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# Investigation of Self-Excited Combustion Instabilities in Two Different Combustion Systems

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The objective of this paper is to characterize dynamic pressure traces measured at self-excited combustion instabilities occurring in two combustion systems of different hardware. One system is a model lean premixed gas turbine combustor and the other a fullscale bipropellant liquid rocket thrust chamber. It is commonly observed in both systems that low frequency waves at around 300Hz are first excited at the onset of combustion instabilities and after a short duration, the instability mode becomes coupled to the resonant acoustic modes of the combustion chamber, the first longitudinal mode for the lean premixed combustor and the first tangential mode for the rocket thrust chamber. Low frequency waves seem to get excited at first since flame shows the higher heat release response on the lower frequency perturbations with the smaller phase differences between heat release and pressure fluctuations. Nonlinear time series analysis of pressure traces reveals that even stable combustion might have chaotic behavior with the positive maximum Lyapunov exponent. Also, pressure fluctuations under combustion instabilities reach a limit cycle or quasi-periodic oscillations at the very similar run conditions, which manifest that a self-excited high frequency instability has strong nonlinear characteristics.

**Key Words :** Combustion Instability, Bifurcation, Nonlinear, Time Series Analysis, Lyapunov Exponent

## Nomenclature

$A$ : Cross sectional area
$D$ : Diameter
$L$ : Length
$P$ : Probability
$p$ : Pressure
$T$ : Temperature
$\Delta t$ : Time step, an inverse of a sampling frequency

### Subscripts

$c$ : Combustion chamber quantity
in : Inlet quantity

### Superscript

: Fluctuating quantity

## 1. Introduction

Combustion instability phenomenon has been one of complicated research subjects having a long history of investigation so far in the fields of engineering for developing various combustion systems including a ram accelerator, an afterburner, a rocket motor, a liquid rocket engine, and recently, a lean premixed gas turbine combustor. All these combustion systems have features of high energy density flow and relatively low acoustic energy dissipation loss in combustion chambers compared with other conventional combustors, which result in more possibilities of energy coupling between heat release oscillations and acoustics. It is very desirable to avoid combustion instabilities from the fact that a conse-

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quence of combustion instabilities can be catastrophic failure of combustors and related hardware due to excessive vibration levels and grossly increased heat transfer. Thus, combustion instabilities occurring in practical situations are being often recognized as abrupt and distinct noise from pressure excitation in combustion systems (Harrje and Reardon, 1972).

One of major problems for the practical understanding of the phenomena is that they always reveal many different aspects of characteristics heavily depending on the types of combustion systems and operating conditions. This unique characteristic makes it so complicated to achieve general solutions for responsible mechanisms that can be applied for describing and predicting these phenomena. It is obvious from the previous F-1 liquid rocket engine development that the task of designing a stable thrust chamber could become very tedious trial-and-error processes (Oefelein and Yang, 1993). In spite of inherent difficulties of the problems, their various aspects have been investigated with significant amount of efforts and thus, decent results have been produced to provide the better understanding. A recent increase of interests in developing a lean premixed gas turbine combustor makes combustion instability become a resurgent technical issue concerned by many researchers long after relative lack of research demands. It is attempted in this paper to review some of remarkable literature that discusses about mechanisms and parameters playing a role in inducing combustion instabilities, especially occurring in lean premixed combustors and liquid rocket thrust chambers.

Experimental studies (Richards and Janus, 1998 ; Lieuwen et al., 2001 ; Seo, 1999 ; Seo, 2003 ; Yang et al., 2003) on combustion instabilities in premixed gas turbine combustors indicated that non-uniform premixing of gaseous fuel and air is considered as a dominant parameter inducing and sustaining heat release oscillations. Incoming mass flow fluctuations by pressure oscillations in an inlet section account for equivalence ratio fluctuations that result in heat release oscillations. When the latter fluctuations become coupled with pressure oscillations, combustion instability may occur from a closed loop of energy transfer mechanism. However, it was also observed that a fully premixed flow, even without equivalence ratio fluctuations, is prone to self-excited pressure oscillations (Seo, 1999).

The identification of mechanisms for initiating combustion instabilities in liquid rocket engines turns out to be a much more difficult task since complexity of combustion processes increases due to atomization and vaporization of liquid droplets burning in thrust chambers. The comprehensive compilation of combustion instability knowledge edited by Harrje and Reardon (1972) suggested that linear and nonlinear instabilities could be selectively considered depending on whether instabilities are spontaneous or triggered by finite disturbances that may occur naturally in combustion processes or may be artificially generated by disturbance devices such as a pulse gun and a bomb. It was argued that a natural trigger for high frequency instabilities at nominal operating conditions, sometimes called pop, might originate from uncontrolled flow or accumulation of propellants in cracks or crevices. A short note on a role of combustion processes for liquid bipropellant combustion instabilities by Anderson et al. (1991) summarized that periodic atomization and burning of propellants are both dominating processes for providing heat release oscillations for initiation and sustaining of combustion instability although any combustion process can be responsible for initiating and sustaining high frequency pressure oscillations in a liquid propellant chamber.

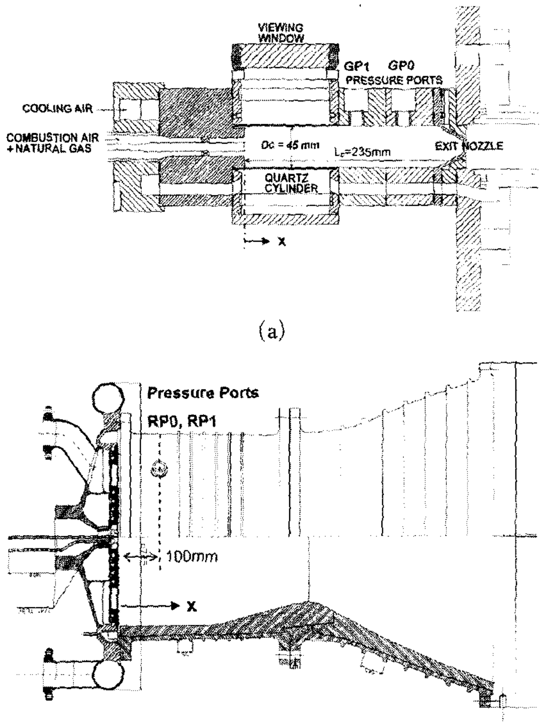
Even though decent amount of research efforts conducted for the investigation of key processes for initiating combustion instability are available up to the present, it may be regarded as natural that there is no unifying mechanism about the phenomena due to their inherent complexity. There are still pieces of information missing about how combustion instability suddenly occurs at certain conditions of combustors and what mechanisms exist behind a coupling between heat release and acoustics in combustors.

The objectives of the present paper are to investigate characteristics of dynamic pressure fluctuations measured at the onset of high frequency combustion instabilities observed in two combustion systems of different hardware and to discuss about nonlinear behavior of pressure fluctuations based on results acquired by the application of nonlinear time series analysis.

## 2. Experiments

The following presentation introduces experimental results of self-excited combustion instabilities occurring in two combustion systems of different hardware. One is a pressurized model gas turbine combustor and the other a fullscale liquid bi-propellant rocket thrust chamber. All the details about the experimental setups were described in the previous literature (Seo, 1999; Lee, 2002) and thus, only brief summaries of each setup are provided in the present paper.

A premixed gas turbine combustor illustrated in Fig. 1(a) is fueled by natural gas (methane> 95% in volume) and air running at a nominal condition of a chamber pressure, $p_c$, at 0.5~0.6 MPa and a preheated combustion air temperature, $T_{in}$, at 600~700 K. Combustion air was fully premixed with natural gas before flowing through the inlet choking venturi, which assures that local equivalence ratio variations due to non-uniform premixing are minimized at combustion region. Swirling mixture flow induced by a vane swirler was introduced into the cylindrical combustion chamber with an aspect ratio, $A_c/A_{in}$, equal to 6.3. The upstream and downstream acoustical boundary conditions of the combustor are well defined by the inlet choking venturi and exit nozzle, respectively. For dynamic pressure measurements in the combustion chamber, conditioned signals from water-cooled and flush-mounted pressure transducers (PCB piezoelectronics, model 113A21) were simultaneously sampled at a frequency of 30 kHz. Pressure transducers labeled as GP0 and GP1 are installed at two different axial locations for the identification of longitudinal resonant modes as described in Fig. 1(a). The natural frequency and rise time of the pressure transducers are 500 kHz and 1 $\mu s$, respectively, which stay well above the characteristic time scales of interest in this study.



(a)

(b)

**Fig. 1** Cross sectional views of (a) a model lean premixed gas turbine combustor with dynamic pressure measurements at GP0 and GP1, and (b) a full-scale liquid rocket thrust chamber fueled by kerosene and liquid oxygen with RP0 and RP1 for dynamic pressure measurements

A schematic view of a fullscale liquid rocket thrust chamber without any instability suppression devices such as an acoustic absorber and a baffle is presented in Fig. 1(b). The thrust chamber burning liquid oxygen and kerosene employs split-triplet (F-OO-F) impinging type injectors. Typical operational conditions are a chamber pressure of 1.44 MPa and an O/F mixture ratio of 2.38. Dynamic pressure signals at the inner surface of the thrust chamber were measured by using two water-cooled, Helium bleed dynamic pressure transducers, RP0 and RP1, (PCB piezoelectronics, model 123A24) installed at the same axial location 100mm away from the injector faceplate and 135 degrees apart in the tangential direction for the identification of tangential

**Table 1** Operating conditions of each test for a model lean premixed gas turbine combustor and a full-scale liquid rocket thrust chamber

| test No. | combustion system | inlet air temperature $T_{in}$ (K) | chamber pressure $p_c$ (MPa) | O/F Ratio | equivalence ratio | fuel/oxidizer |
|---|---|---|---|---|---|---|
| GC#1 | gas turbine | 621 | 0.55 | – | 0.58 | NG/air |
| RC#1 | liquid rocket | – | 1.44 | 2.43 | – | kerosene/LOx |
| RC#2 | liquid rocket | – | 1.43 | 2.39 | – | kerosene/LOx |
| RC#3 | liquid rocket | – | 1.45 | 2.32 | – | kerosene/LOx |

resonant modes (Harrje and Reardon, 1972). The simultaneous sampling frequency was 25. 6kHz per channel. Detailed measurement conditions of experimental data for each combustion system are listed in Table 1.

## 3. Results and Discussion

### 3.1 Dynamic pressure traces

For combustion instability of lean premixed gas turbine combustors, equivalence ratio fluctuations may be a dominant factor introducing heat release oscillations in flows. However, even without the modulation of fuel flow mixed with incoming combustion air, unstable combustion with high frequency was observed in the present study. Time traces of dynamic pressures simultaneously measured at two axial locations are presented in Fig. 2 at the onset of self-excited, high frequency pressure oscillations for fully premixed flow realized by injecting fuel before the inlet choking venturi (See Fig. 1(a)). Examination of dynamic pressures in time series manifests that low frequency wave first appears on the top of which high frequency waves are superimposed at the very beginning of combustion instability. Frequency analysis of the traces shows that the most energetic high frequency wave occurs at 1700 Hz and the low frequency at around 280 Hz. The amplitude of the high frequency wave at $x/L_c=0.78$ close to the exit nozzle appears to have the higher value than that at $x/L_c=0.51$ although the amplitudes of the low frequency wave are in the same order regardless of its measurement locations. This observation can be clearly confirmed by estimating amplitude ratios between both pressure signals for each frequency wave by use of a frequency response function.

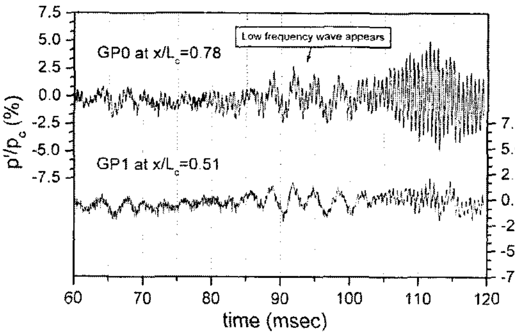


**Fig. 2** Time traces of normalized dynamic pressures measured at two different axial locations when self-excited pressure oscillations occur for GC#1

Figure 3 presents the plot of amplitude ratios and phase differences of the low and high frequency waves between two pressure traces shown in Fig 2. Error for the estimation of phase differences is about six degrees given by $\sqrt{1-\gamma^2_{GP0,GP1}}/(|\gamma_{GP0,GP1}|\sqrt{2n_a})$ where $\gamma_{GP0,GP1}$ is a coherence function between two pressure signals, GP0 and GP1, and $n_a$ a number of ensembles (Bendat and Piersol, 1991). An amplitude ratio of the low frequency wave above 0.9 indicates that the low frequency wave may have the same amplitude in the entire volume of the chamber only with trivial phase differences estimated less than three degrees. For the high frequency wave, the amplitude ratio is around 0.2, which simply reveals that the high frequency wave has amplitude variation throughout the chamber. The phase difference of the high frequency wave within the order of error implies that the node of the high fre-

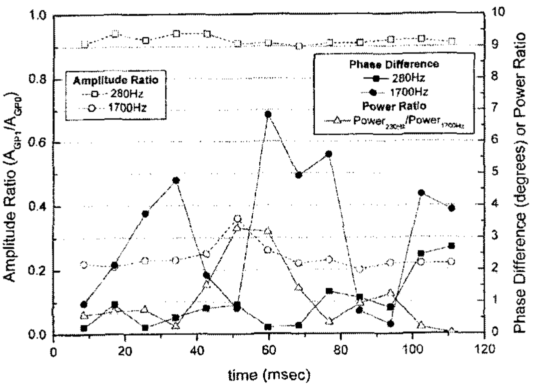


**Fig. 3** Amplitude ratios and phase differences between two measurement locations for low and high frequency waves and their power ratios with respect to time for GC#1

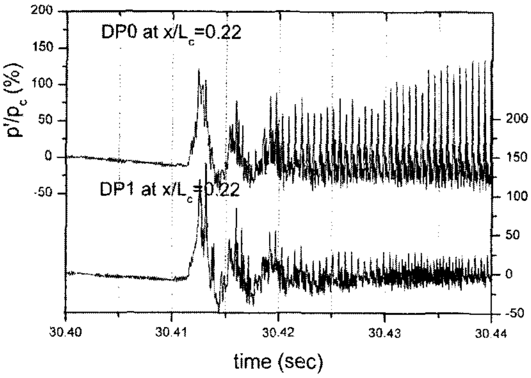


**Fig. 4** Typical time traces of normalized dynamic pressures measured in the thrust chamber at the onset of instability for RC#1

quency wave exists at the upstream of GP1 (see Fig. 1(a)) considering that the high frequency wave corresponds to a standing wave with a wavelength approximately equal to $L_c/2$.

The high frequency wave is identified as the first longitudinal resonant mode standing between the dump plane and exit nozzle, and the low frequency wave as a bulk mode. This low frequency wave happens to be in the vicinity of the Helmholtz frequency of the combustion chamber that yields an estimation of 300 Hz considering the combustion chamber as resonator volume and the inlet section as the inlet pipe of a resonator. From these results, it is observed that Helmholtz type, low frequency pressure fluctuations are excited in the entire region of the combustion chamber just before unstable combustion switches to the resonant high frequency pressure oscillations for the fully premixed lean combustor of the present study.

The experimental results from self-excited unstable combustion of the liquid rocket thrust chamber are also presented along with lean premixed combustion instability occurring in the model gas turbine combustor. Typical dynamic pressures normalized by a static chamber pressure are shown in Fig. 4, which were taken at the onset of self-excited combustion instability without any external perturbations. During stable combustion, normalized pressure fluctuations, $p'/p_c$, are less than 2%. Once instability starts, dynamic pressure abruptly grows even greater than a static chamber pressure and these pressure spikes are comparable to shock waves as observed in the gas rocket experiments (Sirignano and Crocco, 1964). One interesting observation to point out in the time trace plot is that low frequency pressure fluctuations appear at the very beginning of high frequency combustion instability, which is very similar to the results obtained from the case of the lean premixed gas turbine combustor although the normalized magnitude of low frequency oscillations (~100%) in the liquid rocket thrust chamber is excessively greater than that (~2%) recorded for the lean premixed gas turbine combustor. After the attenuation of the low frequency wave in about 10 msec, the amplitude of the high frequency wave starts to increase and finally reaches a steady condition.

Phase and amplitude analysis results for the pressure traces are shown in Fig. 5. Low frequency wave occurs at 306 Hz and high frequency does at 1694 Hz. The low frequency wave is considered as bulk mode type fluctuations since magnitude ratios stay close to one with a negligible phase difference less than five degrees regardless of measurement locations although the high frequency wave is identified as the first tangential resonant mode. The phase difference for the high frequency wave is very close to $\pi$, which signifies that two pressure transducers are

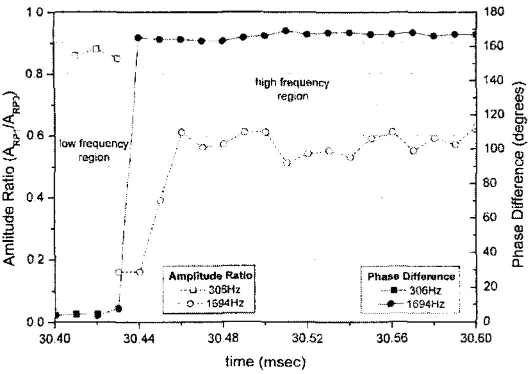


**Fig. 5** Amplitude ratios and phase differences between two different measurement locations for low and high frequency waves with respect to time for RC#1

located on an opposite side against the nodal surface, which exists near RP1 with the estimation of the amplitude ratio around 0.55.

It has been commonly observed in this study that low frequency waves appear at the onset of high frequency combustion instabilities self-excited in two combustion systems although these systems involve different hardware and physical mechanisms of combustion, for instance, liquid droplet combustion and premixed combustion. The previous analytical results by Fleifil et al. (1996) addressed that the lower frequency pressure perturbations result in the higher combustion response fluctuations due to increased flame surface area fluctuations. For a premixed laminar flame, they concluded that a heat release rate is not significantly affected by high frequency oscillations due to maximum time delay, i.e., a large phase difference. Vice versa, low frequency oscillations effectively influence flame surface area fluctuations with relatively small phase differences. The simultaneous measurement of flame emission and pressure in a premixed dump combustor also suggested that phase shift between heat release and pressure fluctuations increase with frequency (Dawson and Fitzpatrick, 2000). All these results support that the lower frequency oscillations considerably induce more heat release rate fluctuations with relatively small phase shift. Although the previous results were not acquired from conditions comparable to the present case, their qualitative explanations can be still valid for encouraging the conclusion that low frequency wave with higher flame response becomes coupled with heat release fluctuations at first before the acoustic mode of combustion instability eventually shifts to high frequency resonant mode.

Relatively small amount of literature is available about the effects of acoustic frequency variation on liquid spray burning in a rocket thrust chamber environment. Vaporization response magnitudes of spray droplets continuously distributed as a function of diameter have their maximum values at the range of ambient pressure oscillation frequency from 100 Hz to 900 Hz, and decrease with frequency according to the analytical calculation (Anderson et al., 1998). Experimental study in the fifties showed that periodic motion of laminar liquid spray in intensified acoustic fields diminishes with an increase of frequency (Miesse, 1955). All these results suggested that vaporization and atomization of spray are affected by change of perturbing frequency.

Normalized dynamic pressures, $p'/p_c$, have significantly different orders of magnitude for each system. One thing noteworthy here is that one may apply a linear approach on the modeling of modest combustion instability as occurs in lean premixed combustors but not on combustion instabilities of a highly energy-dense liquid rocket chamber showing strong nonlinear characteristics. Usually, pressure perturbations on the order of ten percent of chamber pressure or above are considered as nonlinear.

The early excitation of low frequency waves commonly observed in combustion instabilities of two different systems strongly suggests that low frequency perturbations provide a greater impact on heat release oscillations than high frequency ones and thus, they play a substantial role in coupling heat release fluctuations. The initiation of energy coupling consequently leads to the excitation of acoustic resonant modes of combustion chambers. In other words, pressure fluctuations having characteristics of bulk mode and low frequency initiate coherent oscillations in heat release rate rather than high frequency perturbations. As discussed at the beginning, low frequency waves may have various kinds of their origin and formation such as a Helmholtz resonator, a shedding frequency, and an inherent hydraulic low frequency found in turbulent flow.

### 3.2 Nonlinear time series analysis

A conventional Fourier transform has been typically applied for understanding frequency characteristics of dynamic signals measured from combustion instability and linear analysis has been also employed for modeling. However, these methods do not provide enough information for investigating one of the most complicated physical phenomena that show various unique nonlinear characteristics such as limit cycle and bifurcation (Lieuwen, 2002). Attempted was nonlinear time series analysis of dynamic pressure signals in order to better identify nonlinear characteristics inherent in combustion instabilities in the present study.

The power spectrum analysis of pressure signals as plotted in Fig. 6 shows none of distinct peaks of frequencies for two cases of stable combustion in the rocket thrust chamber. Although traditional interpretation of the plots suggests that the pressure signals at stable combustion be regarded as noise, the nonlinear analytic point of view implies that the signals may contain nonlinear characteristics with a broad band of frequencies in the vicinity of 1000 Hz (Fichera et al., 2001).

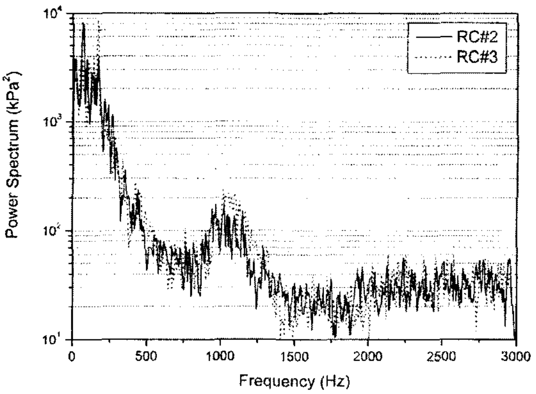


**Fig. 6** Power spectrum plots with $\Delta f = 6.25$ Hz for dynamic pressures at stable conditions of the tests, RC#2 and RC#3

One of basic approaches in nonlinear time series analysis is to construct a phase or state space coordinate. Measurement of discrete signals, $p'(N) = [p'(1), p'(2), p'(3), \cdots, p'(N)]$, in time series can be expressed as a series of a state vector, $y(n)$, in a reconstructed phase space like $y(n) = [p'(n), p'(n+d), \cdots, p'(n+(m-1)d)]$ where $d$ is a time lag accounting for a time difference equal to $d\Delta t$, and $m$ is called an embedding dimension (Abarbanel, 1996). Since the estimation of a time lag and an embedding dimension becomes possible through various kinds of manipulations of time series data, detailed discussions related to their estimation will not be covered in this paper. A phase space plot of time series data allows the identification of a certain subset of data points, which is called the attractor invariant under the dynamical evolution (Hilborn, 2000). The determination of a time lag, $d$, can be made by the estimation of the average mutual information, $I(d)$, that is defined as

$$I(d) = \sum_{p'(n), p'(n+d)} P(p'(n), p'(n+d)) \log_2 \left( \frac{P(p'(n), p'(n+d))}{P(p'(n))\, P(p'(n+d))} \right) \quad (1)$$

where $P(p'(n), p'(n+d))$ is the joint probability density for two measurements having values of $p'(n)$ and $p'(n+d)$. Estimated value of $I(d)$ indicates how much information about measurements at a time lag, $d$, can be learned by the measurements of $p'(n)$. The best choice of a time lag, $d$, can be given by the first local minimum of the average mutual information (Abarbanel, 1996). For the data of Fig. 6, a time lag, $d$, is estimated as three equal to the first minimal value of the mutual information acquired using Eq. (1) as shown in Fig. 7. The time series measurements of signals at a stable condition can be shown in a pseudo phase space as presented in Fig. 8. The projection of state vectors onto the second degree of a freedom space, sometimes called a phase orbit, reveals that dynamic pressures at a stable condition certainly have a distinct structure indicating nonlinearity inherent in signals. The structures shown in phase space plots (see Fig. 8(a) and (b)) are distinguished from a randomly generated noise which shows a cloud-like, circu-

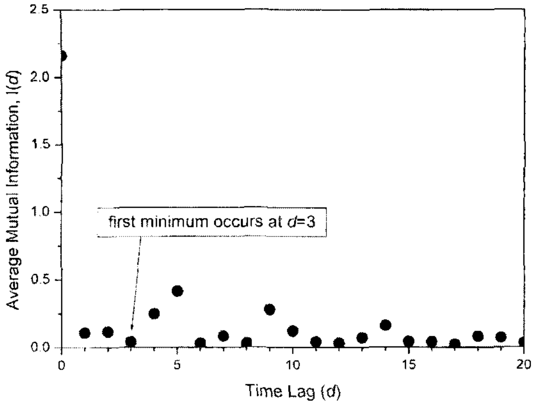


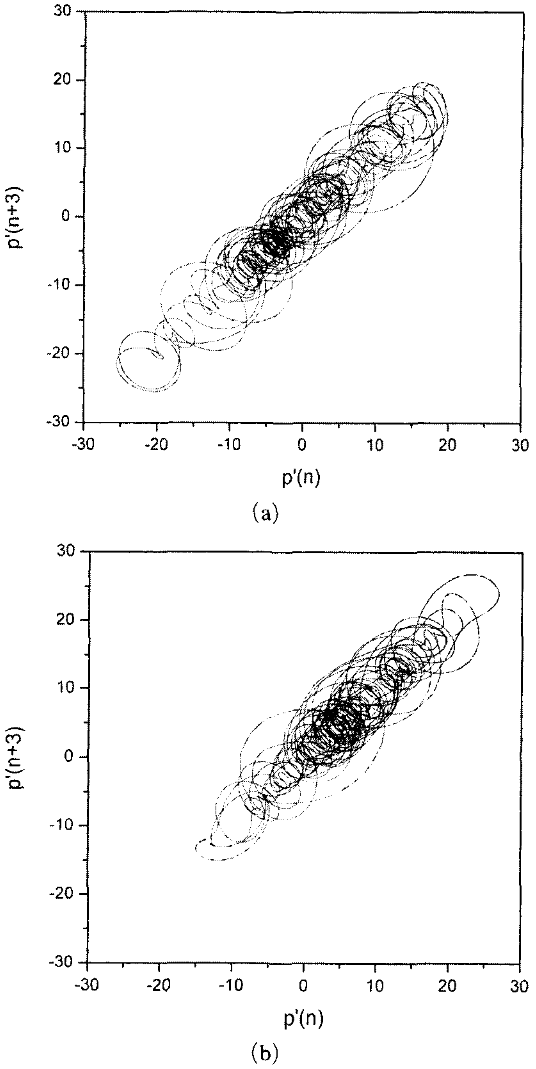
**Fig. 7** Plot of average mutual information estimated for a stable run condition of the test, RC#2

lar shape in a phase space without any bias in direction. This result implies that dynamic pressure traces at a stable condition may contain certain information about their dynamical behaviors although a conventional approach considers that kind of time series data as a noise.

One of important invariants in nonlinear time series analysis is the estimation of the Lyapunov exponents (Skowronski, 2003). The Lyapunov exponent indicates the rate of divergence of two trajectories in a phase space, which is expressed as the average of the natural logarithm of the absolute value of the map function derivatives evaluated at the trajectory points. When the distance between two trajectories is defined as $s(t) = p'(t) - p'_0(t)$, the mathematical description of the Lyapunov exponent, $\lambda$, is as follows,

$$s(t) = s_{t=0}\exp(\lambda t)$$
$$\lambda = \frac{df(p')}{dp'} \quad at \quad p'_0 = 0 \qquad (2)$$

where $f(p') = dp'/dt$ is the time dependent equation assumed for the trajectory. As defined in Eq. (2), the distance between two trajectories diverges at a positive $\lambda$, and converges at a negative $\lambda$. Therefore, the system having at least one positive Lyapunov exponent is defined as a chaotic system (Hilborn, 2000).



(a)

(b)

**Fig. 8** Phase orbit presentation at stable run conditions of (a) RC#2 and (b) RC#3

However, the estimation of all the Lyapunov exponents for practical time series data from experimental measurements becomes barely possible due to noise contamination. The estimation only for the maximum Lyapunov exponent has been usually attempted for the investigation of nonlinear behaviors of time series data. Among various algorithms available, the following practical method has been adapted.

$$S(\Delta n) = \frac{1}{N}\sum_{n_0=1}^{N}\ln\left(\frac{1}{|N(p'_{n_0})|}\sum_{p'_n \in N(p'_{n_0})}|p'_{n_0+\Delta n} - p'_{n+\Delta n}|\right) \qquad (3)$$

In above equation, $N(p'_{n_0})$ is the number of the neighborhood within a diameter, $\varepsilon$. By plotting $S(\Delta n)$ with respect to the number of iteration, $\Delta n$, the linear slope of the curve is estimated as

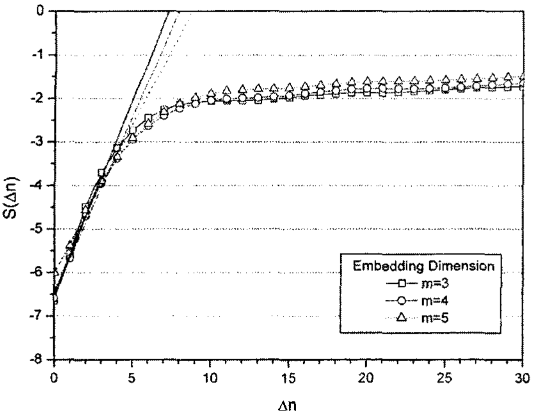


**Fig. 9** Natural logarithm of distances between trajectories as a function of iteration number for three different embedding dimensions at stable run condition of RC#2

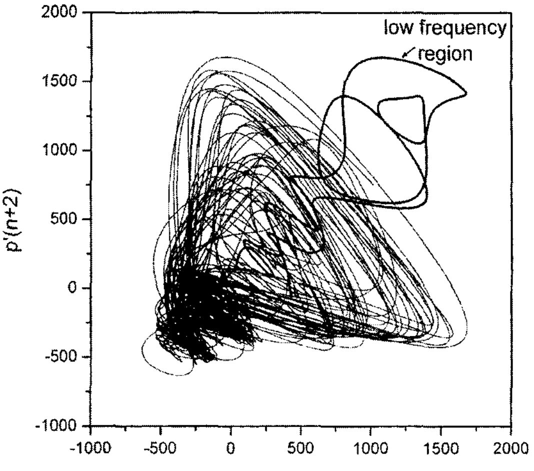


**Fig. 10** Phase orbit presentation of dynamic pressure trace at the onset of instability for RC#1

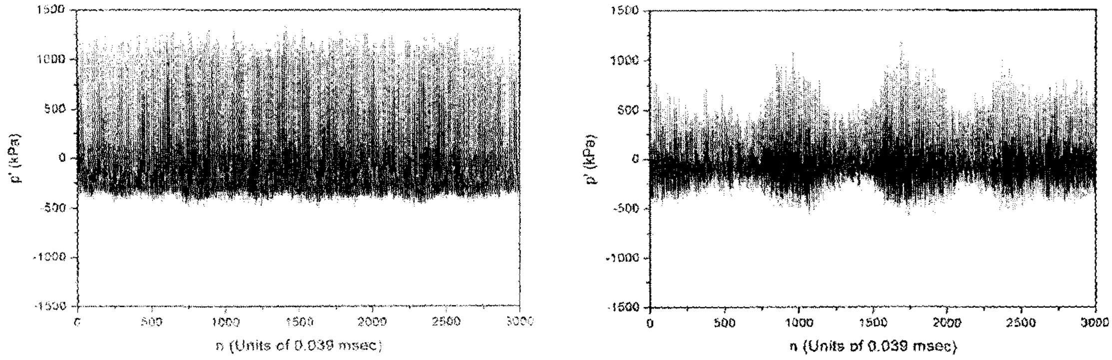
the maximum Lyapunov exponent (Kantz and Schreider, 1997). Using Eq. (3), the plots of $S(\Delta n)$ are presented in Fig. 9 for three different embedding dimensions.

The linear curve fitting in Fig. 9 for each embedding dimension yields the maximum Lyapunov exponents, 0.89, 0.81 and 0.68 for $m=$ 3, 4 and 5, respectively. The positive maximum Lyapunov exponent values reveal that the distance between the nearest neighbors increases with time. Saturation of the curves with an increase of the number of iteration indicates that the distance increases at the same order of an attractor. Eventually, this leads to the speculation that the stable combustion observed by time series of dynamic pressures may manifest a chaotic behavior. However, this needs to be further investigated with serious attention since the sources for an observed nonlinear behavior have not been identified (Culick, 1994).

The appearance of a low frequency wave can be also expressed in a phase space as illustrated in Fig. 10. At the beginning of combustion instability, the low frequency wave (bold line) in a pseudo phase space is described as revealing a unique trajectory. This trajectory in the low frequency region is being attracted to the high frequency region and repeats itself on an attractor featuring characteristics of periodic oscillations. For periodic oscillations, the determination of a time lag, $d$, becomes arbitrary (Abarbanel, 1996).

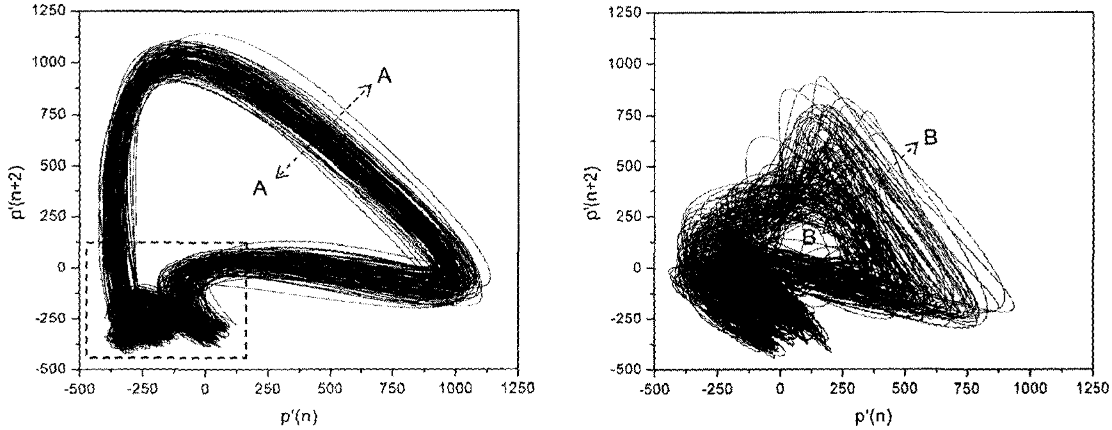
Once combustion instability occurs in the thrust chamber, amplitudes of high frequency oscillations increase and reach a limit cycle, bounded between certain limits as shown in Fig. 11(a). Even at the very similar operating condition, a dynamic pressure time series under unstable combustion occasionally shows that the amplitude of a high frequency wave varies with time as plotted in Fig. 11(b). Examining Fig. 11 (a) and (b), pressure oscillations of combustion instabilities may lead to somewhat different behavior even at the same parameters, which suggest that perturbed acoustics in the rocket thrust chamber shows a characteristic of bifurcation. Time series having a characteristic of varying amplitudes is called quasi periodic and considered as one of routes leading to chaos (Hilborn, 2000).

Phase orbit plots for each set of map-like data shown in Fig. 11 are presented in Fig. 12. For the case of a limit cycle, Fig. 11(a), trajectories in a 2-D phase space follow and stay on the almost same orbit as seen in Fig. 12(a). Contrarily, for a time-varying amplitude case as in Fig. 11(b), trajectories follow different paths when they orbit in the attractor, which results in trajectory routes



(a) (b)

**Fig. 11** Map-like presentation of dynamic pressure time series for unstable conditions of (a) RC#2 and (b) RC#3



(a) (b)

**Fig. 12** Phase orbit plots of dynamic pressure time series for unstable conditions of (a) RC#2 and (b) RC#3. See that structures inside of a dotted-line box are unfolded.

widely spread over although overall shapes of phase orbits look similar to the case of a limit cycle. This fact becomes obvious when comparing widths of A-A and B-B sections for each phase orbit. Considering that a quasi-periodic behavior is one of paths to chaos, it could be presumed that trajectories in Fig. 12(b) will not repeat the very same orbit again with time.

## 4. Conclusions

It is observed for both experimental cases of combustion instabilities that low frequency waves at around 300Hz first appear at the onset of high frequency combustion instabilities, which are coupled to the resonant acoustic modes of the combustion chamber, the first longitudinal mode for the lean premixed combustor and the first tangential mode for the rocket thrust chamber. The appearance of perturbed low frequency waves strongly suggests that they play a role in inducing heat release oscillations at first before the resonant acoustic modes become coupled to heat release rate oscillations with high frequencies on the order of 1000 Hz since flame shows the higher response on the lower frequency wave with the smaller phase differences between heat release and pressure fluctuations. This basically manifests that there may exist a common aspect of high frequency combustion instability mechanisms even for different systems in which quite different physical processes occur.

Results from nonlinear time series analysis of pressure traces reveal that pressure fluctuations at stable conditions, generally regarded as noise in conventional frequency analysis, might have chaotic behavior with the positive maximum Lyapunov exponent. Phase orbit representation is useful for the investigation of characteristic structures intrinsic in time series data and apparently shows the change of structures when low frequency waves perturbed at first switches to the high frequency periodic oscillations. Once combustion instability occurs, pressure fluctuations reach a limit cycle or quasi-periodic oscillations, which indicates that self-excited high frequency instability also shows characteristics of highly nonlinear phenomena such as bifurcation.

## Acknowledgment

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## References

Abarbanel, H. D. I., 1996, *Analysis of Observed Chaotic Data*, Institute for Nonlinear Science, Springer-Verlag, New York, Chap. 1-3, pp. 1~37.

Anderson, W. E., Ryan, H. M. and Santoro, R. J., 1991, "Combustion Instability Phenomena of Importance to Liquid Bi-Propellant Rocket Engines," CPIA Publication 573, Proceeding of the 28th JANNAF Combustion Subcommittee Meeting, San Antonio, TX, CPIA Publication 573, Vol. II, pp. 99~112.

Anderson, W. E., Miller, K. L., Ryan, H. M., Pal, S., Santoro, R. J. and Dressler, J. L., 1998, "Effects of Periodic Atomization on Combustion Instability in Liquid-Fueled Propulsion Systems," *Journal of Propulsion and Power*, Vol. 14, No. 5, pp. 818~825.

Bendat, J. S. and Piersol, A. G., 1991, *Random Data, Analysis and Measurement Procedure*, 2nd ed., John Wiley & Sons, New York, Chap. 9, Sec. 2, pp. 302~317.

Culick, F. E. C., 1994, "Some Recent Results for Nonlinear Acoustics in Combustion Chambers," *AIAA Journal*, Vol. 32, No. 1, pp. 146~169.

Dawson, S. and Fitzpatrick J. A., 2000, "Measurement and Analysis of Thermoacoustic Oscillations in a Simple Dump Combustor," *Journal of Sound and Vibration*, Vol. 230, No. 3, pp. 649~660.

Fichera, A., Losenno, C. and Pagano, A., 2001, "Experimental Analysis of Thermo-Acoustic Combustion Instability," *Applied Energy*, Vol. 70, pp. 179~191.

Fleifil, M., Annaswamy, A. M., Ghoneim, Z. A. and Ghoniem, A. F., 1996, "Response of a Laminar Premixed Flame to Flow Oscillations: A Kinematic Model and Thermoacoustic Instability Results," *Combustion and Flame*, Vol. 106, pp. 487~510.

Harrje, D. T. and Reardon, F. H.(eds.), 1972, "Liquid Propellant Rocket Combustion Instability, Princeton University," SP-194.

Hilborn, R. C., 2000, *Chaos and Nonlinear Dynamics*, 2nd ed., Oxford University Press, Chap. 4, pp. 117~156.

Kantz, H. and Schreider, T., 1997, *Nonlinear Time Series Analysis*, the University Press, Cambridge, UK, Chap. 3, Sec. 1-2, pp. 29~34.

Lee, D. S., 2002, "Final Report on the Development of KSR-III Thrust Chamber Assembly, Korea Aerospace Research Institute," Daejeon, Korea.

Lieuwen, T., Torres, H., Johnson, C. and Zinn, B. T., 2001, "A Mechanism of Combustion Instability in Lean Premixed Gas Turbine Combustors," *Journal of Engineering for Gas Turbines and Power*, Vol. 123, pp. 182~189.

Lieuwen, T. C., 2002, "Experimental Investigation of Limit-Cycle Oscillations in an Unstable Gas Turbine Combustor," *Journal of Propulsion and Power*, Vol. 18, No. 1, pp. 61-67.

Miesse, C. C., 1955, "The Effect of Ambient Pressure Oscillations on the Disintegration and Dispersion of a Liquid Jet," *Jet Propulsion*, October.

Oefelein, J. C. and Yang, V., 1993, "Comprehensive Review of Liquid-Propellant Combustion Instabilities in F-1 Engines," *Journal of Propulsion and Power*, Vol. 9, No. 5, pp. 657~677.

Richards, G. A. and Janus, M. C., 1998, "Characterization of Oscillations During Premix Gas Turbine Combustion," *Journal of Engineering for Gas Turbines and Power*, Vol. 120, pp. 294~302.

Seo, S., 1999, "Parametric Study of Lean Premixed Combustion Instability in a Pressurized Model Gas Turbine Combustor," Ph.D. Thesis, The Pennsylvania State University, University Park, PA.

Seo, S., 2003, "Combustion Instability Mechanism of a Lean Premixed Gas Turbine Combustor," *KSME International Journal*, Vol. 17, No. 6, pp. 906~913.

Sirignano, W. A. and Crocco, L., 1964, "A Shock Wave Model of Unstable Rocket Combustors," *AIAA Journal*, Vol. 2, pp. 1285~1296.

Skowronski, J., Grantham, W. J. and Lee, B., 2003, "Use of Chaos in a Lyapunov Dynamic Game," *KSME International Journal*, Vol. 17, No. 11, pp. 1714~1724.

Yang, Y.-J., Akamatsu, F., Katsuk, M. and Lee, C.-W., 2003, "Characteristics of Self-excited Combustion Oscillation and Combustion Control by Forced Pulsating Mixture Supply," *KSME International Journal*, Vol. 17, No. 11, pp. 1820~1831.