Optimum Convolutional Error Correction Codes for FQPSK-B Signals

Hyung Chul Park* *Regular Member*

ABSTRACT

The optimum convolutional error correction codes for recently standardized Feher-patented quadrature phase-shift keying (FQPSK-B) modulation are proposed. We utilize the continuous phase modulation characteristics of FQPSK-B signals for calculating the minimum Euclidean distance of convolutional coded FQPSK-B signal.

It is shown that the Euclidean distance between two FQPSK-B signals is proportional to the Hamming distance between two binary data sequence. Utilizing this characteristic, we show that the convolutional codes with optimum free Hamming distance is the optimum convolutional codes for FQPSK-B signals.

Key words: Feher patented QPSK or FQPSK, Continuous phase modulation, Euclidean distance, Convolutional code.

I. Introduction

In wireless communication, the channel effects, e.g., noise, fading, Doppler shift, and so forth, cause the performance degradation. The error correction code (ECC) is an attractive method for performance improvement. ECC increases the distance of the encoded signals such that the performance is improved.

In this paper, we propose the optimum convolutional error correction codes for FQPSK-B modulation with minimum Euclidean distance analysis. Multi-year studies by the US Department of Defense (DoD), NASA, AIAA and the Consultative Committee for Space Data Systems (CCSDS) have confirmed that FQPSK technologies [1-5] offer the most spectrally efficient and robust (i.e. smallest degradation from ideal theory) bit error rate (BER) performance of non-linear amplification (NLA)-RF power efficient systems. Based on numerous FQPSK airplane-to-ground, ground-to-ground and satellite tests in the 1Mb/s to 600Mb/s range, the FQPSK has been specified in the new spectrally efficient telemetry standard known as IRIG 106-01 [6] and recommended by CCSDS for use in high speed space communication systems.

For the FQPSK-B modulation, some simulation results have been shown with ECC [7],[8]. However, it wasn't proved whether these ECC are optimum for the FQPSK-B modulation.

In next section, FQPSK-B modulation is reviewed. In Section III, we interpret the FQPSK-B signal as a continuous phase modulation (CPM). The CPM characteristics of the FQPSK-B signal is utilized to calculate the minimum normalized squared Euclidean distance (MNSED) of FQPSK-B signal and derive the optimum convolutional codes for the FQPSK-B signal. The MNSED of FQPSK-B signal is analyzed in Section IV. In Section V, we derive the optimum convolutional codes for the

*SoC team, Hynix Semiconductor Inc., Kang-nam, Seoul. (chori@dimple.kaist.ac.kr)
논문번호 : 040108-0308, 접수일자 : 2004년 3월 9일

FQPSK-B modulation and show that they are equal to the convolutional codes with optimum free Hamming distance. In addition, the BER performances of convolutional coded FQPSK-B signal are shown. Finally, Section VI presents the conclusion.

II. FQPSK-B Modulation

FQPSK modulation is a kind of inter-symbol interference (ISI) and jitter-free OQPSK modulation schemes, which is composed of cross correlator and LPF as shown in Fig. 1(a) [1-5]. In FQPSK-B modulation, modulated signal with inter-symbol interference and jitter-free (IJF) signal encoder, which is designated as FQPSK-1 [3], is written as

$$s_{FQPSK-1}(t) = I_{FQPSK-1}(t)\cos(\omega_c t) + Q_{FQPSK-1}(t)\sin(\omega_c t) \quad for\ nT_s < t < (n+1)T_s \qquad (1)$$

where

$$I_{FQPSK-1}(t) = \sum_{k=0}^{n} x(k)\cdot p(t-k\cdot T_s),$$

$$Q_{FQPSK-1}(t) = \sum_{k=0}^{n} y(k)\cdot p(t-k\cdot T_s - T_s/2),$$

$$p(t) = \begin{cases} \frac{1}{2}\left(1+\cos\left(\frac{\pi t}{T_s}\right)\right), & |t| \le T_s \\ 0, & elsewhere \end{cases}$$

where x(k), y(k) denotes the binary data, ±1. $T_s (= 2T_b)$ denotes the symbol period.

Fig. 1(b) shows the eye-diagram of FQPSK-1 signals. FQPSK-1 signal has a 3dB envelope fluctuation, as shown in Fig. 1(c), leading to causing spectrum regrowth after passing through a non-linear amplifier. However, cross correlator in FQPSK-B modulation reduces the 3dB envelope fluctuation to near 0dB with amplitude parameter A[1)] of $1/\sqrt{2}$, which is designated as FQPSK-KF

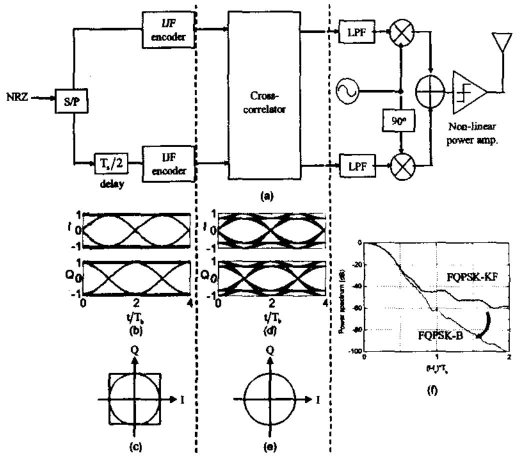

Fig. 1. FQPSK-1, FQPSK-KF and FQPSK-B modulations.

[4]. Eye-diagram and signal constellation of FQPSK-KF signal are shown in Fig. 1(d),(e). Therefore, it allows us to use non-linear amplifier, such as class-C amplifier, without much spectrum regrowth, leading to higher power efficiency.

In order to improve further the spectrum efficiency, a LPF is used at the FQPSK-KF signal, which is designated as FQPSK-B. The spectral sidelobes of FQPSK-B signal are significantly lower than that of FQPSK-KF signal due to the low pass filtering sidelobes, which is shown in Fig. 1(f). It is interesting to note that the analog low pass filtering does not change the ISI/IJF and quasi-constant amplitude characteristics of FQPSK-KF signal.

III. CPM interpretation of the FQPSK-B signal

In general, CPM has following characteristics. First, the CPM signal has a constant envelope. Second, thus, the information is stored in the phase that changes continuously. Since FQPSK-B is a quasi-constant envelope modulation, its phasor

1) Amplitude parameter A is related to the ratio of maximum and minimum amplitude of FQPSK-KF modulated signal, i.e., $\sqrt{2}$ A. When the amplitude parameter A is equal to $1/\sqrt{2}$, the FQPSK-KF signal has quasi-constant envelope (see [4]).

moves along the unit circle in the signal constellation [4],[5]. Since the I and Q signals in the FQPSK-B must be highly correlated to obtain quasi-constant envelope, let us start to look at three consecutive binary data bits, i.e., two consecutive Q data bits with I data bit in between, and vice versa.

3.1 FQPSK-B Modulation with Three Consecutive Binary Data Bits

Let us show that three consecutive binary data bits determine phase point uniquely at the middle of the symbol period. Fig. 2 lists all possible phase points at $t=2nT_b$ for three consecutive binary data bits (Q_{n-1}, I_n, Q_n), which are generated according to the waveform shaping rule for the FQPSK-B signal. When two consecutive Q channels data bits do not change, either $1/\sqrt{2}$ or $-1/\sqrt{2}$ is assigned to their modulated signal at the middle of the symbol period ($t=2nT_b$), depending on whether they are 1 or 0. And, $1/\sqrt{2}$ or $-1/\sqrt{2}$ is assigned to the modulated signal in I channel depending on its data, making

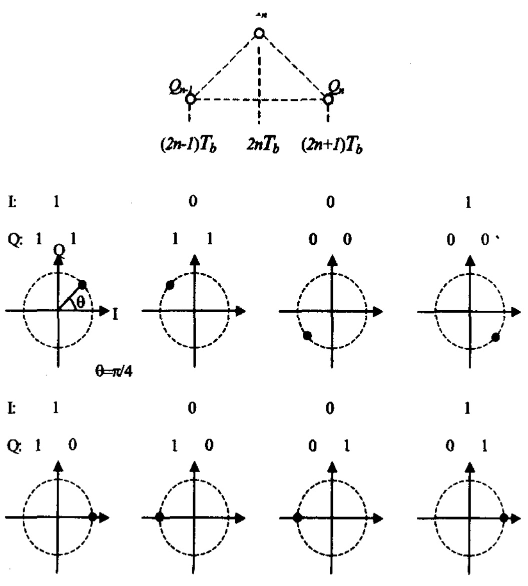

Fig. 2. Allowed phase points (solid dots) of FQPSK-B signal at $t=2nT_b$ for three consecutive binary data bits (Q_{n-1}, I_n, Q_n).

the amplitude given by $\sqrt{I^2(t)+Q^2(t)}$ of the modulated signal to be 1. When two consecutive Q channels data bits change values from 1 to 0 or vice versa, 0 is assigned to the modulated signal in Q channel at the middle of symbol period ($t=2nT_b$) and 1 or -1 is assigned to the modulated signal in I channel depending on its data, making the amplitude of the modulated signal equal to 1 again. Data bits (I_n, Q_n, I_{n+1}) allows a similar set of phase points at $t=(2n+1)T_b$. The only difference is that (I_n, Q_n, I_{n+1}) allows $+\pi/2$, $-\pi/2$ while (Q_{n-1}, I_n, Q_n) allows 0, π. Therefore, three consecutive binary data bits determine 8 allowed phase points at $t=nT_b$ given by $\phi(nT_b)=k(\pi/4),(k=0,\ldots,7)$.

3.2 FQPSK-B Modulation with Four Consecutive Binary Data Bits

For four consecutive binary data bits (Q_{n-1}, I_n, Q_n, I_{n+1}), we can determine two phase points at $t=2nT_b$ and $t=(2n+1)T_b$, respectively, and a single-phase transition path between them. *Single-phase transition* is written as

$$\Delta\phi_{2n}=\phi((2n+1)\cdot T_b)-\phi(2n\cdot T_b),\qquad n=0,1,2,\cdots \tag{2}$$

We can see that the FQPSK-B signal can only have 5 single-phase transition values, which are given as $\pm\pi/2,\pm\pi/4$ and 0. We can find that the phase of the FQPSK-B signal changes continuously from $\phi(2nT_b)$ to $\phi((2n+1)T_b)$.

3.3 FQPSK-B Modulation with Five Consecutive Binary Data Bits

Extending above interpretation, we see that five consecutive binary data bits (Q_{n-1}, I_n, Q_n, I_{n+1}, Q_{n+1}) determine three phase points at symbol time, $t=2nT_b$, $(2n+1)T_b$ and $2(n+1)T_b$, and two-phase transitions. The two-phase transitions are composed of two consecutive single-phase transitions at two consecutive T_b intervals, i.e., $\Delta\phi_{2n}$ and $\Delta\phi_{2n+1}$.

Table 1 shows all allowed two-phase transitions. It is very important to note that the number of allowed two-phase transitions is equal to 15, which is much less than the 25 that is obtained from random combination with 5 allowed single-phase transitions. This means that the FQPSK-B modulation has memory [9]. The aforementioned interpretations of the FQPSK-B signal lead us to interpret FQPSK-B as a kind of CPM.

Table 1. Allowed two-phase transitions.

	$\Delta\Phi_{2n}$	$\Delta\Phi_{2n+1}$		$\Delta\Phi_{2n}$	$\Delta\Phi_{2n+1}$
1	-π/2	-π/2	9	0	+π/4
2	-π/2	-π/4	10	+π/4	-π/4
3	-π/4	-π/2	11	+π/4	0
4	-π/4	-π/4	12	+π/4	+π/4
5	-π/4	0	13	+π/4	+π/2
6	-π/4	+π/4	14	+π/2	+π/4
7	0	+π/4	15	+π/2	+π/2
8	0	0			

IV. Minimum Euclidean Distance of the FQPSK-B Signals

It is well known that the probability of error $P(\varepsilon)$ can be bounded by the union bound [10]

$$P(\varepsilon)=\frac{1}{N}\sum_{i=0}^{N-1}P(\varepsilon|m_i)\le\frac{1}{N}\sum_{i=0}^{N-1}\sum_{\substack{j=0\\(j\neq i)}}^{N-1}Q\left(\sqrt{d_{ij}^2\frac{E_b}{N_0}}\right) \tag{3}$$

where N denotes the total number of signal alternatives or messages, $m_0, m_1, \cdots, m_{N-1}$ and Q(x) denotes $1/2\cdot erfc\left(x/\sqrt{2}\right)$. d_{ij}^2 is the squared Euclidean distance in signal space between m_i and m_j normalized by $2E_b$, which is written as [11-12]

$$d_{ij}^2=\frac{1}{2E_b}\int_0^{\infty}\left(m_i-m_j\right)^2 dt \tag{4}$$

When E_b/N_0 is large, the union bound is dominated by one term such that it is written as

$$P(\varepsilon)\approx C\cdot Q\left(\sqrt{d_{\min}^2\frac{E_b}{N_0}}\right) \tag{5}$$

where C is a constant independent E_b/N_0. $d_{\min}^2$ denotes MNSED, which is written as

$$d_{\min}^2=\min_{m_i,m_j}d_{ij}^2 \tag{6}$$

In order to calculate the MNSED of FQPSK-B signal, it is necessary to consider the transient phase characteristics of FQPSK-B signal. Utilizing the CPM characteristics of FQPSK-B signal, we can find that an input data q_n affects three phase points $\phi_{n-1}, \phi_n, \phi_{n+1}$, which is shown in Fig. 3. The change of three phase points affects the phase transition during $4T_b$. That is, an input data q_n affects the FQPSK-B signal during $4T_b$. For the calculation of MNSED of FQPSK-B signal, it is needed to calculate the distance between two FQPSK-B signals, which have 7 identical input binary data except middle data q_n during $4T_b$. Hence, MNSED of FQPSK-B signal is written as

$$d_{\min}^2=\min_{q_\alpha,q_\beta}\frac{1}{2E_b}\int_0^{4T_b}\left(s(t,\vec{\alpha})-s(t,\vec{\beta})\right)^2 dt \tag{7}$$

where $q_{\vec{\alpha}}$ is equal to $\{q_{n-3}, q_{n-2}, q_{n-1}, +1, q_{n+1}, q_{n+2}, q_{n+3}\}$ and $q_{\vec{\beta}}$ is equal to $\{q_{n-3}, q_{n-2}, q_{n-1}, -1, q_{n+1}, q_{n+2}, q_{n+3}\}$.

Note that the calculated MNSED $d_{\min}^2$ is equal to 1.53.

V. Optimum Convolutional Codes for the FQPSK-B Signal

The distance between the modulated signals is equal to the distance in the signal space, which is proportional to the phase difference for constant amplitude modulation including FQPSK-B. So, the distance increases as the phase difference between two FQPSK-B signals is close to 180°.

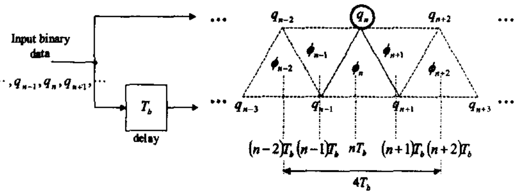

Fig. 3. Phase transition characteristics of FQPSK-B signal.

And, the distance decreases as the phase difference is close to 0°. Hence, if a convolutional code increases the phase difference between two coded FQPSK-B signals, $d^2_{\min}$ is increased such that the BER performance can be improved.

In order to design optimum convolutional codes for the FQPSK-B signal, it is necessary to consider the relationship between the phase difference and Hamming distance. Fig. 4 shows the some cases of phase differences of the FQPSK-B signal and corresponding three consecutive input binary data pair.

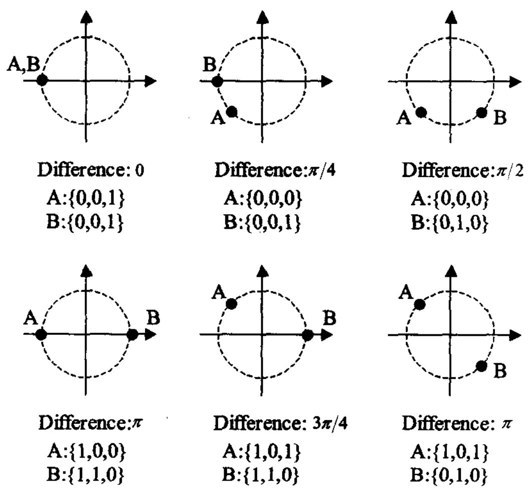

Fig. 4. Examples of phase difference between two phase points of FQPSK-B signal determined by three consecutive binary data {I,Q,I}.

From the total 64 cases of the pair of three consecutive input binary data, we can find that the phase difference is proportional to the Hamming distance between two data sequence. Since the three consecutive binary data is used, the allowed Hamming distance is 0, 1, 2 and 3. Table 2 shows the possible phase difference and occurrence probability for each Hamming distance. It shows that the average phase difference is proportional to the Hamming distance between two three-consecutive binary data.

Table 2. Possible phase differences of FQPSK-B signal and occurrence probabilities for Hamming distance between two three-consecutive binary data.

		Hamming distance between two three-consecutive binary data			
		0	1	2	3
Possible phase difference in FQPSK-B signal	0	100%	0	16.7%	0
	π/4	0	66.6%	0	0
	π/2	0	16.7%	16.7%	0
	3π/4	0	0	66.6%	0
	π	0	16.7%	0	100%
Average phase difference		0	5π/12	7π/12	π

It notes that the convolutional codes with optimum free Hamming distance are the optimum convolutional codes for FQPSK-B signals. Table 3 shows the MNSED of coded FQPSK-B signal with optimum convolutional codes for FQPSK-B modulation, which have rate 1/2 and memory of size 1, 2, 3, 4, 6 and 8. Note that the convolutional codes with optimum free Hamming distance, in which the convolutional code has a larger memory or different code rate, are also optimal for FQPSK-B modulation.

Table 3. Optimal rate 1/2 convolutional codes for FQPSK-B modulation.

Memory size v	d_{free}	Generating polynomials	d^2_{min} of coded FQPSK-B signals
1	3	(1,3), (2,3)	3
2	5	(5,7)	4.65
3	6	(7,13), (7,15), (13,16), (15,16), (13,15), (13,17), (15,17)	4.9
4	7	(23,35)	5.8
6	10	(133,171)	9.1
8	12	(561,753)	9.8

Fig. 5 shows the simulated BER performance of coded FQPSK-B modulation in the AWGN and NLA environments. The convolutional codes with v=1 and 3 have the 2.2dB and 4.5dB, respectively, improvement for a BER of 10^{-4} from the optimum coherent detection of the uncoded FQPSK-B signal.

VI. Conclusions

In this paper, optimum convolutional error correction codes for FQPSK-B signal have been proposed. Utilizing the CPM characteristics of FQPSK-B signal, MNSED of the FQPSK-B signal has been calculated. It has been shown that the phase difference between two FQPSK-B signals is proportional to the Hamming distance between two data sequence. Using the above characteristic, it has been shown that the convolutional codes with optimum free Hamming distance are the optimum convolutional codes for the FQPSK-B signal.

Acknowledgement

The author would like to thank Prof. Kwyro Lee of Korea Advanced Institute of Science and Technology (KAIST) for the helpful discussion and encouragement.

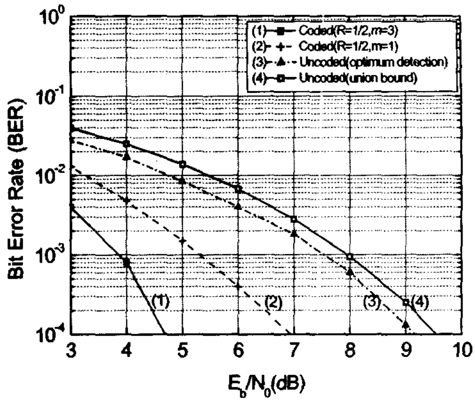

Fig. 5. BER performances of convolutional coded FQPSK-B modulation with Viterbi detection.

References

[1] K. Feher et al., U.S. patents: 4,567,602; 4,644,565; 5,491,457; 5,784,402; 6,198,777, International Patent Cooperation Treaty (PCT) No: PCT US99/17995; International No: WO 00/10272, published Feb. 2000.

[2] K. Feher, *Wireless Digital Communications*. New Jersey: Prentice-Hall, 1995.

[3] T. Le-Ngoc, K. Feher, and H. Pham Van, "New Modulation Techniques for Low-Cost power and Bandwidth Efficient Satellite Earth Stations," *IEEE Trans. Commun.*, vol. COM-30, pp.. 275-283, Jan. 1982

[4] S. Kato and K. Feher, "XPSK: A New Cross-Correlated Phase-Shift Keying Modu- lation Technique," *IEEE Trans. Commun.*, vol. COM-31, pp. 701-707, May 1983.

[5] K. Feher, "Spectrally Efficiency of Commercial Wireless and Telemetry Systems Is Doubled with IRIG 106-00 Standardized FQPSK and Is Quadrupled with FQAM," *Proc. of ITC 2000*, pp. 225~237, Oct. 2000.

[6] "IRIG Standard 106-01: Telemetry Standards", Telemetry Group, Range Commanders Council, U.S. Army, White Sands, NM, USA, Feb. 2001. Detailed information available from : http:// jcs.mil/RCC/mauals/106-01/index.html.

[7] G. Horcher, "Design Considerations for Deve- lopment of an Airborne FQPSK Transmitter," *Proc. of ITC 1999*, Oct. 1999.

[8] J. Lin and K. Feher, "Bandwidth Efficiency and BER Performance of Enhanced and FEC Coded FQPSK," *Proc. of ITC 2000*, pp. 599-608, Oct. 2000.

[9] M. K. Simon and T.-Y. Yan, "Performance Evaluation and Interpretation of Unfiltered Feher-patented Quadrature Phase Shift Keying (FQPSK)," *JPL TMO Progress Report* No. 42-137, May 1999.

[10] J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering*. New York: Wiley, 1965, pp. 264-266.

[11] G. Lindell, C. E. Sundberg, and T. Aulin, "Minimum Euclidean Distance for Combinations of Short Rate 1/2 Convolutional Codes and CPFSK Modulation," *IEEE Trans. Inform. Theory*, vol. IT-30, no. 3, pp. 509-519, May 1984.

[12] T. Aulin and C. E. Sundberg, "Continuous Phase Modulation-Part I: Full Response Signaling," *IEEE Trans. Commun.*, vol. COM-29, no. 3, pp. 196-209, Mar. 1981.

Hyung Chul Park

Feb. 1996: B.S. degree in electrical engineering from Korea Advanced Institute of Science and Technology (KAIST).

Feb. 1998: M.S. degree in electrical engineering from KAIST.

Feb. 2003: Ph.D. degree in electrical engineering and computer science from KAIST.

Feb. 2003 ~ : Hynix Semiconductor Inc. He is a Manager in SoC team.

Research interests: wireless modulation/demodulation algorithm, system design/implementation, interface study between RF/IF stage and digital signal processing part.

He is a member of IEEE, KICS and IEEK.