A Balanced Model Reduction for Fuzzy Systems with Time Varying Delay

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Abstract

This paper deals with a balanced model reduction for T-S(Takagi-Sugeno) fuzzy systems with time varying state delay. We define a generalized controllability gramian and a generalized observability gramian for a stable T-S fuzzy delayed systems. We obtain a balanced state space realization using the generalized controllability and observability gramian and obtain a reduced model by truncating states from the balanced state space realization. We also present an upper bound of the approximation error. The generalized controllability gramian and observability gramian can be computed from solutions of linear matrix inequalities. We demonstrate the efficacy of the suggested method by illustrating a numerical example.

Key Words : T-S fuzzy system, generalized controlability/observability gramian, balanced realization, linear matrix inequality.

I. Introduction

For linear finite dimensional systems with high orders, optimal control techniques such as linear quadratic gaussian and H_∞ control theory, usually produce controllers with the same state dimension as the model. Lower dimensional linear controllers are normally preferred over higher dimensional controllers in control system designs for some obvious reasons : they are easier to understand, implement and have higher reliability. Accordingly the problem of model reduction is of significant practical importance in control system design and has been a focus of a wide variety of studies for recent decades(see [1-6] and the references therein).

The stability analysis and control of linear time delayed systems are problems of practical and theoretical interest since many types of processes such as steel making process and chemical process can be modeled as dynamic systems with time delay. In the last decade, the linear matrix inequality(LMI) based controller design method for delayed systems has been developed remarkably[7-9]. A drawback of the LMI based controller synthesis is that computational requirements increase rapidly as the state dimension increases. Therefore the state dimension must be kept as low as possible.

In recent years, a controller design method for nonlinear dynamic systems modeled as a T-S(Takagi-Sugeno) fuzzy model has been intensively addressed[10-15]. Unlike a single conventional model, this T-S fuzzy model usually consists of several linear models to describe the global behavior of the nonlinear system. Typically the T-S fuzzy model is described by fuzzy IF-THEN rules. Based on this fuzzy model, many researchers use one of control design methods developed for linear parameter varying system. In order to alleviate computational burden in design phase and simplify the designed fuzzy controller, the state dimension of the T-S fuzzy model should be low.

In this paper, using a fuzzy approach we develope a balanced model reduction scheme for T-S fuzzy systems with time varying delay. In section II, we define the T-S fuzzy system with time varying delay. A generalized controllability gramian and a generalized observability gramian are defined and a balanced realization of T-S fuzzy system using the generalized controllability and observability gramian is also presented in section III. A model approximation bound is derived and a suboptimal procedure is described to get a less conservative error bound in section IV. Section V demonstrates a numerical example and finally some concluding remarks are given in section VI.

The notation in this paper is fairly standard. R^n denotes n dimensional real vector space and $R^{n \times m}$ is the set of real $n \times m$ matrices. A^T denotes the transpose of a real matrix A. 0 and I denote zero matrix and identity matrix respectively. $M > 0$ means that M is a positive definite matrix. In a block symmetric matrix, $*$ in (i,j) block means the transpose of (j,i) block. Finally $\| \cdot \|_\infty$ denotes the H_∞ norm of the system.

II. T-S Fuzzy System

We consider the following fuzzy dynamic system with time varying delay $d(t)$.

Plant Rule i ($i = 1, \cdots, r$) :
IF $\rho_1(t)$ is M_{i1} and $\cdots$ and $\rho_g(t)$ is M_{ig}.
THEN

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + A_{d_i} x(t - d(t)) + B_i u(t) \\ y(t) &= C_i x(t) + C_{d_i} x(t - d(t)) + D_i u(t) \end{aligned} \quad (1)$$

where r is the number of fuzzy rules. $\rho_j(t)$ and M_{ij} ($j = 1, \cdots, g$) are the premise variables and the fuzzy set respectively. $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input, $y(t) \in R^p$ is the output variable and A_i, A_{d_i}, B_i,

C_i, C_{d_i}, D_i are real matrices with compatible dimensions. Define $A_{di} = F_i G$, $C_{di} = H_i G$ where $F_i \in R^{n \times k}$, $G \in R^{k \times n}$, $H_i \in R^{p \times k}$. Note that F_i, G, H_i are not necessarily full rank matrices. The easiest choice might be $F_i = A_{di}$, $G = I$, $H_i = C_{di}$. We also assume that time varying delay $d(t)$ satisfies

$$0 \le d(t) < \infty, \quad |\dot{d}(t)| \le m < 1. \tag{2}$$

Let $\mu_i(\rho(t))$, $i = 1, \cdots, r$, be the normalized membership function of the inferred fuzzy set $h_i(\rho(t))$,

$$\mu_i(\rho(t)) = \frac{h_i(\rho(t))}{\sum_{i=1}^{r} h_i(\rho(t))} \tag{3}$$

where

$$h_i(\rho(t)) = \prod_{j=1}^{g} M_{ij}(\rho_j(t)),$$
$$\rho(t) = [\rho_1(t) \ \rho_2(t) \ \cdots \ \rho_g(t)]^T. \tag{4}$$

In this paper, we assume for all $t \ge 0$,

$$h_i(\rho(t)) \ge 0 \ (i = 1, 2, \cdots, r), \quad \sum_{i=1}^{r} h_i(\rho(t)) > 0. \tag{5}$$

Then, we obtain

$$\mu_i(\rho(t)) \ge 0 \quad i = 1, 2, \cdots, r, \quad \sum_{i=1}^{r} \mu_i(\rho(t)) = 1. \tag{6}$$

For simplicity, by defining

$$\mu_i = \mu_i(\rho(t)), \ (i = 1, \cdots, r), \quad \mu^T = [\mu_1 \ \cdots \ \mu_r], \tag{7}$$

the fuzzy system (1) can be written as follows :

$$\begin{aligned}
\dot{x}(t) &= A(\mu)x(t) + F(\mu)w(t) + B(\mu)u(t) \\
&= \sum_{i=1}^{r} \mu_i(A_i x(t) + F_i w(t) + B_i u(t)) \\
y(t) &= C(\mu)x(t) + H(\mu)w(t) + D(\mu)u(t) \\
&= \sum_{i=1}^{r} \mu_i(C_i x(t) + H_i w(t) + D_i u(t)) \\
z(t) &= Gx(t) \\
w(t) &= \Theta(t)z(t) = z(t - d(t))
\end{aligned} \tag{8}$$

where $\Theta(t)$ is a delay operator.

In a packed matrix notation, we express the fuzzy system (8) as follows :

$$G = \left[\begin{array}{c|c|c} A(\mu) & F(\mu) & B(\mu) \\ \hline G & 0 & 0 \\ \hline C(\mu) & H(\mu) & D(\mu) \end{array} \right]. \tag{9}$$

III. Generalized Controllability and Observability Gramians

In order to define controllability and observability gramians, we present following lemma 1 and 2.

Lemma 1 : Suppose that there exist symmetric positive definite matrices Q and R such that following LMI holds for each $i = 1, \cdots, r$.

$$L_{oi} = \begin{bmatrix} A_i^T Q + Q A_i + G^T R G & * & * \\ F_i^T Q & -(1-m)R & * \\ C_i & H_i & -I \end{bmatrix} < 0. \tag{10}$$

When $u(t) \equiv 0$ for all $t \ge 0$ in the system (9), the output energy is bounded above as follows:

$$\int_0^\infty y^T(\tau)y(\tau)d\tau \tag{11}$$
$$< x(0)^T Q x(0) + \int_{-d(0)}^0 x(\tau)^T R x(\tau)d\tau$$

(proof) We define a Lyapunov function candidate $V(x(t))$ as follows :

$$V(x(t)) = x(t)^T Q x(t) + \int_{t-d(t)}^{t} x(\tau)^T R x(\tau)d\tau \tag{12}$$

Since LMI (10) holds,

$$\overline{L}_o = \left[\begin{array}{cc} A^T(\mu)Q + QA(\mu) + C^T(\mu)C(\mu) + G^T R G & \\ F(\mu)^T Q + H(\mu)^T C(\mu) & \\ & * \\ & H(\mu)^T H(\mu) - (1-m)R \end{array} \right] < 0 \tag{13}$$

holds for all μ satisfying (6). Moreover the fuzzy system (9) is quadratically stable. Accordingly $\lim_{t \to \infty} V(x(t), t) = 0$. The output energy becomes

$$\begin{aligned}
&\int_0^\infty y^T(\tau)y(\tau)d\tau \\
&= \int_0^\infty y^T(\tau)y(\tau) + \frac{dV(x(\tau))}{d\tau} d\tau + V(x(0)) \\
&\le \int_0^\infty \eta^T(\tau)\overline{L}_o\eta(\tau)d\tau + V(x(0)) \\
&< V(x(0))
\end{aligned} \tag{14}$$

where $\eta(t) = \begin{bmatrix} x(t) \\ Gx(t - d(t)) \end{bmatrix}$.

Lemma 2 : Suppose that there exist symmetric positive matrix P and S such that following LMI (15) holds for each $i = 1, \cdots, r$.

$$L_{ci} = \begin{bmatrix} A_i P + P A_i^T & * & * & * \\ S F_i^T & -(1-m)S & * & * \\ GP & 0 & -S & * \\ B_i^T & 0 & 0 & -I \end{bmatrix} < 0. \tag{15}$$

Then the input energy required to transit the state from $x(-\infty) = 0$ to $x(0)$ is bounded below as follows :

$$\int_{-\infty}^0 u^T(\tau)u(\tau)d\tau \tag{16}$$
$$> x(0)^T P^{-1} x(0) + \int_{-d(0)}^0 x(\tau)^T S^{-1} x(\tau)d\tau$$

(proof) We define a Lyapunov function candidate $V(x(t))$ as follows :

$$V(x(t)) = x(t)^T P^{-1} x(t) + \int_{t-d(t)}^{t} x(\tau)^T S^{-1} x(\tau)d\tau. \tag{17}$$

As in the proof of lemma 1,

each $i = 1, \cdots, r$.

$$\overline{L}_c = \begin{bmatrix} PA(\mu)^T + A(\mu)P + PG^TS^{-1}GP & * & * \\ SF(\mu)^T & -(1-m)S & * \\ B(\mu)^T & 0 & -I \end{bmatrix} < 0 \quad (18)$$

holds for all μ satisfying (6).

From the Lyapunov function candidate $V(x(t))$, we obtain

$$\begin{aligned}
0 &= \int_{-\infty}^{0} (u(\tau)^T u(\tau) - u(\tau)^T u(\tau)) d\tau \\
&= \int_{-\infty}^{0} (u(\tau)^T u(\tau) - u(\tau)^T u(\tau) + \frac{dV(x(\tau),\tau))}{d\tau}) d\tau \\
&\quad - V(x(0),0) \\
&\leq \int_{-\infty}^{0} u(\tau)^T u(\tau) d\tau - V(x(0)) + \int_{-\infty}^{0} \zeta(\tau)^T \overline{L}_c \zeta(\tau) d\tau \\
&< \int_{-\infty}^{0} u(\tau)^T u(\tau) d\tau - V(x(0))
\end{aligned} \quad (19)$$

where $\zeta(t) = \begin{bmatrix} P^{-1}x(t) \\ S^{-1}Gx(t-d(t)) \\ u(t) \end{bmatrix}$. This completes the proof.

As in [6], we say Q and P, solutions of LMI's (10) and (15), as the generalized observability gramian and controllability gramian respectively. While the observability and controllability gramian in linear time invariant systems are unique, the generalized gramians of the fuzzy delayed system (9) are not unique. But the generalized gramians are related to the input and output energy as can be seen in lemma 1 and lemma 2.

Using the generalized gramians, we suggest a balanced realization of the fuzzy system (9). We obtain a transformation matrix T and U satisfying

$$\begin{aligned}
T^{-1}PT^{-T} &= T^T QT = \Sigma = diag(\sigma_1, \sigma_2, \cdots, \sigma_n), \\
&\qquad\qquad \sigma_1 \geq \sigma_2 \cdots \geq \sigma_n \quad (20) \\
U^{-1}SU^{-T} &= U^T RU = \Pi = diag(\pi_1, \pi_2, \cdots, \pi_k)
\end{aligned}$$

With T and U defined in (20), the change of coordinates in the fuzzy system (9) gives

$$\begin{aligned}
G &= \left[\begin{array}{c|c|c} A_b(\mu) & F_b(\mu) & B_b(\mu) \\ \hline G_b & 0 & 0 \\ \hline C_b(\mu) & H_b(\mu) & D_b(\mu) \end{array}\right] \\
&= \left[\begin{array}{c|c|c} T^{-1}A(\mu)T & T^{-1}F(\mu)U & T^{-1}B(\mu) \\ \hline U^{-1}GT & 0 & 0 \\ \hline C(\mu)T & H(\mu)U & D(\mu) \end{array}\right] \\
&= \sum_{i=1}^{r} \mu_i \left[\begin{array}{c|c|c} A_{b,i} & F_{b,i} & B_{b,i} \\ \hline G_b & 0 & 0 \\ \hline C_{b,i} & H_{b,i} & D_{b,i} \end{array}\right] \\
&= \sum_{i=1}^{r} \mu_i \left[\begin{array}{c|c|c} T^{-1}A_i T & T^{-1}F_i U & T^{-1}B_i \\ \hline U^{-1}GT & 0 & 0 \\ \hline C_i T & H_i U & D_i \end{array}\right] \quad (21)
\end{aligned}$$

One can easily observe that the state space realization of (21) satisfy following LMI's (22) and (23).

$$L_{ob} = \begin{bmatrix} A_b(\mu)^T\Sigma + \Sigma A_b(\mu) + C_b(\mu)^T C_b(\mu) & \\ F_b(\mu)^T\Sigma + H_b(\mu)^T C_b(\mu) & \\ \Pi G_b & \end{bmatrix}$$
$$\begin{bmatrix} * & * \\ H_b(\mu)^T H_b(\mu) - (1-m)\Pi & * \\ 0 & -\Pi \end{bmatrix} < 0 \quad (22)$$

$$L_{cb} = \begin{bmatrix} A_b(\mu)\Sigma + \Sigma A_b(\mu)^T + B_b(\mu)B_b(\mu)^T & \\ \Pi F_b(\mu)^T & \\ G_b\Sigma & \end{bmatrix}$$
$$\begin{bmatrix} * & * \\ -(1-m)\Pi & * \\ 0 & -\Pi \end{bmatrix} < 0 \quad (23)$$

From this reason, we say that the realization (21) is a balanced realization of the fuzzy system (9) and Σ is a balanced gramian.

IV. Balanced Model Reduction

In this section, we develop a balanced model reduction scheme using the balanced gramian defined in section III. we also derive an upper bound of model approximation error resulting from the balanced model reduction. We assume that the fuzzy system (9) is already balanced and partitioned as follows :

$$\begin{aligned}
G &= \left[\begin{array}{cc|c|c} A_{11}(\mu) & A_{12}(\mu) & F_1(\mu) & B_1(\mu) \\ A_{21}(\mu) & A_{22}(\mu) & F_2(\mu) & B_2(\mu) \\ \hline G_1 & G_2 & 0 & 0 \\ \hline C_1(\mu) & C_2(\mu) & H(\mu) & D(\mu) \end{array}\right] \\
&= \sum_{i=1}^{r} \mu_i \left[\begin{array}{cc|c|c} A_{i,11} & A_{i,12} & F_{i,1} & B_{i,1} \\ A_{i,21} & A_{i,22} & F_{i,2} & B_{i,2} \\ \hline G_1 & G_2 & 0 & 0 \\ \hline C_{i,1} & C_{i,2} & H_i & D_i \end{array}\right] \quad (24)
\end{aligned}$$

where $A_{11}(\mu) \in R^{(n-r)\times(n-r)}$, $A_{22}(\mu) \in R^{r\times r}$ and the other matrices are compatibly partitioned. From (24) we obtain a reduced order model by truncating r states as follows :

$$G_r = \left[\begin{array}{c|c|c} A_{11}(\mu) & F_1(\mu) & B_1(\mu) \\ \hline G_1 & 0 & 0 \\ \hline C_1(\mu) & H(\mu) & D(\mu) \end{array}\right] = \sum_{i=1}^{r} \mu_i \left[\begin{array}{c|c|c} A_{i,11} & F_{i,1} & B_{i,1} \\ \hline G_1 & 0 & 0 \\ \hline C_{i,1} & H_i & D_i \end{array}\right] \quad (25)$$

In order to derive an upper bound of model approximation error, we need the bounded real lemma for fuzzy systems with time varying delay.

Lemma 3 : Suppose that there exist symmetric positive definite matrices X and R satisfying LMI (26) for all μ satisfying (6).

$$\begin{bmatrix} XA(\mu)^T + A(\mu)X & * & * & * & * \\ RF(\mu)^T & -(1-m)R & * & * & * \\ B(\mu)^T & 0 & -I & * & * \\ \gamma^{-1}C(\mu)X & \gamma^{-1}H(\mu)R & \gamma^{-1}D(\mu) & -I & * \\ GX & 0 & 0 & 0 & -R \end{bmatrix} < 0 \quad (26)$$

Then the fuzzy delayed system (9) satisfies $\| G \|_\infty \leq \gamma$.
(proof) One can easily prove lemma 3 using the method of [7]. Hence we omit the detailed proof.

Theorem 4 : The reduced order system (25) is quadratically stable and balanced. The model approximation error is given by

$$\| G - G_r \|_\infty \leq 2 \sum_{i=n-r+1}^{n} \sigma_i. \tag{27}$$

(proof) We partition the balanced gramian Σ as $\Sigma = diag(\Sigma_1, \Sigma_2)$ where $\Sigma_1 \in R^{(n-r)\times(n-r)}$, $\Sigma_2 \in R^{r \times r}$. Then the reduced order system G_r satisfies LMI's (28) and (29).

$$\begin{bmatrix} A_{11}(\mu)^T\Sigma_1 + \Sigma_1 A_{11}(\mu) + C_1(\mu)^T C_1(\mu) \\ F_1^T\Sigma_1 + H(\mu)^T C_1(\mu) \\ \Pi G_1 \\ \\ * \qquad * \\ H(\mu)^T H(\mu) - (1-m)\Pi \quad * \\ 0 \qquad -\Pi \end{bmatrix} < 0 \tag{28}$$

$$\begin{bmatrix} A_{11}(\mu)\Sigma_1 + \Sigma_1 A_{11}(\mu)^T + B_1(\mu)B_1(\mu)^T \\ \Pi F_1(\mu)^T \\ G_1\Sigma_1 \\ \\ * \qquad * \\ -(1-m)\Pi \quad * \\ 0 \qquad -\Pi \end{bmatrix} < 0 \tag{29}$$

Hence the reduced order system G_r is quadratically stable and balanced. Without loss of generality we assume that $r = 1$. Thus, $\Sigma_2 = \sigma_n$. A state space realization of the error system $G_e = G - G_r$ can be written by

$$G_e = \left[\begin{array}{ccc|ccc} A_{11}(\mu) & 0 & 0 & F_1(\mu) & 0 & B_1(\mu) \\ 0 & A_{11}(\mu) & A_{12}(\mu) & 0 & F_1(\mu) & B_1(\mu) \\ 0 & A_{21}(\mu) & A_{22}(\mu) & 0 & F_1(\mu) & B_2(\mu) \\ \hline G_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_1 & G_2 & 0 & 0 & 0 \\ \hline -C_1(\mu) & C_1(\mu) & C_2(\mu) & -H(\mu) & H(\mu) & 0 \end{array}\right] \tag{30}$$

We define the coordinate transformation T as follows :

$$T = \begin{bmatrix} \frac{1}{2}I & \frac{1}{2}I & 0 \\ \frac{1}{2}I & \frac{-1}{2}I & 0 \\ 0 & 0 & I \end{bmatrix} \tag{31}$$

With T defined in (31), the change of coordinate in the error system G_e gives (32).

$$G_e = \left[\begin{array}{cc|c} A(\mu) & F_e(\mu) & B_e(\mu) \\ \hline G_e & 0 & 0 \\ \hline C(\mu) & H(\mu) & 0 \end{array}\right] = \left[\begin{array}{ccc|cc|c} A_{11}(\mu) & 0 & A_{12}(\mu)/2 & F_1(\mu)/2 & F_1(\mu)/2 & B_1(\mu) \\ 0 & A_{11}(\mu) & -A_{12}(\mu)/2 & F_1(\mu)/2 & -F_1(\mu)/2 & 0 \\ A_{21}(\mu) & -A_{21}(\mu) & A_{22}(\mu) & 0 & F_1(\mu) & B_2(\mu) \\ \hline G_1 & G_1 & 0 & 0 & 0 & 0 \\ G_1 & -G_1 & G_2 & 0 & 0 & 0 \\ \hline 0 & -2C_1(\mu) & C_2(\mu) & -H(\mu) & H(\mu) & 0 \end{array}\right] \tag{32}$$

In order to prove $\| G_e \|_\infty \leq 2\sigma_n$, we will show that there

exist symmetric positive definite matrices X_e and Π_e satisfying following LMI (33).

$$L := \begin{bmatrix} L_{11} \\ \sigma_n^{-2}H_e(\mu)H_e(\mu)^T C_e(\mu)X_e + \Pi_e F_e(\mu)^T \\ G_e X_e \\ \\ * \qquad * \\ \sigma_n^{-2}\Pi_e H_e(\mu)^T H_e(\mu)\Pi_e - (1-m)\Pi_e \quad * \\ 0 \qquad -\Pi_e \end{bmatrix} < 0 \tag{33}$$

where

$$L_{11} = A_e(\mu)X_e + X_e A_e(\mu)^T + \sigma_n^{-2}X_e C_e(\mu)^T C_e(\mu)X_e + B_e(\mu)B_e(\mu)^T$$

Using Schur complement in LMI (26), LMI (33) can be derived. Lets choose X_e and Π_e as,

$$X_e = \begin{bmatrix} \Sigma_1 & 0 & 0 \\ 0 & \sigma_n^2\Sigma_1^{-1} & 0 \\ 0 & 0 & 2\sigma_n \end{bmatrix}$$
$$\Pi_e = \begin{bmatrix} \Pi + \sigma_n^2\Pi^{-1} & \Pi - \sigma_n^2\Pi^{-1} \\ \Pi - \sigma_n^2\Pi^{-1} & \Pi + \sigma_n^2\Pi^{-1} \end{bmatrix} \tag{34}$$

Then LMI (33) can be expressed as follows :

$$L = \begin{bmatrix} M_1^T & 0 & 0 \\ 0 & M_2^T & 0 \\ 0 & 0 & M_3^T \end{bmatrix} L_{cb} \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}$$
$$+ \begin{bmatrix} N_1^T & 0 & 0 \\ 0 & N_2^T & 0 \\ 0 & 0 & N_3^T \end{bmatrix} L_{ob} \begin{bmatrix} N_1 & 0 & 0 \\ 0 & N_2 & 0 \\ 0 & 0 & N_3 \end{bmatrix} < 0 \tag{35}$$

where

$$M_1 = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \end{bmatrix}, \quad M_2 = M_3 = [I \quad \Pi], \quad N_1 = \begin{bmatrix} 0 & \sigma_n\Sigma_1^{-1} & 0 \\ 0 & 0 & -I \end{bmatrix},$$
$$N_2 = N_3 = [\sigma_n\Pi^{-1} \quad -\sigma_n\Pi^{-1}]$$

Since X_e and Π_e defined in (34) satisfy LMI (33), we can conclude $\| G_e \|_\infty \leq 2\sigma_n$. This completes the proof.

In theorem 4, we have proved that the model reduction error is bounded above by $2 \sum_{i=n-r+1}^{n} \sigma_i$. In order to get a less model reduction error bound, it is necessary for $\sigma_{n-r+1}, \cdots, \sigma_n$ to be small. Hence we choose a cost function as $J = trace(PQ) = \sum_{i=1}^{n} \sigma_i^2$. Thus, we minimize the non-convex cost function subject to the convex constraints (10) and (15). Since this optimization problem is non-convex, the optimization problem is very difficult to solve it. So we suggest a suboptimal procedure using an iterative method. We summarize an iterative method to solve a suboptimal problem.

step 1 : Set $i = 0$. Initialize Q_i, R_i, P_i and S_i such that $J = trace(P_i + Q_i)$ is minimized subject to LMI's (10) and (15).

step 2 : Set $i = i + 1$.

1) Minimize $J_i = trace(P_i Q_{i-1})$ subject to LMI (15).

2) Minimize $J_i = trace(P_i Q_i)$ subject to LMI (10).

step 3 : If $|J_i - J_{i-1}|$ is less than a small tolerance level, stop iteration. Otherwise, go to step 2.

V. A Numerical Example

We consider a T-S fuzzy system with time varying delay given by

Plant Rule 1 :

$$\dot{x}(t) = A_1 x(t) + A_{d1} x(t - d(t)) + B_1 u(t)$$
$$y(t) = C_1 x(t) + C_{d1} x(t - d(t)) \tag{36}$$

Plant Rule 2 :

$$\dot{x}(t) = A_2 x(t) + A_{d2} x(t - d(t)) + B_2 u(t)$$
$$y(t) = C_2 x(t) + C_{d2} x(t - d(t)) \tag{37}$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -17 & -17 & -7 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -6 & -17 & -17 & -7 \end{bmatrix},$$

$$B_1^T = B_2^T = [0 \ 0 \ 0 \ 1], \ C_1 = C_2 = [1 \ 0 \ 0 \ 0],$$

$$A_{d1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0.1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.1 & 0 \end{bmatrix},$$

$$C_{d1} = C_{d2} = [0.1 \ 0 \ 0 \ 0], \ |\dot{d}(t)| \le 0.1$$

We choose $F_1 = A_{d1}$, $F_2 = A_{d2}$, $H_1 = H_2 = C_{d1}$, $G = I$ for convenience. Using the iterative method described in section IV, we obtain

$$P = 10^{-3} \begin{bmatrix} 8.97 & -0.431 & -2.71 & 0.341 \\ -0.431 & 2.86 & -0.181 & -6.56 \\ -2.71 & -0.181 & 6.98 & -0.963 \\ 0.341 & -6.56 & -0.963 & 102.0 \end{bmatrix},$$

$$Q = \begin{bmatrix} 3.27 & 2.90 & 1.11 & 0.154 \\ 2.90 & 3.12 & 1.27 & 0.180 \\ 1.11 & 1.27 & 0.538 & 0.0765 \\ 0.154 & 0.180 & 0.0765 & 0.0115 \end{bmatrix} \tag{38}$$

Using (20) we also obtain the balanced gramian $\Sigma = diag(0.1718, 0.0467, 0.0092, 0.0051)$. Accordingly we can expect the model reduction error is bounded by 0.0286 when we truncate last 2 state variables. Using (25) the reduced system can be obtained as follows :

Plant Rule 1 :

$$\dot{x}(t) = A_{1,11} x(t) + F_{1,1} G_1 x(t - d(t)) + B_{1,1} u(t)$$
$$= \begin{bmatrix} -0.1381 & 0.3879 \\ -0.3973 & -0.8332 \end{bmatrix} x(t)$$
$$+ \begin{bmatrix} 0.0005 & 0.0053 \\ 0.0005 & 0.0063 \end{bmatrix} x(t - d(t)) + \begin{bmatrix} 0.2142 \\ 0.2525 \end{bmatrix} u(t)$$
$$y(t) = C_{1,1} x(t) + H_1 G_1 x(t - d(t)) \tag{39}$$
$$= [0.1934 \ -0.2272] x(t)$$
$$+ [0.0193 \ -0.0227] x(t - d(t))$$

Plant Rule 2 :

$$\dot{x}(t) = A_{2,11} x(t) + F_{2,1} G_1 x(t - d(t)) + B_{2,1} u(t)$$
$$= \begin{bmatrix} -0.2209 & 0.4852 \\ -0.4950 & -0.7184 \end{bmatrix} x(t)$$
$$+ \begin{bmatrix} 0.0007 & 0.0087 \\ 0.0009 & 0.0102 \end{bmatrix} x(t - d(t)) + \begin{bmatrix} 0.2142 \\ 0.2525 \end{bmatrix} u(t) \tag{40-1}$$

$$y(t) = C_{2,1} x(t) + H_2 G_1 x(t - d(t))$$
$$= [0.1934 \ -0.2272] x(t) \tag{40-2}$$
$$+ [0.0193 \ -0.0227] x(t - d(t))$$

When $d(t) = 1$, the model reduction error is depicted in Fig.1. In Fig.1, the solid line represents the model reduction error of the plant rule 1 and the dotted line represents that of the plant rule 2.
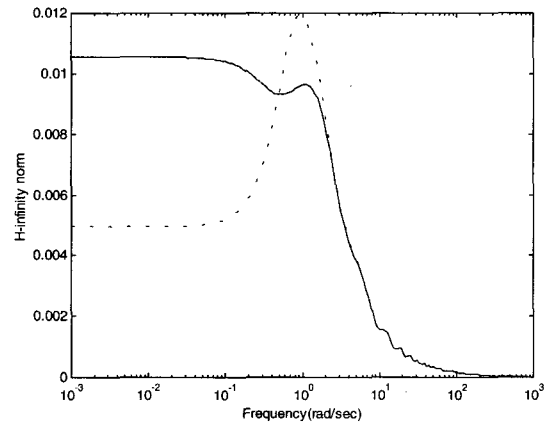

Fig.1 H_∞ norm of model reduction error

VI. Concluding Remark

In this paper, we have studied a balanced model reduction problem for T-S fuzzy systems with time varying delay. For this purpose, we have defined generalized controllability/ observability gramians for the fuzzy system with delay. This generalized gramians can be obtained from solutions of LMI problem. Using the generalized gramians, we have derived a balanced state space realization. We have obtained the reduced model of the fuzzy system by truncating some state variables and also suggested an upper bound of model reduction error. In order to get a less conservative reduction error bound, we presented an iterative sub-optimization procedure for non-convex optimization. Finally, a numerical example is presented to demonstrate the efficacy of our method.

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