On the Limitation of Telegrapher's Equations for Analysis of Nonuniform Transmission Lines

Se-Yun Kim

Abstract

The limitation of telegrapher's equations for analysis of nonuniform transmission lines is investigated here. It is shown theoretically that the input impedance of a nonuniform transmission line cannot be derived uniquely from the Riccati equation only except for the exponential transmission line of a particular frequency-dependent taper. As an example, the input impedance of an angled two-plate transmission line is calculated by solving the telegrapher's equations numerically. The numerical results suffer from larger deviation from its rigorous solution as the plate angle increases.

Key words : Nonuniform Transmission Line, Telegrapher's Equations, TEM Mode, Angled Two Plates.

Ⅰ. Introduction

The estimation of the input impedance of transmission lines(TLs) is one of the canonical topics in high-frequency circuits. As a leading tool for treating the problem, the telegrapher's equations have worked well in analysis and synthesis of uniform TLs[1]. In microwaves, nonuniform or tapered TLs have been widely used in the design of impedance matching networks, filters, couplers, circulators, and resonators[2],[3]. Recently we have investigated the electromagnetic coupling to the uniform[4] and nonuniform TLs[5] by employing the telegrapher's equations. The purpose of this paper is to criticize the validity of the telegrapher's equations for analyzing nonuniform TLs.

Walker and Wax[6] investigated the reflection coefficients of a nonuniform TL by solving the Riccati equation, which was derived from the telegrapher's equation. Later, Ahmed[7] derived the same Riccati equation for the input impedance of a nonuniform TL as follows. At first, a nonuniform TL is discretized into a number of piecewise sections short enough to consider each section as an uniform TL. The input impedance is then calculated by cascading all the sections sequentially. This implies that Ahmed's approach also relies on the concept of the telegrapher's equations.

The telegrapher's equations are derived under a heuristic assumption that only a planar TEM mode propagates along the nonuniform TL regardless of nonuniform structure. The planner TEM wave is the unique type of wave which propagates along an uniform TL without attenuation. However, there may be other types such as cylindrical and spherical TEM waves as well as planar TEM wave. In view of a rigorous mode theory[8], the transverse type of the TEM mode supported by a nonuniform TL should be specified uniquely according to the longitudinal variation of its structure.

The limitations of the telegrapher's equations for analyzing a nonuniform TL are discussed here in two aspects. In a theoretical point, the input impedance of a nonuniform TL cannot derive uniquely from the Riccati equation only except for the exponential TL of a particular frequency-dependent taper. The other is such a typical case as the input impedance of an angled two-plate TL, because its rigorous solution is well known by adopting a cylindrical TEM mode[9]. The input impedance calculated by solving the telegrapher's equations numerically reveals larger deviation from its rigorous value as the plate angle increases.

Ⅱ. Theoretical Aspect

Consider Fig. 1, in which the characteristic impedance of a nonuniform TL, $Z_c(x)$ is an arbitrary function of the longitudinal length. x. To apply the telegrapher's equations[6], one may construct an equi-

Manuscript received February 4, 2004 ; revised March 22, 2004. (ID No. 20040204-004J)
Imaging and Media Research Center, Korea Institute of Science and Technology, Seoul, Korea.

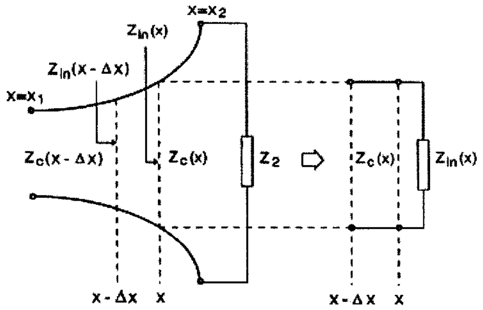


Fig. 1. An equivalent modeling on a nonuniform TL by cascading a number of piecewise uniform TLs sequentially.

valent model on the nonuniform TL by cascading a large number of piecewise equi-sections, as shown in Fig. 1. Then the input impedance, $Z_{in}(x-\Delta x)$ was expressed by (1)[7]

$$Z_{in}(x-\Delta x) = Z_c(x)\frac{Z_{in}(x)+jZ_c(x)\tan k\Delta x}{Z_c(x)+jZ_{in}(x)\tan k\Delta x} \quad (1)$$

where k denotes the free-space wavenumber. In (1), the section of the nonuniform TL in $[x-\Delta x, x]$ is assumed to be an uniform TL with the characteristic impedance $Z_c(x)$. For small Δx, one may transform (1) into the well-known Riccati equation (2)[7] as

$$\frac{d}{dx}Z_{in}(x) = -jkZ_c(x) + jk\frac{Z_{in}^2(x)}{Z_c(x)} \quad (2)$$

Without loss of generality, the section in $[x-\Delta x, x]$ can be replaced by another uniform TL with the characteristic impedance $Z_{in}(x-\tau\Delta x)$ for $0\leq\tau\leq 1$. Then one may derive a more general form of (2) as

$$\frac{d}{dx}Z_{in}(x) = -jkZ_c(x-\tau\Delta x) + jk\frac{Z_{in}^2(x)}{Z_c(x-\tau\Delta x)}, \quad (3)$$

for $0\leq\tau\leq 1$

To assure the uniqueness of the Ahmed's approach, one may assume that the Riccati equation in (2) is equal to (3). Subtracting (3) from (2), and then dividing both sides by $\tau\Delta x$ yield

$$\frac{Z_c(x)-Z_c(x-\tau\Delta x)}{\tau\Delta x} = Z_{in}^2(x)\frac{1}{\tau\Delta x}\left[\frac{1}{Z_c(x)} - \frac{1}{Z_c(x-\tau\Delta x)}\right] \quad (4)$$

For infinitesimal Δx , (4) can be simply rewritten as

$$\left[1+\frac{Z_{in}^2(x)}{Z_c^2(x)}\right]\frac{d}{dx}Z_c(x) = 0 \quad (5)$$

There are two types of solutions to (5). One is trivial because $\frac{d}{dx}Z_c(x)=0$ means an uniform TL. The other solution satisfies the following relation as

$$Z_{in}(x) = \pm jZ_c(x) \quad (6)$$

Applying (6) into (2) yields

$$\frac{d}{dx}Z_c(x) = \pm 2kZ_c(x) \quad (7)$$

From (7), one may obtain the following solution

$$Z_c(x) = Z_c(0)\exp(\pm 2kx) \quad (8)$$

Since two Riccati equations in (2) and (3) are definitely different for short range of $[x-\Delta x, x]$, those accumulated values of $Z_{in}(z)$ are also different in general. However, if $Z_c(x)$ becomes constant or exponential function of x as (8), both right-hand terms of (2) and (3) become equal even in the infinitesimal section $[x-\Delta x, x]$. In conclusion, the result in (8) illustrates that the input impedance of a nonuniform TL can be derived from the Riccati equation uniquely only for the exponential TL of a particular frequency-dependent taper.

Ⅲ. Numerical Aspect

Fig. 2 depicts an angled two-plate TL with intersection angle θ_c terminated by a load impedance Z_L at $\rho=\rho_2$. The input impedance $Z_{in}(\rho)$ at $\rho=\rho_1$ is expressed rigorously as[9]

$$Z_{in}(\rho_1) = -jZ_c(\rho_1)\frac{\Gamma_2 H_1^{(1)}(k\rho_1)+H_1^{(2)}(k\rho_1)}{\Gamma_2 H_0^{(1)}(k\rho_1)+H_0^{(2)}(k\rho_1)} \quad (9)$$

where $H_m^{(n)}$ denotes the m-th order Hankel function of the n-th kind for n=1 or 2. The characteristic impedance $Z_c(\rho)$ at $\rho=\rho_1$ and reflection coefficients

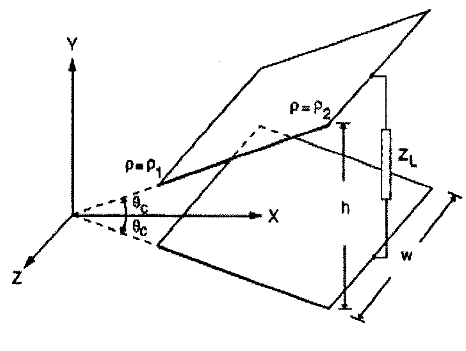


Fig. 2. The geometry of an angled two-plate TL.

Γ_2 at $\rho=\rho_2$ are given by

$$Z_c(\rho_1) = Z_0 \frac{\theta_c \rho_1}{w} \tag{10}$$

$$\Gamma_2 = -\frac{Z_L H_0^{(2)}(k\rho_2) + jZ_c(\rho_2) H_1^{(2)}(k\rho_2)}{Z_L H_0^{(1)}(k\rho_2) + jZ_c(\rho_2) H_1^{(1)}(k\rho_2)} \tag{11}$$

where w and Z_0 denote the width of the plate and the free-space characteristic impedance, respectively.

A test geometry is chosen by the angled two-plate TL of $h=\frac{1}{3}\lambda$, $\rho_2-\rho_1=\frac{1}{3}\lambda$, and Z_L=37.7 Ω in Fig. 2. To calculate $Z_{in}(\rho_1)$ by following the Ahmed's approach, the angled two-plate is divided into N number of equi-sections. If N is larger than 200, the curves of $Z_{in}(\rho_1)$ converge, as shown in Fig. 3. It should be noted that as θ_c increases, both ρ_1 and ρ_2 decrease because the height h is fixed. $Z_{in}(\rho_1)$ obtained by solving the telegrapher's equations numerically is compared to the rigorous solution. The exact curve of $Z_{in}(\rho_1)$ can be obtained by using (9). Fig. 4 illustrates that the input impedance curves calculated by following the Ahmed's approach with N=200 are affected by large deviation from the exact curve as θ_c increases from 0° to 25°. It implies that the telegrapher's equations themselves are erroneous for θ_c>10° even if those solutions are highly accurate for θ_c>10°. One may argue that the deteriorating accuracy for large tilt angle in Fig. 4 arises from the propagation of higher modes. It is well recognized that the characteristics of a TL are specified only by its propagation constant and characteristic impedance. For example, assume that the dominant TEM mode and two higher modes propagate along the longitudinal axis of the TL without attenuation.

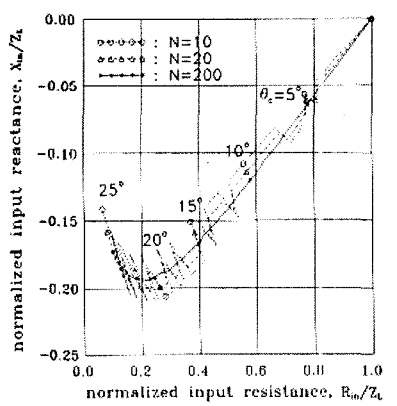


Fig. 3. Convergence of normalized input impedances calculated by using (2) as N increases from 10 to 200.

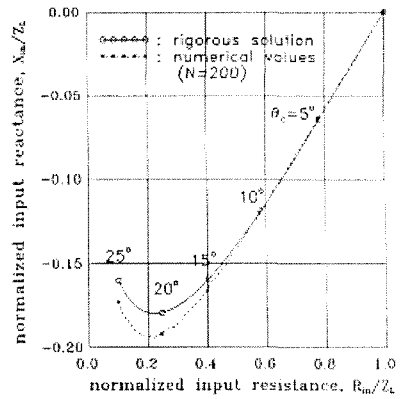


Fig. 4. Comparison between normalized input impedances of an angled two-plate TL calculated by using (2) and (9).

Then three different TL models are required to represent all of the propagation modes into the equivalent circuits. However our interest is to calculate the input impedance of a nonuniform TL only for the dominant TEM mode. Hence the propagation of some higher modes cannot distort the results in Fig. 4. It leads us to conclude that in spite of a canonical tool for analysis and synthesis of nonuniform TLs, the accuracy of the input impedance calculated by solving the telegrapher's equations numerically is not assured.

Ⅳ. Conclusions

The validity of the telegrapher's equations for analysis of nonuniform TLs was investigated here. The input impedance of a nonuniform TL cannot derive uniquely from the Riccati equation only except for the exponential TL of a particular frequency-dependent taper. The inaccuracy of the telegrapher's equations aries from the heuristic assumption that a planar TEM wave is always propagated along nonuniform TLs. This defect was assured by showing that the input impedance of an angled two-plate TL calculated by solving the telegrapher's equations numerically was erroneous as its intersection angle increases.

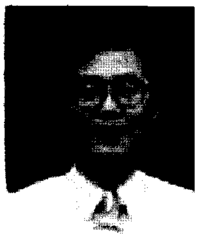
References

[1] F. Romeo, M. Santomauro, "Time-domain simulation of n coupled transmission lines", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 131-137, Feb. 1987.

[2] H. Kaufman, "Bibliography of nonuniform trans-

mission lines", *IRE Trans. Antennas Propagat.*, vol. AP-3, pp. 218-220, Oct. 1955.
[3] D. C. Youla, "Analysis and synthesis of arbitrarily terminated lossless nonuniform lines", *IEEE Trans. Circuit Theory*, vol. CT-11, pp. 363-372, Sep. 1964.
[4] S. Y. Kim, H. T. Ha, "Transient response of nonlinear protective devices for suppression of lighting surges on a pair-wire line", *IEICE Trans. Part A*, vol. E74, pp. 3018-3023, Oct. 1991.
[5] S. Y. Hong, J. W. Ra and S. Y. Kim, "Electromagnetic coupling to a nonuniform transmission line consisting of two angled wires", *Microwave Optical Technol. Lett.*, vol. 5, pp. 736-739, Dec. 1992.
[6] L. R. Walker, N. Wax, "Nonuniform transmission lines and reflection coefficients", *J. Appl. Phys.*, vol. 17, pp. 1043-1045, 1946.
[7] M. J. Ahmed, "Impedance transformation equations for exponetial, cosine-squared, and parabolic tapered transmission lines", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 67-68, Aug. 1981.
[8] R. E. Collin, *Field Theory of Guided Waves,* New York: McGraw-Hill, ch. 3, 1960.
[9] S. Ramo, J. R. Whinnery and T. V. Duzer, *Fields and Waves in Communication Electronics*, New York: Wiley, ch. 9, 1984.

Se-Yun Kim



received the B.S. degree in electrical engineering from the Seoul National University in 1978, the M.S. and Ph.D. degrees in electrical engineering from the Korea Advanced Institute of Science and Technology in 1980 and 1984, respectively. From 1984 to 1986, he was a post-doctoral fellow in the Korea Advanced Institute of Science and Technology. In 1986, he joined the Korea Institute of Science and Technology, where he is now a principal researcher. His research interests include electromagnetic diffraction, microwave imaging, underground probing, and EMI/EMC.