Journal of the Korea Society of Mathematical Education Series D: Research in Mathematical Education
Vol. 8, No. 2, June 2004, 69–80

韓國數學教育學會誌 시리즈 D: <數學教育研究>
제 8 권 제 2 호 2004 년 6 월, 69–80

Education and Professional Development of Mathematics Teachers in Korea[1]

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(Received June 8, 2004)

It is undeniable that teachers play the principal roles in education. This is why education and professional development of teachers are so important. Some of recent works have made this fact clearer. In America, in particular, many reports and research papers have recently been published on these problems. In this paper, we first introduce briefly the current system of education, employment, and professional development of mathematics teachers in Korea. And then we mention a research project on education of mathematics teachers. The final report of the project contains some suggestions for curriculums of the department for education of mathematics teachers. We describe one example of extended syllabus which implements those suggestions. The example is on Modern Algebra I.

Keywords: professional development, in-service teacher, teacher education, Teachers Employment Test (TET).
ZDM Classification: B59, C79
MSC2000 Classification: 97B50, 97C70

EDUCATION OF MATHEMATICS TEACHERS

In Korea, there are three kinds of institutes where prospective teachers are educated. We remark that any candidate having teaching certificate needs to pass a national

[1] This paper will be presented as a Regular Lecture at the 10th International Congress on Mathematical Education (ICMI-10), Copenhagen, Denmark, July 4–11, 2004.

examination to be a classroom teacher of the national or public schools[2].

1. Teacher educating institutes

These are the typical institutes for educating teachers. Every graduate is offered the teaching certificate. There are three types of university (or college) for teacher education.

(1) Universities only for educating elementary school teachers
There are 11 such universities which are national. Ewha Womans University is not only for educating teachers. It is the unique private university that is also educating the elementary school teachers.

(2) College only for educating secondary teachers
Many universities (national, private) have a college for educating secondary teachers[3].

(3) Korea National University of Education
This is the unique national university only for educating teachers for all levels of schools. This university is educating teachers for kindergarten, primary schools, and secondary schools.

2. General universities

The department of mathematics of general universities is not for teacher preparation. Usually, however, these departments have some students who want to be teachers. A small portion of them is selected. They take the required pedagogical courses including field experience. At graduation, they are given the teaching certificate. One weak point of this case is that mathematical courses they take at the department are usually separated from school mathematics.

3. Graduate schools of education

There are many graduate schools of education. Many of them are for professional development for in-service teachers. But some of them are educating the prospective teachers. The students in this program are the graduates of mathematics major. They take the required pedagogical courses including field experience. At graduation, they are given the teaching certificate with Masters degree of education. The problem that the mathematical courses they took at undergraduate schools were quite separated from

[2] However, every private school has the full authority of employing the teachers who have the certificate.

[3] In fact, there are 31 department of mathematics education in Korea. Ewha Womans University is educating the elementary teachers as well as the secondary teachers.

pedagogy need to be improved during two years of teacher preparation.

EMPLOYMENT OF TEACHERS

As mentioned before, to be a teacher of national or public schools every prospective teacher with teaching certificate should pass a national examination, called Teachers Employment Test (TET). TET consists of two phases. The first phase is paper examination. The questions of the test can be categorized into three areas: mathematics (50%), mathematics education (20%), and general education (30%). If an examinee is successful in the first phase, she/he can join the second phase[4]. The second phase is various depending on the provincial education bureau. The performance and teaching ability as a teacher is the main concern of this phase. If you are successful in TET, you can enter the classroom as a teacher after a short period of orientation organized by each provincial education bureau. Obviously, TET has serious impacts on the curriculum of teacher educating institutes.

PROFESSIONAL DEVELOPMENT OF IN-SERVICE TEACHERS[5]

The official programs for professional development of teachers are offered by graduate school of education or by re-training center of each provincial education bureau. Unfortunately, these programs don't seem to be very effective. As a result, professional development through the small study group consisting of fellow in-service teachers is strongly recommended. Nowadays the Korean Society of Mathematical Education invites various study groups for workshops to encourage such activities and to offer opportunities of sharing ideas and experiences.

We have recently developed a questionnaire to see the level of mathematical contents knowledge of in-service mathematics teachers of secondary schools (Shin 2004). It consists of 50 questions on the following algebra-related areas: Foundations of Mathematics, Linear Algebra, Number Theory, Modern Algebra, Applied Algebra, History of Mathematics, and Philosophy of Mathematics. Some examples in the questionnaire are following:

[4] The interval between two phases is usually a couple of weeks.

[5] Recently Korea has much interest in gifted education (Shin & Han 2000). As a result many institutes for gifted education have been established. However, professional development of teachers as well as developing teaching materials for the institutes are strongly demanded.

1. Which of the following concepts or claims has the least relationship with Russell's paradox?

(1) Axiom of Choice.
(2) The collection of the beautiful flowers is not a set.
(3) System of Zermelo-Skolem-Fraenkel.
(4) Paradox of the barber of Seville.

2. The following is the process of obtaining the least multiple of 12, 30 and 70:

$$
\begin{array}{r|rrr}
2 & 12 & 30 & 70 \\ \hline
 & 6 & 15 & 35
\end{array}
$$

$$\downarrow$$

$$
\begin{array}{r|rrr}
3 & 6 & 15 & 35 \\ \hline
 & 2 & 5 & 35
\end{array}
$$

$$\downarrow$$

$$
\begin{array}{r|rrr}
5 & 2 & 5 & 35 \\ \hline
 & 2 & 1 & 7
\end{array}
$$

At the first step, we divide all the three numbers by 2. However, at the second and the third step we divide only two numbers by 3 and 5, respectively. When we get the least multiple of four or more numbers, we use the same algorithm. Can you explain why the algorithm works?

(1) I know the algorithm, and can explain why it works.
(2) I know the algorithm, and can explain not clearly but roughly why it works.
(3) I know the algorithm, and don't know why it works.
(4) I don't remember even the algorithm.

3. When we determine whether a given number is a multiple of 9, we check whether the sum of all digits is a multiple of 9. Which of the following concepts is used in the algorithm ?

(1) the least common multiple
(2) the greatest common divisor
(3) prime number
(4) congruence equation

4. Usually the concept "the greatest common multiple" is not defined. Why do you think so?

(1) If we try to define it, we come up with a contradiction.
(2) It is not simple to define it.
(3) There is little mathematical significance.
(4) It is not defined uniquely.

5. Which of the following explanations about the claim '$2^0 = 1$' is most reasonable?

(1) It is just an axiom.
(2) We can prove it.
(3) It is sometimes claimed that '$2^0 = 0$'.
(4) The claim '$2^0 = 0$' can be also proved.

6. What is the fundamental concept that can be used to refute the following wrong argument:

To get the value of the infinite series

$1+(-1)+1+(-1)+1+(-1)+1+(-1)+\cdots$, we note that

$$1+(-1)+1+(-1)+1+(-1)+1+(-1)+\cdots$$
$$=1-\{1+(-1)+1+(-1)+1+(-1)+1+(-1)+\cdots\}.$$

Now we let

$$1+(-1)+1+(-1)+1+(-1)+1+(-1)+\cdots = S, \text{ then } S = 1-S.$$

Therefore, $S = \dfrac{1}{2}$.

(1) density (2) continuity
(3) field (4) convergence

7. We are given the general linear equation: $ax = b$. If $a = 0$ and $b = 0$, then it has infinitely many solutions. If $a = 0$ and $b \neq 0$, then it doesn't have any solution. Now if $a \neq 0$, then it has unique solution

$$a^{-1}b = \frac{b}{a}.$$

From the above argument we can see that the number of the equation depends on the value of a. Now we consider a system of two linear equations with two variables. The system can be expressed as $AX = B$. Here A is a 2×2 matrix and X and B are 2×1 matrices. Now what of A determines the number of solutions of the given system?

(1) determinant (2) trace
(3) eigen-values (4) eigen-vectors

8. When we solve the linear equation $x+3+x=2+3$, which of the following concepts or properties is not used?

(1) identity (2) inverse element
(3) commutativity (4) scalar multiple

We did a pilot test using the questionnaire and found out that the university mathematics courses are quite separated from school mathematics. We are scheduled to collect data from more than 100 mathematics teachers. The data is expected to enable us to develop better curriculum and courses for teacher re-training institutes.

A RESEARCH ON EDUCATION OF MATHEMATICS TEACHERS

Recently the Korea Research Foundation has launched a project for developing the curriculum and teaching/learning materials for the departments of mathematics education at the various teacher-educating universities. The products are following:

(1) Extensive survey on related literatures
(2) Suggestions for and developing curriculum of mathematics department of teacher educating universities
(3) Extended syllabuses of 26 key courses of mathematics department of teacher educating universities. Each syllabus consists of about 100 pages of description.
(4) A list of about 300 non-professional books which are recommended for prospective or in-service teachers of mathematics.
(5) A list of about 200 web-sites which are recommended for prospective or in-service teachers of mathematics.

The syllabuses have been developed to accept the following suggestions which are proposed in the final report of the project:

(1) To help the students to understand the foundation of mathematics.
(2) To help the students to enjoy the mathematics.
(3) To help the students to know and utilize the effectiveness and usefulness of mathematics.
(4) To help the students to do mathematics in real life.
(5) To help the students to connect each course with school mathematics.
(6) To help the students to integrate all branches of mathematics.
(7) To stimulate the intellectual interest of students to induce self investigation.
(8) To incorporate technology in some courses.

(9) To show a good teaching model.

The following is the proposed course schedule for the department for educating elementary school teachers. The number in the parenthesis is the number of credits.

Grade	Courses
1	Understanding of Mathematics for Elementary School Teachers (2)
2	Mathematics Education for Elementary School Teachers I: Theory (2) Probability and Statistics for Elementary School Teachers (2) Geometry for Elementary School Teachers (3)
3	Mathematics Education for Elementary School Teachers II: Practice (3) Foundations of Mathematics for Elementary School Teachers (2) Assessment of Mathematics for Elementary School Teachers (2)
4	Research on Mathematics Education for Elementary School Teachers (2) Psychology in Mathematics Learning (3) Teaching and Learning of Mathematics (3) Algebra for Elementary School Teachers (3)

The following is the proposed course schedule for the department for educating secondary school mathematics teachers. The number of credits of all courses is 3.

Grade	Courses for the First Semester	Courses for the Second Semester
1	Calculus I Set Theory	Calculus II Discrete Mathematics
2	Linear Algebra and Its Applications Geometry I Analysis I	Number Theory and Its Applications Geometry II Analysis II
3	Modern Algebra I Differential Equations General Topology Probability and Statistics Foundations of Mathematics Education	Modern Algebra II Complex Analysis Differential Geometry Applied Mathematics Teaching and Learning of Mathematics Field Experience I
4	Algebra for School Mathematics Geometry for School Mathematics History of Mathematics for School Teachers Problem Solving in Mathematics Field Experience II	Analysis for School Mathematics Probability and Statistics for School Mathematics Philosophy of Mathematics for School Teachers School Mathematics and Modern Mathematics Assessment in Mathematics

AN EXAMPLE

We describe the example of the syllabus on Modern Algebra I[6]. This course is for 45 hours. Each class hour is 50 minutes.

1. The main ideas of this course are following:

(1) The courses is focused on 3 algebraic structures: group, ring, vector space. The basic examples are **Z, Q**, $\mathbf{R}^2$, $\mathbf{R}^3$, $\mathrm{Mat}_2(\mathbf{R})$, $\mathbf{Q}[x]$, $\mathbf{R}[x]$. The definitions, examples, and basic properties of these algebraic structures are introduced in parallel. Various contents of school mathematics are mentioned. Some examples of them will be given later. This course is neither for group theory nor for ring theory. Understanding important algebraic structures through basic examples and school mathematics is the main purpose.

(2) Substructures (subgroup, subring, and subspace), quotient structures (with normal subgroup and ideal), and direct sums are introduced in parallel. Some concepts (subset, partition by an equivalence relation, Cartesian product, $n\mathbf{Z}$, $\mathbf{Z}_n$, and matrix and determinant, for examples) in set theory, number theory, and linear algebra are naturally mentioned.

(3) The homomorphism and the linear mapping are introduced in parallel. Various functions (function in set theory and continuous function in calculus, for examples) are mentioned.

(4) Lagrange Theorem is proved. It'd be helpful for the students to compare the theorem with the following facts;
   (i) In set theory, $|X \cup Y| = |X| + |Y| - |X \cap Y|$.
   (ii) In linear algebra, $\dim(X+Y) = \dim X + \dim Y - \dim(X \cap Y)$.
   (iii) In ring (field) theory, $[L:F]=[L:K][K:F]$ for $F \le K \le L$. This fact will be proved later in this course.

(5) The basic properties of maximal ideals are proved and some finite fields (of order 4, 8, 9) are constructed. The prime fields $\mathbf{Z}_p, p$: prime, are mentioned.

(6) A topic which needs number theory, group theory, and ring theory. The following is such an example: For a ring R we define R^* to be the set of all units. Note that R^* forms a multiplicative group, called the group of units. Now discuss the group Z_n^* with respect to following concepts:
   (i) Euler function
   (ii) Finite abelian group

[6] Even though we don't introduce here, Linear Algebra and Its Applications is deeply interwoven with algebraic coding theory, and Number Theory and Its Applications with cryptography.

(iii) Cyclic group
(iv) The existence of primitive elements
(v) Isomorphism between $\mathbf{Z}_8^*$ and $\mathbf{Z}_{12}^*$.

(7) Field extension and constructible numbers are studied briefly. The co-work of algebra and geometry is explained. Some examples will be given later.
(8) Some students will take the advanced course (Modern Algebra 2 for Teachers). The course mainly deals with Galois theory. To be ready for the course the students will be given some home-works for vacation.

2. It is worth pointing out that commutativity, associativity, and distributive property are deeply connected with finiteness and convergence. The following examples would be helpful:

(1) Discuss the following solution:

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots\frac{1}{n\cdot(n+1)}+\cdots$$
$$=(1-\frac{1}{2})+(\frac{1}{2}-\frac{1}{3})+\cdots+(\frac{1}{n-1}-\frac{1}{n})+(\frac{1}{n}-\frac{1}{n+1})+\cdots$$
$$=1-\frac{1}{2}+\frac{1}{2}-\cdots-\frac{1}{n}+\frac{1}{n}+\cdots$$
$$=1.$$

(2) Check the associativity and commutativity in the series:

$$1-1+1-1+\cdots+1-1+\cdots.$$

(3) Discuss the following solution:

$$\lim_{n\to\infty}\frac{1+2+\cdots n}{n^2}$$
$$=\lim_{n\to\infty}(\frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{n}{n^2})$$
$$=\lim_{n\to\infty}\frac{1}{n^2}+\lim_{n\to\infty}\frac{2}{n^2}+\cdots+\lim_{n\to\infty}\frac{n}{n^2}$$
$$=0.$$

(4) Discuss the following arguments:

(i) If we let $x=0.999\cdots$, then $10x=9.999\cdots$. So, $9x=9$. Thus $x=1$.

(ii) If we let $x=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}$, then $x=1+\frac{1}{x}$.

Solving this equation, we can get $x=\frac{1+\sqrt{5}}{2}$.

3. It is worth noting the similarity among the solutions of linear equation, a system of linear equations, and linear congruence equation.

(1) Discuss the solution of $ax = b$.
  (i) In the case that $a = 0$. We have two cases. If $b = 0$, there are infinitely many solutions. But if $b \neq 0$, there is no solution.
  (ii) In the case that $a \neq 0$, there exists unique solution $x = \frac{b}{a} = a^{-1}b$.

(2) Solution of $AX = B$, where A is a square matrix.
  (i) In the case that A is singular (in other words, $|A| = 0$), there exist(s) infinitely many or no solution(s) depending on B.
  (ii) In the case that A is non-singular (in other words, $|A| \neq 0$), there exists unique solution $X = A^{-1}B$.

(3) Compare the above results with the case of linear congruence equation $ax \equiv b \pmod{n}$.
  (i) When does the congruence equation have solutions?
  (ii) If the equation has solutions, how many are there? And what are they?

4. Algebraic structures in school mathematics

(1) $\mathbf{Q}(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in \mathbf{Q}\}$ forms a field. This fact is connected with the rationalization of the denominator of

$$\frac{1}{1 + 3\sqrt{2}}.$$

(2) The set $\mathbf{C}$ of all complex number forms a field. This fact is connected with the realization of the denominator of

$$\frac{1}{1 + 2i}.$$

(3) Discuss commutativity, associativity, identity, inverse element in the following problems.
  (i) Solve: $x + 3 + x = 2 + 3$.
  (ii) Compute $123 \times 345 \times 0$.
  (iii) Compute $123 \times 345 \times \frac{1}{123}$.
  (iv) Compute $123 \times 345 + (-345)$.
  (v) Compute $123 \times 345 + (-123)$.

(4) Applying the properties of algebraic structures, argue the following claim: (negative number) × (negative number) = (positive number).

5. Discuss field extension in school mathematics through following problems:

(i) Geometric construction.
(ii) Algebraic construction.
(iii) Paper-folding.

6. Discuss co-work of algebra and geometry through following topics:

   (i) Some examples in analytic geometry.
   (ii) Erlangen Program by Felix Klein in 1872.
   (iii) A proper picture explaining the equality: $(a+b)^2 = a^2 + 2ab + b^2$.
   (iv) Construction of numbers.
   (v) Proper pictures explaining the following equalities:

$$1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots = 2,$$

$$1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots = \frac{4}{3}.$$

7. All the students are required to read the following books[7]:
   - Fermats Last Theorem (S. Singh 1997).
   - The Man Who Loved Only Numbers (P. Hoffman 1998).
   - My Brain Is Open (B. Schechter 1998).
   - Knowing and Teaching Elementary Mathematics (L. Ma 1999).
   - Mystery of Aleph (A. D. Aczel 2000).
   - The Elegant Universe (B. Greene 1999).
   - Beautiful Mind (S. Nasar 1998).
   - In Code (S. Flannery & D. Flannery 2000).
   - Flatland (E. Abbott, Annotated by I. Stewart 2002).

Of course some students might read some of the above books when they took Set Theory, Linear Algebra and Its Applications, or Number Theory and Its Applications.

CONCLUSIONS

It is said that the first word that a foreign business person in Korea learns is 'bballi, bballi' which means 'hurry up, hurry up.' The bballi-bballi tradition is sometimes good in business. The problem is that it is also quite prevalent even in education. It is clear that this situation is undesirable in mathematics education. In other words, mathematics education cannot be successful with a bballi-bballi spirit. There are also other education-

[7] All of these books have been translated into Korean.

al problems that stem from this hurried culture.

The fever for children education in Korea is quite remarkable (KSICMI 2004). Although the educational fever combined with the bballi-bballi tradition may come up with some negative results[8], it will surely be a tremendous power for Korean education if the fever is used effectively. But we need to note that it is also the teachers that should organize the fever of parents well to find solutions to the various problems in mathematics education of Korea.

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[8] The abnormal popularity of private education is a typical result.