

## 시변 시간지연 함수를 위한 시뮬레이션 객체의 구성

최 순 만†

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### Configuration of Simulation Object for Time Varying Time Delay Functions

Soon-Man Choi†

**Abstract** : Time delays are included in most of actual systems, and some of which are shown as time varying. To analyze the time varying time delay system in the time domain, a useful delay module to express the function as a tool is much helpful to get corresponding outputs under given conditions. A method is proposed here to design the algorithm of time delay module for simulation or control purposes, including the problems of initializing and reallocating data in buffer. After classifying the time varying time delay into the distributed mode and lumped mode, an object to describe delay module is configured and tested under the defined input signal and given time delay variation. The simulation results show that the output of module matches reasonably with the case of real system.

**Key words** : Time varying time delay, Object, Distributed mode, Lumped mode

#### 1. Introduction

It is inevitable that the transmission of information or material between different parts or places includes time delay. Piping systems, transportation systems, communication systems and power systems are typical examples of time-varying time-delay systems. One of those cases is easily experienced when the length of a pipe line is abruptly changed

due to the operation of line valves or the flow speed of fluid in pipes is varied.

Time-delay in system causes serious deterioration on stability and performance of the system, so considerable research has been devoted to control problems with time-delay systems or time-varying time-delay (TVTD) systems<sup>(1)-(5)</sup>. For example, the smith controller among them tried to solve the stability problem with time delay by including time delay

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blocks in itself<sup>(6)</sup>. But in case of TVTD, the modeling of time delay block is not easy to be generalized on the time space because lots of application conditions should be considered case by case.

To manage the expression of TVTD block, this paper classifies the cases of TVTD into 3 modes such as the distributed delay mode, the lumped delay mode on output side and the lumped delay mode on input side, based on their dynamic properties.

And an algorithm for modelling TVTD as an object is suggested according to the given 3 modes including the matters of the data initialization and the data reallocation in memory buffer. To confirm the function effectiveness of this TVTD module, the results of simulation curves are illustrated.

## 2. Modelling TVTD

### 2.1 Model block

Consider the systems with time delays which are time-varying in the states and control inputs.

$$\dot{\mathbf{x}}(t) = \mathbf{A}_0\mathbf{x}(t) + \mathbf{A}_1\mathbf{x}(t - \mathbf{L}_x(t)) + \mathbf{B}_0\mathbf{u}(t) + \mathbf{B}_1\mathbf{u}(t - \mathbf{L}_u(t)) \quad (1)$$

$$\mathbf{y}(t) = \mathbf{C}_0\mathbf{x}(t) + \mathbf{C}_1\mathbf{x}(t - \mathbf{L}_x(t)) + \mathbf{D}_0\mathbf{u}(t) + \mathbf{D}_1\mathbf{u}(t - \mathbf{L}_u(t)) \quad (2)$$

where the  $n$ -dimensional vector  $\mathbf{x}$  represents the state, the  $m$ -dimensional vector  $\mathbf{u}$  is the control input. The matrices  $\mathbf{A}_0$ ,  $\mathbf{A}_1$ ,  $\mathbf{B}_0$ ,  $\mathbf{B}_1$ ,  $\mathbf{C}$  and  $\mathbf{D}$  are constant matrices of appropriate dimensions.  $\mathbf{L}_x$  and  $\mathbf{L}_u$  are the time delay matrices for the state and the control

input respectively.

We can show the initial values as Eq. (3) within time delay period for a time delayed input, and Eq.(4) is for a delayed state.

$$u(\sigma_u) = \phi_u(\sigma_u), \quad \sigma_u \in [-L_u(\tau), \tau], \quad 0 \leq \tau \leq t \quad (3)$$

$$x(\sigma_x) = \phi_x(\sigma_x), \quad \sigma_x \in [-L_x(\tau), \tau], \quad 0 \leq \tau \leq t \quad (4)$$

where,  $\phi_u$  and  $\phi_x$  are the initialization functions which decides the output values during the time delay period. And from the properties, these functions need to be initialized again and be able to change the possible output value whenever the given time delay is changed. Fig.1 and Fig.2 show the relations of input, output, given time delay and initializing function at the block of time varying time delay.

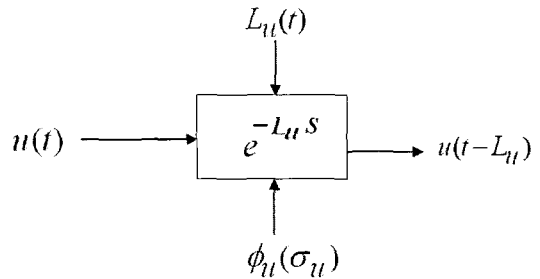


Fig. 1 Time delayed input by  $L_u(t)$

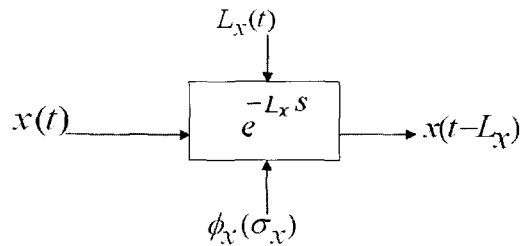


Fig. 2 State delayed block by  $L_x(t)$

If the time delays of state and control

input are independent each other, both time-varying delays can exist at the same time. And Eq. (3) and Eq. (4) include the expression of the initial values at  $-L \leq \sigma \leq 0$  and  $t-L \leq \sigma \leq t$  like illustrated at Fig. 3 and Fig. 4. From the figures,  $L_u(0)$  and  $L_x(0)$  are static values, but  $L_u(t)$  and  $L_x(t)$  are variable values, being changed dynamically according to the system condition.

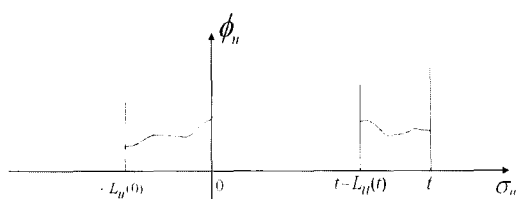


Fig. 3 Initializing function by  $L_u$

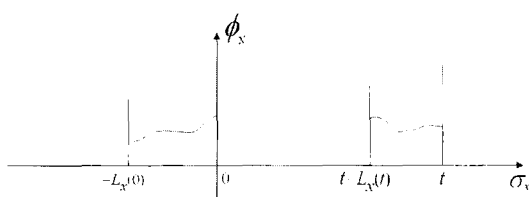


Fig. 4 Initializing function by  $L_x$

Here, whether the functions of  $\phi_u$  and  $\phi_x$  be time varying or not is decided by the system conditions. Due to the role of  $\phi(\phi_x$  or  $\phi_u)$ , every variation of the value  $L(t)$  or the set  $\sigma$  shall make the function  $\phi$  activate again for compensating renewed time delay within TVTD block.

### 2.2 Conditions of object

If the block for TVTD is configured by an object oriented program, it needs to include initialization functions and also data buffers enough to contain the

designed maximum time delay. In case the block is given a new value of time delay, it should re-allocate the data in memory buffer before the next sampling period begins. And how to transform or allocate the data is only dependent on the property of given process or system.

From the consideration of Fig. 1 - Fig. 4, TVTD object can be used as time delay blocks for both control inputs and control states if it is designed to have much flexibility. Also based on the delay properties, the initializing function  $\phi$  in TVTD object can be grouped into two kinds of modes, namely, the distributed delay mode and the lumped delay mode.

## 3. Modes of TVTD

### 3.1 Distributed mode

An example of distributed mode time delay is the case where the transfer delay is varied if the flow rate is adjusted up or down in pumping system. In this mode, when the time delay value is abruptly changed at  $\sigma_x=t$ , all the data in the memory buffer in TVTD block are affected uniformly by it.

Any filling default values as initial data or removing existing data from memory buffer is not necessary during this mode. And the data in buffer need to be stored by the data type which includes two floating numbers to be able to describe a value with the time.

In distributed mode, renewing the time delay value causes that the time factor of all data in buffer are multiplied by the ratio which is decided between the last time delay and updated time delay. If

input signal of triangle function was entered to a time delay block with time delay  $L_x(t)$  and the time delay of the block is increased abruptly at time  $t$ , the stored data in the block are transformed like the Fig.5, changing  $L_x(t)$  to the greater value of  $L'_x(t)$  if the velocity of flow is lowered in case of pumping system. The value of  $L'_x(t)/L_x(t)$  corresponds to the ratio of the latest flow velocity to the preceding when the pumping rate changes. In Fig. 5, we see that there is not any sharp discontinuity within the

interval by the variation of time delay.

### 3.2 Lumped mode

Lumped mode of time delay can occur in pumping or piping system when any length of its pipe line is added up or removed from the existing pipe line by valve operation.

In this mode, when the time delay value is increased abruptly at  $\sigma_x = t$ , filling up buffer with any initializing data is needed for the additional time interval. And all the data in the memory buffer

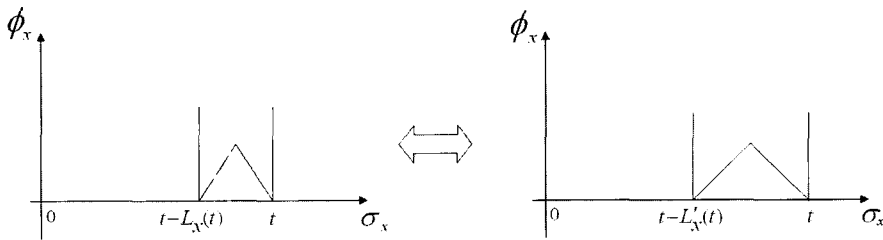


Fig. 5 Transformation of existing values (distributed mode)

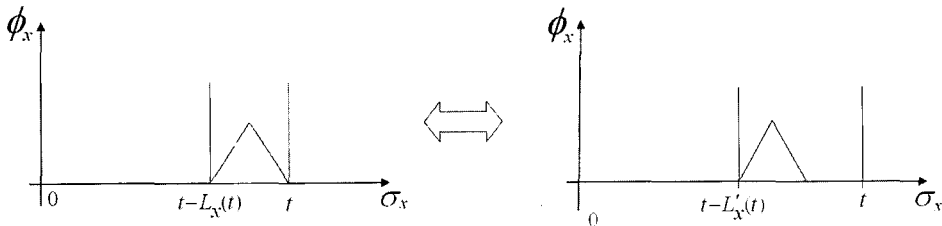


Fig. 6 Transformation of data in buffer (lumped delay on output side)

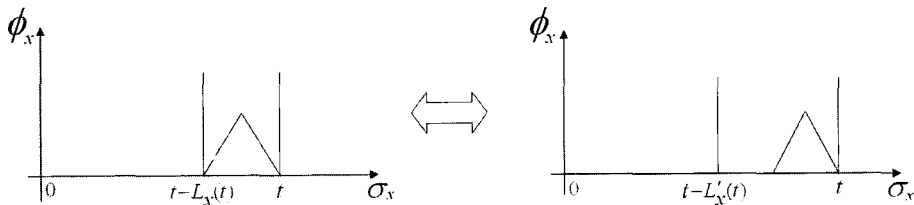


Fig. 7 Transformation of data in buffer (lumped delay on input side)

should be shifted as much as the difference of time delay. According to the modelled plant, inserting the additional pipe line can occur on any position of the pipe system. Therefore, filling-up initial data for the newly inserted pipe line in memory buffer shall depend on the position of actual pipe line. Fig. 6 shows the case that a pipe line is inserted newly just before the output side of time delay block. And Fig. 7 is the

case inserted in the input side.

#### 4. Configuration of TVTD block

Fig. 8 shows the concept to configure a general object to be able to execute the necessary functions as time-varying time delay. For the initializing functions to respond dynamically to time varying condition, an analyzer has been adopted in the model.

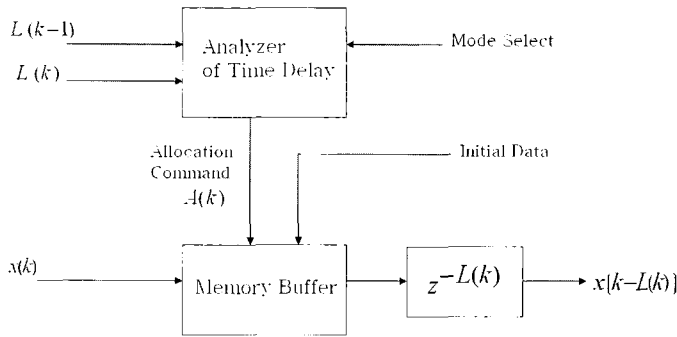


Fig. 8 Configuration of variable time delay object

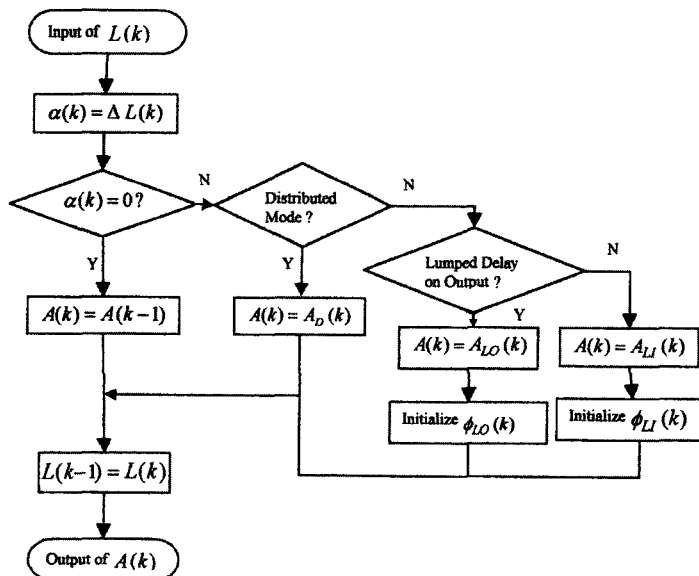
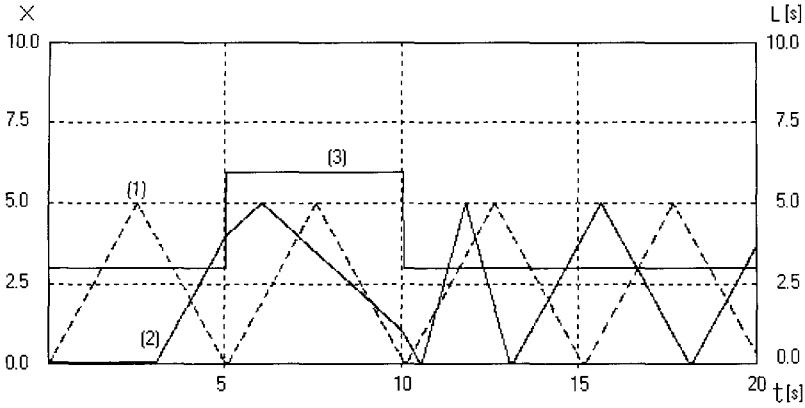
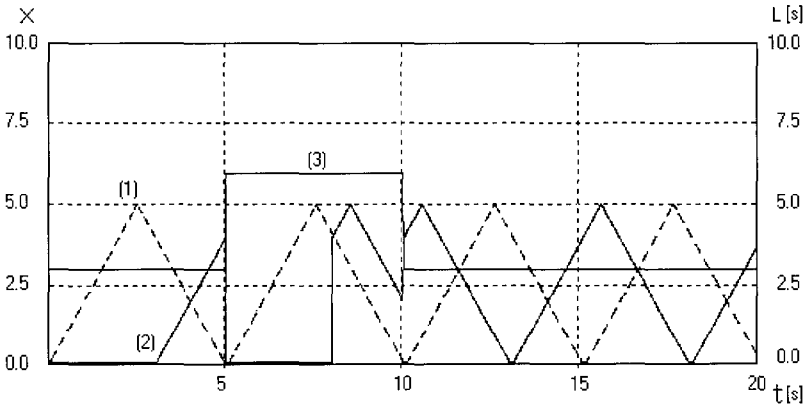


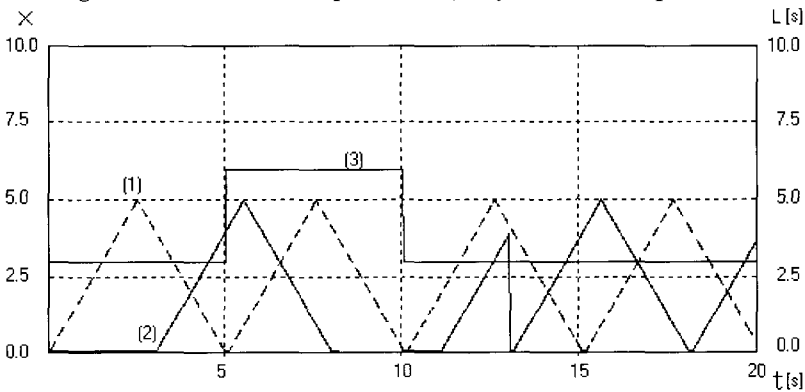
Fig. 9 Procedures in module according to time-delay mode



**Fig. 10 The result of distributed delay mode**



**Fig. 11 The result of lumped mode (delayed on the output side)**



**Fig. 12 The result of lumped mode (delayed on the input side)**

The block of analyzer decides how to re-allocate the data in buffer every sampling period after checking the time delays  $L(k)$  and  $L(k-1)$ . By the allocation

command of analyzer, the data in buffer are shifted or transformed based on the selected mode which is set in advance. Fig. 9 shows the detailed procedure of

time delay block, where  $A(k)$  indicates the instance of allocation function at the  $k$ th sampling. And the expressions of  $A_D$ ,  $A_{LO}$  and  $A_{LI}$  are given for the allocation case of distributed mode, lumped mode(at output side) and lumped mode(at input side) respectively.

## 5. Simulation results

To confirm the functions of proposed delay object, a series of triangle input (a saw wave with period of 5 seconds) is applied to the input of the object like the curve (1) in the Fig. 10. The result of Fig. 10 is the case of distributed delay mode.

And the curve (3) of it shows the variation of the time delay which begins with the value of 3.0[s], abruptly increased to the value of 6.0[s] at time 5.0[s] and returned to the original value of 3.0[s] again at time 10.0[s]. The curve (2) is the output of the object which comes out of the module through the given time delay at time  $t$ . From the relation of 3 curves, we can see that the slope of the output is changed to smaller one at time 5.0[s], becomes greater at time 10.0[s] and restored again to origin at time 13.0[s].

Fig. 11 is the result of the lumped delay mode where the delay is applied on the output side of the object. This case shows that the output is sharply cut to the level of initial value 0 between 5.0[s] and 8.0[s]. And from 3.0[s] to 10.0[s], a partial of triangle is revealed, but without the change of the slope. After 10.0[s], the output is restored to the normal time delay state.

Fig. 12 is the result of the lumped delay mode where the delay is applied on the input side of the object. The output becomes zero abruptly as initial value at time 8.0[s]. The default value 0 as initial value is kept from 8.0[s] to 11.0[s]. Then the curve shows a partial of triangle at time between 11.0[s] and 13.0[s]. And after 13.0[s], the output is restored to the normal time delay state.

## 6. Conclusions

In this paper, TVTD was grouped into distributed mode and lumped mode based on the delay properties, and the lumped mode divided again into the cases of input delayed and output delayed for the given memory buffer.

Also a generalized module to manage these modes of TVTD was proposed and designed with reasonable algorithm including the initialization functions. This block showed that it re-allocated and transformed data of time delay buffer reasonably whenever the time delay is changed. Follows are the results of simulation with defined conditions given.

1. At distributed mode, the insertion of initial values is not necessary even when the time delay is lengthened. And the output on curves revealed the reasonable continuity without any sharp cut.
2. At lumped mode, the insertion of initial values is to be given when the time delay is lengthened. The output of proposed module was reasonable and the discontinuity was confirmed timely at these lumped modes.
3. The output of given initial values at

abrupt change of time delay was reasonable and followed well the case of real system.

4. The overall functions of proposed TVTD block were well matched with the design concept and revealed it is available to the application.

### References

- [1] Akira Kojima., Kenko Uchida Etsujiro Shimemura., Robust stabilization problem for a system with delays in control. *SICE*. Vol. 29, No.3, pp.315-325, 1993.
- [2] Shafiee Z. and Shenton AT., Fequency-domain design of PID controllers for stable and unstable systems with time delay . *Automatica* 33(12), pp.2223-2232, 1997.
- [3] Jong-Hae Kim., Eun-Tae Jeung. and Hong-Bae Park., Robust Control for Parameter Uncertain Delay Systems in State and Control Input. *Automatica*, Vol 32, No. 9, pp.1337-1339, 1996.
- [4] Miller K M., Witt N A J. and Russell M P., Mathematical and Scale Model Platforms for Manoeuvring Trials, *International Symposium on Manoeuvrability of Ships at Low Speed*, Ilawa, Poland, Oct. 1995.
- [5] H H Choi. and M J Chung., Comments on quadratic stabilization of continuous time systems with state-delay and norm-bounded time-varying uncertainties, *IEEE Transactions on Automatic Control*, Vol 42, No. 12, pp. 1740-1742, 1997.
- [6] O.J.M Smith., A controller to overcome dead time , *ISA. J.*, Vol. 6, No.2, pp.28-33, 1959.

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