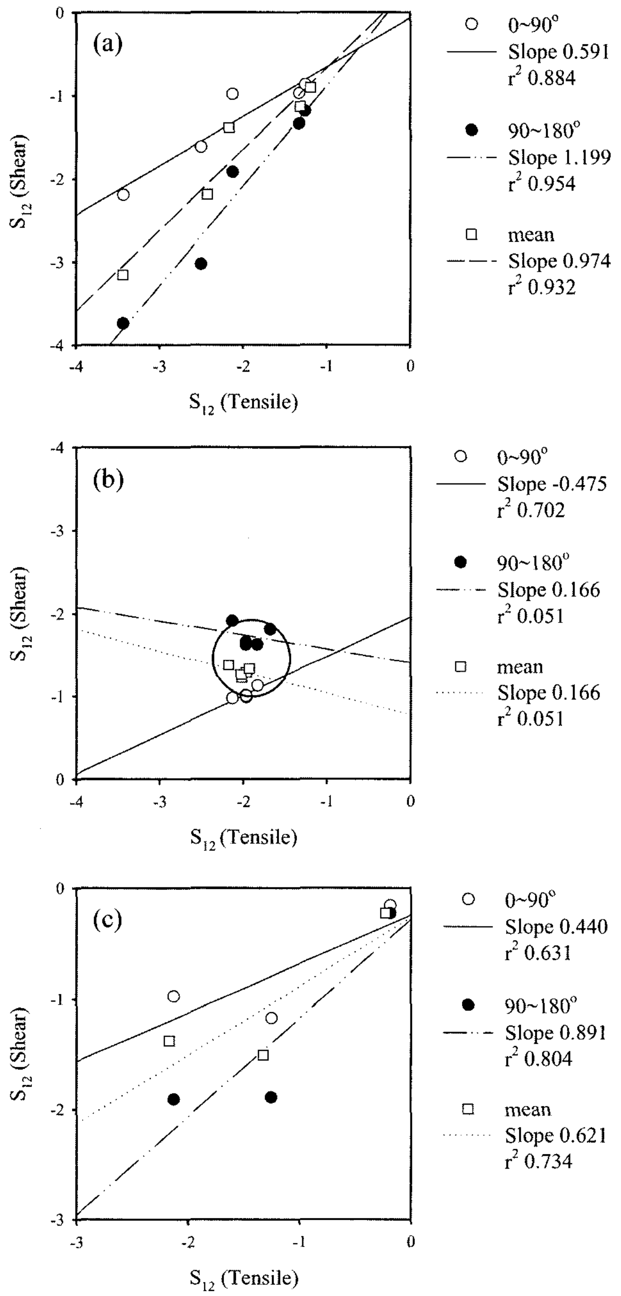

Figure 6. (a) Compatibility parameter (S_{12}) for the nonwovens produced in temperature series, (b) Compatibility parameter (S_{12}) for the nonwovens produced in pressure series, and (c) Compatibility parameter (S_{12}) for the nonwovens produced in bond area series.

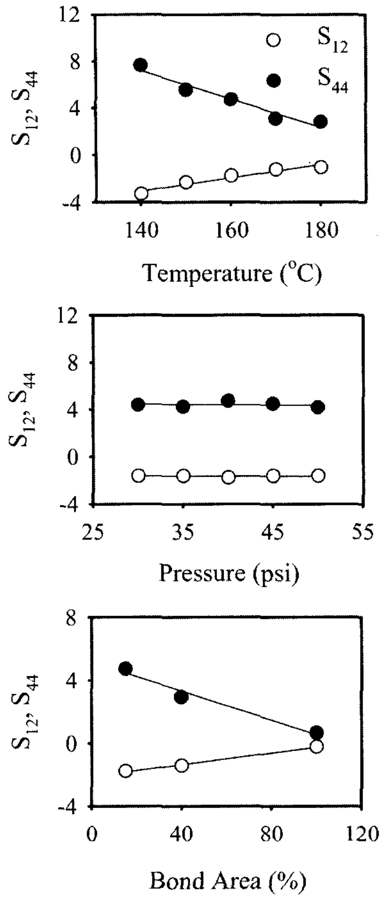

Figure 7. Two compliance constants computed from the Polynomial regression analysis for the nonwovens produced in three different processing parameters (top: temperature, middle: pressure, bottom: bond area series).

mean value for the pressure series. The same excellent level of agreement was observed for the temperature and bonded area series.

Through the polynomial regression analysis using the functional form of equation (4), S_{44} and one combination of $S_{11}+S_{22}-2S_{12}-S_{44}$ are obtained. Thus S_{12} could be calculated respectively from the $2S_{12}+S_{44}$ from tensile and $S_{11}+S_{22}-2S_{12}-S_{44}$ from shear. It is important that the reliability of S_{12} should be explored prior to using the value in a practical application. The compatibility of S_{12} calculated from tensile was explored by comparing with those obtained from shear measurements at the limited shear angles of ±0.5 ° and its mean value. The slope of S_{12} from tensile and from shear was used as compatibility parameters as shown in Figure 6(a)~(c). For ideal orthotropic material, the slope is expected as 1. Figure 6(a) of temperature series shows very high co-relation coefficient in each case and almost same S_{12} values are shown in the case of the mean values from shear. It is emphasized that the mean shear modulus is not the statistical mean but the mechanical mean which includes the combined effects of tension and compression together for the constituent fibers. This idea is well matched to the complexity of tensile process, even though the detail mechanism might be different. The compliance obtained from tensile measurement includes tension and compression modes of the constituent fibers together by the applied tensile load and simultaneously following lateral force. The fraction of the mechanical modes may be changed according to the applied load direction and mode, and the fiber orientation distribution. Figure 6(b) of the pressure series is well representing the tendency that does not show significant performance changes in the tensile and shear moduli as shown in Figure 1(b). The bond area series show linear relations but need more data for ensuring more statistically meaningful information (Figure 6(c)).

The mean S_{12} and S_{44} are respectively reported in the Figure 7 with three different processing conditions and clearly show the meaningful performance changes of nonwoven samples as mentioned above.

Conclusions

The attainment of more statistically meaningful constants is accomplished by employing the polynomial regression method based on the orthotropic theory. The valid method to obtain the four compliance is established by exploring the compatibility of the values from tensile and shear measurements. The compliance S_{12} from tensile and shear (employing mean value of shear moduli at +0.5 ° and −0.5 °) measurements are well matched and also the mean values at the shear meet the condition for the symmetry of the orthotropic theory. The mean value from shear can be successfully used in determining the four compliances and those four constants can give meaningful information in engineering a textile product for a specific end use. However when it is used in a practical application, it should be carefully noted that the four compliances containing the linear and nonlinear behaviors of the constituent fibers are determined.

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