An Escalator Structure-Based Adaptation Algorithm for Channel Equalization with Eigenvalue Spread-Independency

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Abstract

In this paper we introduce a new escalator(ESC) structure-based adaptation algorithm. The proposed algorithm is independent of eigenvalue spread ratio(ESR) of channel and has faster convergence speed than that of the conventional ESC algorithms. This algorithm combines the fast adaptation ability of least square methods and the orthogonalization property of the ESC structure. From the simulation results the proposed algorithm shows superior convergence speed and no slowing down of convergence speed when we increase the ESR of the channel.

Key words : Escalator, Orthogonalization, Channel Equalizer, Least Square.

Ⅰ. Introduction

Many researchers have studied various adaptive equalizer structures and coefficient-adjustment algorithms. The tapped delay line(TDL) equalizer structure using the least mean square(LMS) algorithm(LMS-TDL) of Widrow and Hoff[1] has been being widely used due to its simplicity in realization. One drawback of the LMS algorithm is that its convergence speed decreases as the ratio of the maximum to the minimum eigenvalues of the input autocorrelation matrix increases. To cope with this problem, it has been proposed to orthogonalize the input signal using Gram-Schmidt orthogonalization procedure which can be implemented by the escalator(ESC) structure[2]. Due to the complete orthogonalization property of the ESC structure, its convergence speed is independent of the eigenvalue spread ratio(ESR) of the input signals. Though the ESC structure completely orthogonalize the input signals, its coefficients adaptation algorithms are not sufficiently studied for faster convergence speed. LMS algorithm for the ESC in which mean squared local estimation errors are minimized is currently used in the ESC filtering problems[3],[4]. In this paper we present a new fast adaptation algorithm for the ESC structure by introducing least square(LS) criterion to the local errors of the structure.

The performance index for the mean squared error (MSE) criterion is defined as the expected value of the squared difference $e(k)$ between the desired information symbol $d(k)$ and the estimated information symbol $y(k)$ at time k

$$MSE = E[e^2(k)] \tag{1}$$

In the MSE criterion the tap weight coefficients of a linear equalizer are adjusted to minimize the MSE. The derivation of the algorithms for adjusting the coefficients of the linear equalizer to minimize the MSE is based on a statistical approach. Instead of a statistical average the performance index can be expressed in terms of a time average. We can determine the coefficients of the equalizer that minimize the time-average weighted square error

$$LS = \sum_{n=0}^{k} w^{k-n} e^2(n) \tag{2}$$

The LS approach deals directly with the received data whereas MSE-based approaches minimize the expected value of the squared error. By adopting the LS criterion to the ESC structure, minimizing LS with respect to ESC coefficients and combining steepest descent method with it, we propose a new ESC-coefficients adaptation algorithm which has ESR-independent and faster convergence speed.

Ⅱ. Escalator Equalizer Structure[2]

Given a symmetric matrix R, there exists a unit lower triangular(ULT) matrix W, such that WRW^T is a diagonal matrix. The ULT matrix W can be computed

Manuscript received March 17, 2004 ; revised May 10, 2004. (ID No. 20040317-009J)
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in the form of $W = W_N W_{N-1} \cdots W_1$. The ULT transformation $Y(k)=W \cdot X(k)$ means that system W generates the uncorrelated output vector $Y(k)$ for the input vector $X(k)$ where its symmetric autocorrelation matrix $R=E[X(k)X^T(k)]$. If we define $X(k)$ as an input vector augmented with the desired sample $d(k)$, $X(k)=[x(k-N+1), x(k-N+2), \cdots, x(k), d(k)]^T$ and $Y(k)=[y(k-N+1), y(k-N+2), \cdots, y(k), e(k)]^T$ as an output vector augmented with the error sample $e(k)$, $y(k)=W \cdot X(k)$, becomes filtering process of ESC structure.

We can realize the ULT transformation sequentially like $Y_1(k)=W_1 \cdot X(k)$, $Y_2(k)=W_2 \cdot Y_1(k)$ and $Y_3(k)=W_3 \cdot Y_2(k)$ etc. The final stage's output vector $Y_N(k)$ becomes $Y(k)$. The corresponding ESC filter realization for N=3 is shown in Fig. 1 and the general ESC filter equations are

$$y_{i,j}(k-m) = y_{i-1,j+1}(k-m) - \alpha_j^i(k) y_{i-1,1}(k-n),$$
$$y_{0,N+1}(k+1) = d(k), y_{0,j}(\cdot) = x(\cdot), y_{i,j}(k+1) = e_i(k), \qquad (3)$$

for $1 \le i \le n$, $1 \le j \le N-i+1$, $m = N-i-j$ and $n=N-i$.

For the escalator structure the global minimization of the output energy can be accomplished as a sequence of local minimization problems, one at each stage of the escalator filters. Fig. 2 shows the part of the escalator filter which corresponds to the weight $\alpha_j^i(k)$. We can see that part of escalator filter can be considered as one-tap coefficient TDL filter.

Ⅲ. Escalator Filter Coefficients Adaptation by MSE Criterion

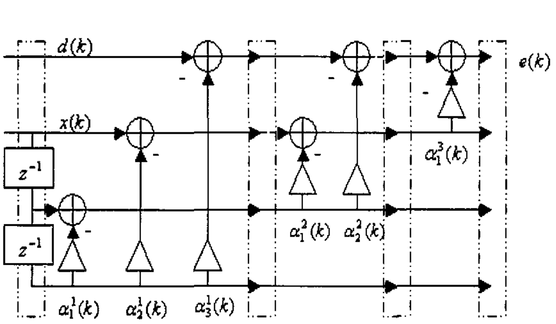

Fig. 1. ESC filter realization for N=3.

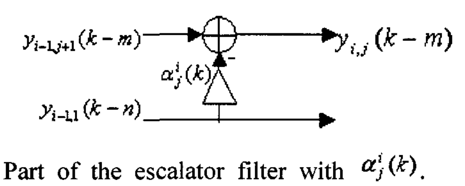

Fig. 2. Part of the escalator filter with $\alpha_j^i(k)$.

The escalator weight $\alpha_j^i(k)$, which can be considered as the coefficient of one-tap TDL, can be optimized according to a MSE criterion or by employing the method of least squares. Suppose we adopt the MSE criterion and select the parameter to minimize the sum of the mean-square errors where the error is $y_{i,j}(k-m)$. In other words, the MSE is given as

$$MSE = E[y^2_{i,j}(k-m)]$$
$$= E[(y_{i-1,j+1}(k-m) - \alpha_j^i(k) y_{i-1,1}(k-n))^2] \qquad (4)$$

Differentiation of MSE with respect to $\alpha_j^i(k)$ yields the solution

$$\alpha_j^i(k) = \frac{E[y_{i-1,j+1}(k-m) y_{i-1,1}(k-n)]}{E[y_{i-1,1}^2(k-n)]} \qquad (5)$$

It's instructive to note that each $\alpha_j^i(k)$ defined above is in the form of ration whose numerator and denominator are cross-correlation and autocorrelation(variance) terms, respectively.

Assuming that the variance of a random variable $f(k)$ changes slowly, one common method for estimating the variance, $E[f^2(k)]$, is to use a single-pole low-pass filter, i.e.,

$$\sigma_f^2(k) = \beta \sigma_f^2(k-1) + (1-\beta) f^2(k) \qquad (6)$$

where $0 \le \beta \le 1$ is a smoothing parameter which controls the bandwidth and time constant of the system whose transfer function with it's input $f^2(k)$ is given by

$$S(z) = (1-\beta)\frac{z}{z-\beta} \qquad (7)$$

Similarly the instantaneous cross-correlation function of two random variables $f(k)$ and $g(k)$ can be estimated using a simple low-pass filter; i.e.,

$$r_{fg}(m,k) = E[f(k)g(k-m)]$$
$$= \beta r_{fg}(m,k-1) + (1-\beta) f(k) g(k-m) \qquad (8)$$

Thus the denominator and numerator terms in (5) can be updated via the a single-pole low-pass system (6) and (8) to yield adaptive escalator filter weights.

$$\alpha_j^i(k) = \frac{c_{i,j}(k)}{v_y{}^i(k)} \qquad (9)$$

$$c_{i,j}(k) = \beta c_{i,j}(k-1) + (1-\beta) y_{i-1,j+1}(k-m) y_{i-1,1}(k-n) \qquad (10)$$

$$v_y{}^i(k) = \beta v_y{}^i(k-1) + (1-\beta) y_{i-1,1}^2(k-n) \qquad (11)$$

where $0 \le \beta \le 1$, and the initial values of $c_{i,j}(k)$ and

$v_i^i(k)$ are usually zero.

Instead of estimating the cross-correlation functions by using the simple low-pass filter, we can use the steepest descent method with time-varying convergence parameter $\mu_i(k)$ as the following[4]:

$$\alpha_j^i(k+1) = \alpha_j^i(k) + \mu_i(k) y_{i,j}(k-m) y_{i-1,1}(k-n) \tag{12}$$

where $\mu_i(k) = 2\mu / v_y^i(k)$ and $v_y^i(k)$ is estimated using a recurrence relation given by (11) for $0 \le \beta \le 1$. Here, the time constant of the LPF is determined by β, whereas the average time constant of the adaptation process of the LMS-type algorithm is dependent on μ. As β becomes smaller, the time constant of the LPF decreases, which yields faster estimation of the signal power but a larger estimation noise in the steady state. For convenience's sake, we call the algorithm (12) LMS-ESC in this paper.

Ⅳ. Escalator Filter Coefficients Adaptation by LS Criterion

In this section we propose to adopt a least-square (LS) criterion to the local ESC filter structure for updating $\alpha_j^i(k)$. The performance index to be minimized is

$$\begin{aligned} LS &= \sum_{p=0}^{k} w^{k-p} y_{ij}^2(p-m) \\ &= \sum_{p=0}^{k} w^{k-p} [y_{i-1,j+1}(p-m) - y_{i-1,1}(p-n) \cdot \alpha_j^i(p)]^2 \end{aligned} \tag{13}$$

Minimization of LS with respect to $\alpha_j^i(k)$ yields the solution

$$\alpha_j^i(k) = \frac{\sum_{p=0}^{k} w^{k-p} y_{i-1,j+1}(p-m) y_{i-1,1}(p-n)}{\sum_{p=0}^{k} w^{k-p} y_{i-1,1}^2(p-n)} \tag{14}$$

The numerator and denominator in (14) can be updated recursively in time as follows:

$$A(k) = w \cdot A(k-1) + y_{i-1,j+1}(k-m) y_{i-1,1}(k-n) \tag{15}$$

$$B(k) = w \cdot B(k-1) + y_{i-1,1}^2(k-n) \tag{16}$$

Then $$\alpha_j^i(k) = \frac{A(k)}{B(k)} \tag{17}$$

The numerator and denominator in (14) or (17) can be considered the time-average weighted auto and cross-correlation functions. They are similar to MSE criterion case, where it's numerator and denominator are the statistical autocorrelation function of the received signal components and the cross-correlation function between the desired symbol and the received signal components. As in (12) we can use the steepest descent approach to the numerator in (14) or (17). Since the desired $\alpha_j^i(k)$ minimizes the error $y_{i,j}(k-m)$ in the Fig. 2, the error $y_{i,j}(k-m)$ will be orthogonal to the input $y_{i-1,1}(k-n)$. This suggests the following adaptation algorithm for ESC structure.

$$\alpha_j^i(k+1) = \alpha_j^i(k) + \frac{y_{i,j}(k-m) y_{i-1,1}(k-n)}{B(k)} \tag{18}$$

where $B(k)$ is computed recursively as in (16).

Ⅴ. Simulation Results

In this section the performance of three adaptive equalizer algorithms for two channels that have different eigenvalue spread ratios is compared. The three algorithms considered are: the LMS-TDL algorithm[5], LMS-ESC algorithm[4] and the proposed algorithm (18) for the ESC structure. The impulse response h_i of the channel model is

$$h_i = \frac{1}{2}\{1 + \cos[2\pi(i-2)/B]\}, \quad i = 1,2,3 \tag{19}$$

where the parameter B determines the channel bandwidth and controls the eigenvalue spread ratio of the correlation matrix of the inputs in the equalizer[5]. The eigenvalue spread ratio increases with B. The experiment is carried out in two channels that are intended to evaluate the convergence performance of the three algorithms using TDL and ESC structure to changes in the eigenvalue spread.

Channel 1: B=3.1, ESR=11.12, Channel 2: B=3.3, ESR=21.71

The LMS-TDL equalizer has 11 tap coefficients. The LMS-ESC and the proposed algorithm consist of 11 stages. A zero mean white Gaussian noise sequence with variance 0.001 was added to yield the equalizer input. The convergence parameter for the LMS-TDL was 2μ=0.02. The convergence parameter 2μ and smoothing parameter β for LMS-ESC are 0.02 and 0.99, respectively. These values of convergence parameters for those algorithms were determined experimentally so that the related steady-state mean squared errors were the same. We see from Fig. 3 and Fig. 4

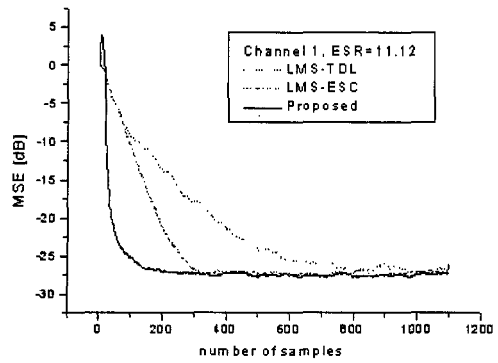

Fig. 3. Convergence performance for eigenvalue spread ratio(ESR)=11.12.

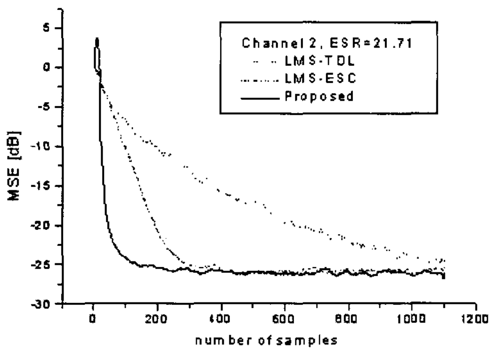

Fig. 4. Convergence performance for eigenvalue spread ratio(ESR)=21.71.

that increasing the ESR has the effect of increasing the steady-state MSE value of the three algorithms and slowing down the rate of the convergence of the LMS-TDL equalizer algorithm. In the channel 1 approximately 900 samples are required for the LMS-TDL to converge and in the channel 2 LMS-TDL requires more than 1100 samples to converge. On the other hand, the algorithms having ESC structure show no decrease in their convergence speed. In both cases, the proposed has shown more rapid convergence than the LMS-ESC. The LMS-ESC converges after about 300 samples but the proposed algorithm requires about 150 samples to converge.

Ⅵ. Conclusions

In this paper we proposed a new escalator-coefficient adaptation algorithm that is independent of eigenvalue spread ratio and has faster convergence speed than the conventional ESC algorithms. This algorithm adopts the least square criterion to the ESC structure and combines steepest descent method with the LS method. It exploits the fast adaptation ability of LS method and the orthogonalization property of the ESC structure. The simulation results show that the proposed algorithm has 2 times faster convergence speed than for the LMS-ESC algorithm and it shows no performance degradation in convergence speed when we increase the eigenvalue spread ratio of the channel. The improved performance indicates that the proposed algorithm does appear to be an attractive alternative to the MSE-criterion ESC algorithms for adaptive channel equalization applications with large eigenvalue disparity.

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