Spectral Element Analysis for an Axially Moving Viscoelastic Beam

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In this paper, a spectral element model is derived for the axially moving viscoelastic beams subject to axial tension. The viscoelastic material is represented in a general form by using the one-dimensional constitutive equation of hereditary integral type. The high accuracy of the present spectral element model is verified first by comparing the eigenvalues obtained by the present spectral element model with those obtained by using the conventional finite element model as well as with the exact analytical solutions. The effects of viscoelasticity and moving speed on the dynamics of moving beams are then numerically investigated.

Key Words : Viscoelastic Beam, Moving Beam, Spectral Element Model, Vibration, Critical Moving Speed

1. Introduction

Above a certain critical moving speed, the moving structures such as the moving belts used in power transmissions may experience severe vibrations resulting in structural failures. Thus, the dynamics and stability of such structures have been studied extensively to ensure that they are under stable working conditions (Wickert and Mote, 1988). In most existing literatures, the axially moving one-dimensional structures have been considered to be elastic and represented by the string models or the beam models.

With the advancement of material technologies, new materials such as plastics, metallic or ceramic reinforced composite materials and polymeric material are now widely used for moving belts. Such materials exert inherently viscoelastic behavior which can be modeled by integral or differential type of constitutive equations. However, there have been very limited number of studies on the moving structures of which material is viscoelastic. Fung et al. (1997) is the first to investigate the transverse vibration by using a standard linear solid model of hereditary integral type. Zhang and Zu (1998) adopted a Kevin model of differential type for the viscoelastic material of moving belt. Le-Ngoc and McCallion (1999) developed a dynamic stiffness model for the axially moving elastic string. Hou and Zu (2002) used a standard linear solid model of differential type to investigate nonlinear free oscillations. In the aforementioned references, the moving viscoelastic belts were all represented by string models. The beam model was used by Marynowski and Kapitaniak (2002) to investigate the stability and oscillations of an axially moving viscoelastic web by using the differential type of constitutive equations for the viscoelastic material.

In the literature, various solution methods have been presented for the vibration analysis of linear viscoelastic structures ; for instance, Galerkin method (e.g., Fung et al., 1997), Runge-Kutta method (e.g., Marynoski and Kapitaniak, 2002), Laplace transform method (e.g., Flugge, 1975), the correspondence and superposition

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principles (e.g., Findley et al., 1976), Fourier transform method (e.g., Christensen, 1982), finite element method (e.g. White, 1986), and so forth.

The spectral element method (SEM) has been well recognized as an exact solution method for the dynamic analysis of structures (Doyle, 1997 ; Lee and Lee ; 1998 ; Lee et al., 2001 ; Oh et al., 2004). The spectral element matrix is formulated by using the frequency-dependent shape functions satisfying structural dynamic equations. Thus, one can use only one finite element for a uniform structure, regardless of its length. The conventional finite element assembly procedure can be equally used in SEM to obtain the global system equation for a complete structure. In SEM, the dynamic responses in frequency- and time-domains are computed very efficiently by using the forward-FFT (simply, FFT) and inverse-FFT (simply, IFFT) algorithms. Because the SEM is a frequency-domain method, it seems to be best fit for the viscoelastic structures of which material properties are most often extracted indirectly from the experimental forced vibration responses given in the form of receptance frequency response functions (FRF) (Dalenbring, 2003).

Thus, the purposes of this paper are to develop a spectral element model for the axially moving viscoelastic beams subject to axial tension, and to investigate the effects of viscoelasticity and moving speed on the vibration of an example moving viscoelastic beam.

2. Equation of Motion

2.1 Constitutive equation

The three-dimensional constitutive equation for an integral type anisotropic linearly viscoelastic material is given by (Christensen, 1982)

$$\begin{aligned}\sigma_{ij}(t) \int_{-\infty}^{t} r_{ijkl}(t-\tau)\,\dot{\varepsilon}k_l(\tau)\,d\tau \\ = r_{ijkl}(t) * d\varepsilon_{kl}(t)\end{aligned} \tag{1}$$

where $\sigma_{ij}(t)$ is the stress history tensor, $\varepsilon_{ij}(t)$ is the strain tensor, r_{ijkl} (t) is the fourth order tensor of relaxation function, and ($*$) denotes the Stieltjes convolution between $r_{ijkl}(t)$ and $d\varepsilon_{ij}(t)$. The dot ($\cdot$) denotes the derivative with respective to time t. For the one dimensional isotropic linearly viscoelastic material, Eq. (1) can be reduced to

$$\sigma(t) = \int_{-\infty}^{t} r(t-\tau)\,\dot{\varepsilon}(\tau)\,d\tau = r(t) * d\varepsilon(t) \tag{2}$$

In the frequency-domain, Eq. (2) can be expressed as

$$\boldsymbol{\sigma}(\omega) = i\omega R(\omega)\,\varepsilon(\omega) \tag{3}$$

where $i=\sqrt{-1}$ is the imaginary unit, ω is the circular frequency, and $\sigma(\omega)$, $\varepsilon(\omega)$ and $R(\omega)$ are the Fourier transforms of the stress history $\sigma(t)$, strain history $\varepsilon(t)$, and relaxation function $R(t)$, respectively.

2.2 Equation of motion

Consider a uniform straight viscoelastic beam of length L, which travels at constant transport speed c. The equation of motion and the relevant boundary conditions can be derived form the extended Hamilton's principle (Abolghasemi, 2003):

$$\int_{t_1}^{t_2} (\delta K - \delta V + \delta W)\,dt = 0 \tag{4}$$

where K and V are the kinetic and potential energies, respectively, given by

$$\begin{aligned} K &= \frac{1}{2}\int_0^L \rho A\{c^2 + (\dot{w} + cw')^2\}dx \\ V &= \frac{1}{2}\int_0^L (Mw'' + Pw'^2)\,dx \end{aligned} \tag{5}$$

In Eq. (5), $w(x, t)$ is the transverse deflection, ρA the mass per length, M the resultant bending moment, and P the axial tension. The dot ($\cdot$) and the prime ($'$) denote the partial derivatives with respective to the time t and the spatial coordinate x, respectively. The virtual work δW is given by

$$\begin{aligned}\delta W = \int_0^L f(x, t)\,\delta x + M_1\delta\theta_1 + M_2\delta\theta_2 \\ + Q_1\delta w_1 + Q_2\delta w_2\end{aligned} \tag{6}$$

where $f(x, t)$ is the external force, and M_1, Q_1 and θ_1 are the bending moment, the transverse shear force, and the slope at $x=0$, while M_2, Q_2 and θ_2 are those at $x=L$. The slopes θ_1 and θ_2

are related to the transverse deflection as

$$\theta_1(t)=w'(0,\ t),\ \theta_2(t)=w'(L,\ t) \tag{7}$$

Introducing Eqs. (5) and (6) into Eq. (4), and then applying the integral by parts gives

$$\begin{aligned}&\int_{t_1}^{t_2}\int_0^L[-M''-\rho A(c^2w''+2c\dot{w}'+\ddot{w})+Pw''+f(x,\ t)]\delta w dxdt\\&+\int_{t_1}^{t_2}\{Q(x,\ t)\delta w|_0^L+Q_1\delta w_1+Q_2\delta w_2\}dt\\&+\int_{t_1}^{t_2}\{-M(x,\ t)\delta\theta|_0^L+M_1\delta\theta_1+M_2\delta\theta_2\}dt=0\end{aligned} \tag{8}$$

where $M(x,\ t)$ and $Q(x,\ t)$ are the resultant bending moment and the transverse shear force, respectively, defined by

$$\begin{aligned}M(x,\ t)&=\int_A-\sigma z dA\\Q(x,\ t)&=M'+\rho Ac(\dot{w}+cw')+Pw'\end{aligned} \tag{9}$$

where A represents the cross-sectional area of the moving beam. From Eq. (8), the equation of motion for the moving viscoelastic beam can be obtained as

$$\begin{aligned}&I\{R(t)*\dot{w}''''\}\\&+\rho A(c^2w''+2c\dot{w}'+\ddot{w})-Pw''=f(x,\ t)\end{aligned} \tag{10}$$

with the boundary conditions given by

$$\begin{aligned}w(0,\ t)&=w_1 \text{ or } Q(0,\ t)=Q_1\\\theta(0,\ t)&=\theta_1 \text{ or } M(0,\ t)=M_1\\w(L,\ t)&=w_2 \text{ or } Q(L,\ t)=-Q_2\\\theta(L,\ t)&=\theta_2 \text{ or } M(L,\ t)=M_2\end{aligned} \tag{11}$$

Substituting Eq. (2) into Eq. (9) gives the relations:

$$\begin{aligned}M(x,\ t)&=I(R*\dot{w}'')\\Q(x,\ t)&=I(R*\dot{w}''')+\rho Ac(\dot{w}+cw')-Pw'\end{aligned} \tag{12}$$

3. Spectral Element Formulation

To formulate the spectral element, the general solution of the free vibration problem is represented in the discrete Fourier transform (DFT) form as

$$w(x,\ t)=\sum_{n=1}^{N-1}W_n(x)\,e^{i\omega_n t} \tag{13}$$

Heve, $W_n(x)$ is the spectral components (or Fourier coefficients) corresponding to discrete frequencies $\omega_n=2\pi n/T$ ($n=0,\ 1,\ 2,\ \cdots,\ N-1$), where N is the number of spectral components to be taken into account in the analysis, and T is the time window related to N as

$$N=2f_{\text{NYQ}}T \tag{14}$$

where f_{NYQ} is the highest frequency called Nyquist frequency. The spectral components are arranged to satisfy $W_{N-n}=W_n^*$, where W_n^* is complex conjugate of W_n by the theory of DFT. The summation and subscripts used in (13) are so obvious that they will be omitted in the following equations for brevity.

By substituting Eq. (13) into Eq. (10), with $f(x,\ t)=0$, one can obtain

$$\begin{aligned}&i\omega R(\omega)IW''''-(P-\rho Ac^2)W''\\&+21\omega\rho AcW'-\rho A\omega^2W=0\end{aligned} \tag{15}$$

The general solution of Eq. (15) is assumed in the form as

$$W(x)=Ce^{ikx} \tag{16}$$

where k is the wavenumber. Substituting Eq. (16) into Eq. (15) yields a dispersion relation as

$$\begin{aligned}&i\omega R(\omega)Ik^4+(P-\rho Ac^2)k^2\\&-2\omega\rho Ack-\rho A\omega^2=0\end{aligned} \tag{17}$$

From Eq. (17), four roots k_r ($r=1,\ 2,\ 3,\ 4$) can be obtained. Then the general solution of Eq. (15) can be expressed in the form

$$W(x)=\sum_{r=1}^{4}C_re^{ik_rx} \tag{18}$$

Now, consider a finite beam element of length l as shown in Fig. 1. The spectral nodal degrees of freedom (DOFs) are defined by

$$\begin{aligned}W_1&=W(0),\ \Theta_1=W'(0)\\W_2&=W(l),\ \Theta_2=W'(l)\end{aligned} \tag{19}$$

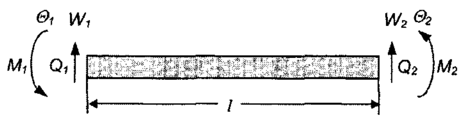


Fig. 1 Sign convention for the moving finite viscoelastic beam element

Substituting Eq. (18) into Eq. (19) gives a relation between the spectral nodal DOFs vector $\boldsymbol{d}$ and the constants vector $\boldsymbol{C}$ as

$$\boldsymbol{d}=\boldsymbol{Y}(\omega)\boldsymbol{C} \tag{20}$$

where

$$\boldsymbol{d}=\{W_1\ \Theta_1\ W_2\ \Theta_2\}^{\mathrm{T}}$$
$$\boldsymbol{C}=\{C_1\ C_2\ C_3\ C_4\}^{T}$$
$$\boldsymbol{Y}(\omega)=\begin{bmatrix} 1 & 1 & 1 & 1 \\ \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4 \\ e_1 & e_2 & e_3 & e_4 \\ e_1\varepsilon_1 & e_2\varepsilon_2 & e_3\varepsilon_3 & e_4\varepsilon_4 \end{bmatrix} \tag{21}$$

with the definitions of

$$\varepsilon_r=ik_r,\ e_r=e^{ik_r l} \tag{22}$$

Assume that the shear force $Q(x, t)$ and bending moment $M(x, t)$ can be represented in the DFT forms as

$$\begin{aligned} Q(x, t)&=\sum_{n=0}^{N-1} Q_n(x)\, e^{i\omega_n t} \\ M(x, t)&=\sum_{n=0}^{N-1} M_n(x)\, e^{i\omega_n t} \end{aligned} \tag{23}$$

Applying Eq. (13) into Eq. (12) and using Eq. (23) give the spectral components of $Q(x, t)$ and $M(x, t)$ as follows:

$$\begin{aligned} M(x)&=i\omega R(\omega)\, W'' \\ Q(x)&=i\omega R(\omega)\, W'''+(\rho Ac^2-p)\, W'+i\omega\rho Ac W \end{aligned} \tag{24}$$

where the subscript n is omitted for brevity. The spectral nodal shear forces and bending moments shown in Fig. 1 are defined by

$$\begin{aligned} Q_1&=Q(0),\ M_1=-M(0) \\ Q_2&=-Q(l),\ M_2=M(l) \end{aligned} \tag{25}$$

Substituting Eq. (18) into Eq. (24) and applying the results into Eq. (25) yield a relation between the nodal forces vector $\boldsymbol{f}$ and the constants vector $\boldsymbol{C}$ as

$$\boldsymbol{f}=\boldsymbol{X}(\omega)\boldsymbol{C} \tag{26}$$

where

$$\boldsymbol{f}=\{Q_1\ M_1\ Q_2\ M_2\}^{\mathrm{T}}$$
$$\boldsymbol{X}(\omega)=\begin{bmatrix} -g_1 & -g_2 & -g_3 & -g_4 \\ -h_1 & -h_2 & -h_3 & -h_4 \\ e_1g_1 & e_2g_2 & e_3g_3 & e_4g_4 \\ e_1h_1 & e_2h_2 & e_3h_3 & e_4h_4 \end{bmatrix} \tag{27}$$

with

$$\begin{aligned} g_r&=-i[i\omega R(\omega)\, k_r^3+(\rho Ac^2-P)\, k_r+\omega\rho Ac] \\ h_r&=-i\omega R(\omega)\, k_r^2 \end{aligned} \tag{28}$$

The constants vector $\boldsymbol{C}$ can be readily eliminated from Eqs. (20) and (26) to obtain the relation between the spectral nodal forces vector and the spectral nodal DOFs vector as follows:

$$\boldsymbol{f}=\boldsymbol{S}(\omega)\boldsymbol{d} \tag{29}$$

where $\boldsymbol{S}(\omega)$ is the symmetric frequency-dependent spectral element matrix defined by

$$\boldsymbol{S}(\omega)=\boldsymbol{X}(\omega)\,\boldsymbol{Y}(\omega)^{-1} \tag{30}$$

Assembling the spectral elements in a completely similar way to that used in the conventional FEM and then applying the relevant boundary conditions yield a global system equation in the form:

$$\boldsymbol{S}_g(\omega)\,\boldsymbol{d}_g=\boldsymbol{f}_g \tag{31}$$

where $\boldsymbol{S}_g(\omega)$ is the global spectral matrix, $\boldsymbol{d}_g$ the global spectral nodal DOFs vector, and $\boldsymbol{f}_g$ the global spectral nodal forces vector.

The eigenvalues can be computed from the condition that the determinant of global spectral matrix $\boldsymbol{S}_g(\omega)$ is zero as follows:

$$\det \boldsymbol{S}_g(\omega)=0 \tag{32}$$

To obtain the dynamic responses in the time-domain, we first compute $\boldsymbol{f}_g$ from the external forces transformed into the frequency-domain by using the forward-FFT algorithm. Next we solve Eq. (31) for $\boldsymbol{d}_g$ and apply the results into Eq. (20) to compute the spectral components of response from Eq. (18). Finally, based on the DFT theory of Eq. (13), the inverse-FFT algorithm is used to obtain the vibration response in the time-domain. One may remind that, because there have been made no restriction on $R(\omega)$, the spectral element matrix of Eq. (30) can be applied to the moving beams made of any viscoelastic material.

4. Numerical Results and Discussion

Numerical studies have been conducted to evaluate the present spectral element model as well as to investigate the effects of viscoelastcity and moving speed on the dynamics of moving viscoelastic beams. For the numerical studies, two beam models shown in Fig. 2 are considered: one-span beam (Fig. 2(a)) and two-span beam (Fig. 2(b)). The one-span beam is simply supported at both ends and the two-span beam is constructed by connecting two equal one-span beams so that its total length is twice of the length of the original one-span beam. The mid-point and the two ends of the two-span beam of are all simply-supported. Each span of beam has the length $L=1$ m, width $b=0.2$ m, thickness $h=0.0015$ m, and mass density $\rho=7800$ kg/m^3.

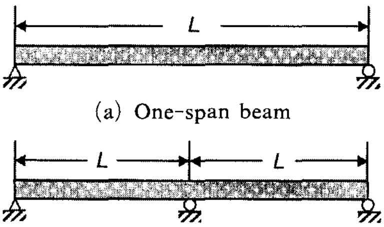
As shown in Fig. 3, the viscoelasticity of the beam material is represented by the Kelvin-Voigt model as

$$\sigma(t)=E\varepsilon(t)+\eta\dot{\varepsilon}(t) \tag{33}$$

Thus the Fourier transform of the relaxation function of the viscoelastic material can be obtained from Eq. (33) as (Christensen, 1982)

$$i\omega R(\omega)=E+i\omega\eta \tag{34}$$

For numerical illustrations in the next section, Young's modulus $E=2\times10^{11}$ N/m^2 is used while varying the magnitude of viscoelasticity η.



(a) One-span beam

(b) Two-spans beam

Fig. 2 Two stationary beams with simply supported boundary conditions

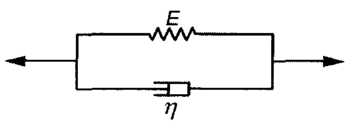


Fig. 3 Kelvin-Voigt model of the viscoelastic beam material

4.1 Evaluation of the spectral element model

The high accuracy of the present spectral element model is verified first by comparing the eigenvalues obtained by using the present SEM with those by the conventional FEM as well as with the exact analytical results from (Karnovsky, 2001). For this end, we assume that two example beams shown in Fig. 2 are stationary (i.e., $c=0$ m/s), but not subject to the axial tension (i.e., $P=0$ N).

To formulate the finite element model used for the FEM results, the displacement fields within a finite element of length l are assumed as (Petyt, 1990)

$$w(x,t)=\boldsymbol{N}(x)\,\boldsymbol{d}(t) \tag{35}$$

where $\boldsymbol{N}(x)$ is the shape function matrix given by

$$\boldsymbol{N}(x)=\left[1-3\left(\frac{x}{l}\right)^2+2\left(\frac{x}{l}\right)^3,\ x\left(\frac{x}{l}-1\right)^2,\ \left(\frac{x}{l}\right)^2\left(3-2\frac{x}{l}\right),\ x\left[\left(\frac{x}{l}\right)^2-\left(\frac{x}{l}\right)\right]\right] \tag{36}$$

The finite element equation can be derived in the form as

$$\boldsymbol{M}\ddot{\boldsymbol{d}}+(\boldsymbol{C}_G+\boldsymbol{C}_V)\,\dot{\boldsymbol{d}}+\boldsymbol{K}\boldsymbol{d}=\boldsymbol{f} \tag{37}$$

where $\boldsymbol{d}$ and $\boldsymbol{f}$ are the spectral nodal DOFs vector and the nodal forces vector defined by Eq. (21a) and Eq. (27a), respectively. The finite element matrices in Eq. (37) are given by

$$\mathbf{M}=\frac{\rho Al}{42}\begin{bmatrix}156 & 22l & 54 & -13l\\ & 4l^2 & 13l & -3l^2\\ & & 156 & -22l\\ symm & & & 4l^2\end{bmatrix}$$

$$\mathbf{C}_G=\frac{\rho Ac}{30}\begin{bmatrix}0 & 6l & 30 & -6l\\ -6l & 0 & 6l & -l^2\\ -30 & -6l & 0 & 6l\\ 6l & l^2 & -6l & 0\end{bmatrix}$$

$$\mathbf{C}_V=\frac{1}{l^3}\begin{bmatrix}12\eta I & 6\eta Il & -12\eta & 6\eta Il\\ & 4\eta Il^2 & -6\eta Il & 2\eta Il^2\\ & & 12\eta I & -6\eta Il\\ symm & & & 4\eta Il^2\end{bmatrix} \tag{38}$$

$$\mathbf{K}=\frac{EI}{30l^3}\begin{bmatrix}K_{11} & K_{12} & K_{13} & K_{14}\\ & K_{22} & K_{23} & K_{24}\\ & & K_{33} & K_{34}\\ symm & & & K_{44}\end{bmatrix}$$

where

$$\begin{aligned} K_{11} &= -K_{13} = K_{33} = 360 + 36r - 36s \\ K_{12} &= -K_{14} = K_{34} = 180 + 3rl - 3sl \\ K_{24} &= 60l^2 - rl^2 + sl^2 \\ K_{22} &= -K_{44} = 120l^2 + 4rl^2 - 4sl^2 \\ r &= \frac{Pl^2}{EI},\ s = \frac{\rho A c^2 l^2}{EI} \end{aligned} \tag{39}$$

Table 1 and Table 2 compare the lowest three eigenvalues (i.e., λ_1, λ_2, λ_3) of the one-span beam and the two-span beam, respectively, for two cases. The first case is when the viscoelasticity is not taken into account (i.e., $\eta=0$) and the second case is when the viscoelasticity is taken into account (i.e., $\eta=6.8\times10^{-4}E$, where $E=2\times10^{11}$ N/m^2). Physically, $\eta=0$ means that the beams are pure elastic, while $\eta\neq0$ means that the beams are viscoelastic. For the SEM results, only one finite element is used for one-span beam while two finite elements for two-span beam. Table 1 and Table 2 show that the present SEM results are identical to the exact analytical results (Karnovsky, 2001). When the beams are pure elastic (i.e., $\eta=0$), the FEM results indeed converge to the SEM results or to the exact analytical results as the total number of finite elements increases for both example beams. This is also true for the viscoelastic beams with $\eta=6.8\times10^{-4}E$. This observation certainly confirms the extremely high accuracy of the present spectral element model.

Table 1 The lowest three eigenvalues of the simply-supported stationary one-span beam obtained by SEM, FEM and the exact theory (Karnovsky, 2001)

η	Method	N	λ_1	λ_2	λ_3
0	Exact		3.444 i	13.777 i	30.998 i
	SEM	1	3.444 i	13.777 i	30.998 i
	FEM	10	3.444 i	13.800 i	31.014 i
		20	3.444 i	13.778 i	30.999 i
		50	3.444 i	13.777 i	30.998 i
$6.8\times10^{-4}E$	SEM	1	$-0.025+3.444\ i$	$-0.405+13.771\ i$	$-2.052+30.930\ i$
	FEM	10	$-0.025+3.444\ i$	$-0.407+13.794\ i$	$-2.055+30.946\ i$
		20	$-0.025+3.444\ i$	$-0.405+13.772\ i$	$-2.052+30.931\ i$
		50	$-0.025+3.444\ i$	$-0.405+13.771\ i$	$-2.052+30.930\ i$

Note: (1) N=Number of finite elements, (2) $E=2\times10^{11}$ N/m^2

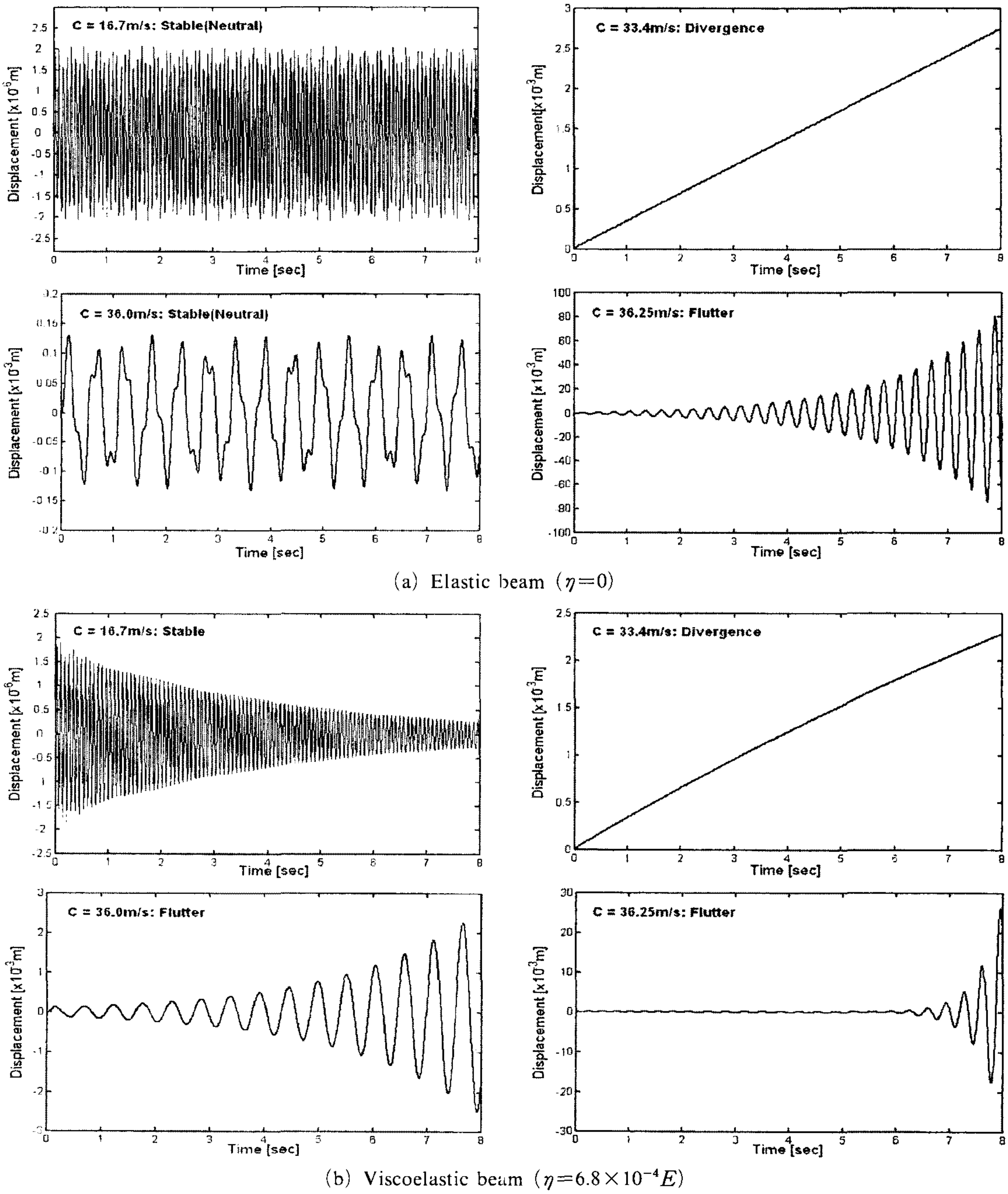
Table 2 The lowest three eigenvalues of the simply-supported stationary two-span beam obtained by SEM, FEM and the exact theory (Karnovsky, 2001)

η	Method	N	λ_1	λ_2	λ_3
0	Exact		3.444 i	5.381 i	13.777 i
	SEM	2	3.444 i	5.381 i	13.777 i
	FEM	10	3.446 i	5.382 i	13.800 i
		20	3.444 i	5.381 i	13.778 i
		50	3.444 i	5.381 i	13.777 i
$6.8\times10^{-4}E$	SEM	2	$-0.025+3.444\ i$	$-0.062+5.380\ i$	$-0.405+13.771\ i$
	FEM	10	$-0.025+3.445\ i$	$-0.062+5.382\ i$	$-0.407+13.794\ i$
		20	$-0.025+3.444\ i$	$-0.062+5.380\ i$	$-0.405+13.772\ i$
		50	$-0.025+3.444\ i$	$-0.062+5.380\ i$	$-0.405+13.771\ i$

Note: (1) N=Number of finite elements, (2) $E=2\times10^{11}$ N/m^2

4.2 Effects of the moving speed and viscolasticity

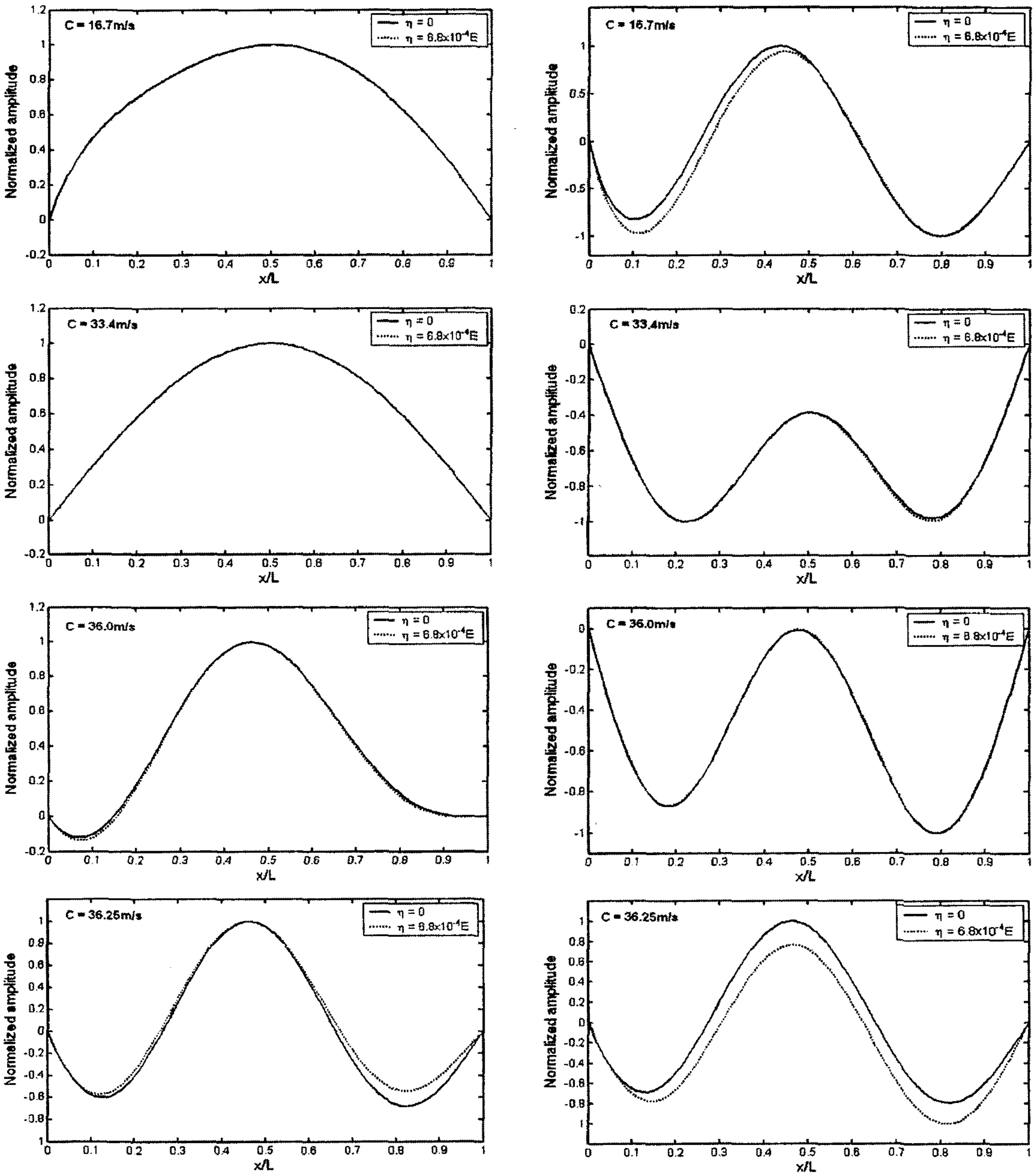
Figures 4(a) and 4 (b) show the dynamic responses of the pure elastic one-span beam and the viscoelastic one-span beam, respectively, at various moving speeds: $c=16.7$ m/s, $c=33.4$ m/s, $c=36.0$ m/s, and $c=36.25$ m/s. The dynamic response of the pure elastic beam is neutral at 16.7 m/s, unstable by the divergence at 33.41 m/s ($>c_D=33.40$ m/s), neutral at 36.0 m/s, and



(b) Viscoelastic beam ($\eta=6.8\times10^{-4}E$)

Fig. 4 Dynamic responses of the moving one-span beam subject to axial tension $P=2.5$ kN/m at various moving speeds unstable by the flutter at 36.26 m/s ($>c_F=36.25$ m/s), where c_D and c_F denote the divergence speed and the flutter speed, respectively. However, the dynamics of the viscoelastic beam is found to be somewhat different from that of the pure elastic beam. For instance, the neutral zone below c_D becomes stable and the neutral zone $c_D \leq c \leq c_F$ becomes unstable by the flutter due to the viscoelasticity effect.

Figure 5 compares the first natural mode



(a) First mode (b) Second mode

Fig. 5 Effect of viscoelasticity and moving speed on the first and second natural mode shapes of the moving viscoelastic one-span beam subject to axial tension $P=2.5$ kN/m shapes of the moving one-span beam subject to the axial tension $P=2.5$ kN/m, at four moving speeds, depending on the degree of viscoelasticity η: when $\eta=0$ and $\eta=6.8\times10^{-4}E$. The viscoelasticity tends to slightly distort the original natural mode shape of pure elastic beam (i.e., when $\eta=0$). The changes of natural mode shapes due to the viscoelasticity are found to be relatively larger at $c=36.25$ m/s when compared with those at the lower moving speeds. In general, however the effect of viscoelasticity on the change of natural modes seems to be not so significant.

5. Conclusions

In this paper, the spectral element model has been developed for the transverse vibration of an axially moving viscoelastic beam subject to axial tension. The viscoelasticity of the beam material was represented in a general form by using the one-dimensional constitutive equation of hereditary integral type of viscoelastic material. The present spectral element model was then evaluated by comparing its solutions with those obtained by the conventional FEM as well as with the results by exact theory. Numerical studies were conducted to investigate the effects of the moving speed and the viscoelasticity on the dynamics and natural mode shapes of axially moving viscoelastic beams. It has been found that the effect of viscoelsticity changes the stability of the viscoelastic beam. For instance, the neutral zone below the divergence speed becomes stable and the neutral zone below the flutter speed becomes unstable by the flutter due to the effect of viscoelsticity.

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