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Mathematical Modeling of Zone Drawing Process

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Abstract: To provide guidelines and a basic understanding of static and continuous zone drawing processes, we propose two different mathematical models in terms of the processing conditions and material parameters. Although the models are not finely tuned, because of assumptions made, they are still useful for the analysis of the process and for predicting the processibility.

Keywords: mathematical model, zone drawing.

Introduction

Zone drawing process has been studied as a method to impart superior mechanical properties to fibers or films produced from polymers. When fiber or film has been heated locally by heat band under applied load, the heated part of the specimen will be stretched. With the extension, the degree of polymer chain orientation in the heated zone will be increased depending on the processing condition and material properties. When the polymer chain is flexible and the heating time is long enough to transfer heat from the band to the material, the chain will be fully extended. As a result, the polymer obtains high strength and high modulus.

To reveal the relations between the processing conditions and the polymer structural change, zone drawing was applied to various polymers such as polyolefins, nylon, polyester, PVA and so on.[1-14] The researches found that degree of orientation in amorphous region changed significantly during the process. These structure changes result in the zone-drawn polymers. Although these researches showed that zone drawing is one of the most effective methods to create high performance polymers, the process has not been employed in fiber manufacturing due to low speed and batch type process. Recently, continuous zone drawing process has been developed and researched focused on the processibility, polymer structural change and the mechanical properties of the products.[15,16]

However, the process mechanism has not been studied theoretically in terms of material properties and the processing conditions. In this study, we establish theoretical models for static and continuous zone drawing processes using heat transfer analysis and constitutive models for polymers. Although the models are not fine tuned, they are expected to provide the guideline and basic understanding of the processes. In addition, the models can be used to predict the mechanical properties of the resulted fiber or film by calculating the inherent draw ratio using the process conditions and material parameters.

Models

In this study, we propose two different theoretical models according to zone drawing mechanism; one is for static process and the other is for continuous process. In static process, one end of polymer film or fiber is fixed and constant load is applied at the other end. The heat band moves along the specimen with a constant speed. In continuous process, the specimen moves between two fixed points separated by the length L with increasing velocity from v_0 to v_L. The heat band is located between the two fixed points.

Static Process. The schematic diagram of the static pro-

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cess is shown in Figure 1. Although this process is not suitable for mass production, it is still useful to understand the effect of the processing conditions on the physical properties of the product. In this study, we develop a model using heat transfer analysis and rheological concepts.

To simplify the process, the followings are assumed.

(1) Heat transfer along the specimen is negligible. Only heat transfer is along the thickness direction of the specimen.

(2) Plastic deformation does not occur where the heat band does not contact. Outside of heating zone, deformation is recoverable because the specimen deforms like elastic body. The only area of specimen heated by the heat band is plastically drawn.

From the assumption 1, heat transfer during the process is one-dimensional phenomenon. Thus, we have the solution as follows:

$$T = T_H - 2(T_H - T_0)\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{\beta_n}\exp(-\beta_n^2\tau)\cos\frac{\beta_n x}{b} \quad (1)$$

$$\beta_n = \frac{2n-1}{2}\pi, \text{ and } \tau = \frac{D_H}{b^2}t \quad (2)$$

where T_H is the processing temperature (the temperature of the heat band), T_0 is the temperature of the specimen before heating, T is the temperature of the part of the specimen during heating, b is the half of the thickness of the specimen, D_H is the thermal diffusion constant of the material which can be expressed in terms of the density ρ, the heat capacity C_p, and thermal conductivity of the material as follows:

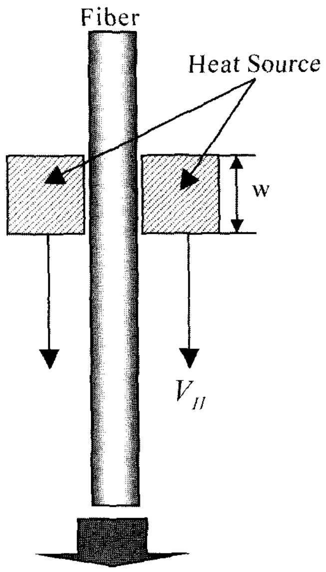

Figure 1. The geometry of static zone-drawing process.

$$D_H = \frac{\kappa}{\rho C_p} \quad (3)$$

Since the heat band moves with constant speed V_H and width of the heat band is w, every part of specimen is heated for heating time t_H, which can be expressed by

$$t_H = \frac{w}{V_H} \quad (4)$$

In order to simplify the problem, we consider the average temperature at $x = 0$ as follows:

$$\bar{T} \equiv \frac{1}{t_H}\int_0^{t_H} T(t, 0)dt \quad (5)$$

It is convenient to consider $\bar{T}$ as a processing variable because diameter of fiber (or thickness of film) is extremely small compared to the specimen length. Substitution of eq. (1) into eq. (5) yields

$$\bar{T} = T_H - \frac{2}{t_H}(T_H - T_0)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\beta_n}f_n \quad (6)$$

where

$$f_n \equiv \int_0^{t_H}\exp\left(-\frac{\beta_n^2 D_H}{b^2}t\right)dt = \frac{b^2}{\beta_n^2 D_H}\left[1 - \exp\left(-\frac{\beta_n^2 D_H}{b^2}t_H\right)\right] \quad (7)$$

The average temperature depends on the heat band speed. The high speed case is not taken into account because the specimen would not be drawn as a result of insufficient heat transfer.

If the heat band speed is slow enough that the specimen can be heated for sufficient time, the average specimen temperature will be equilibrate to the heat band temperature.

$$\bar{T} = T_H - \frac{2b^2\phi}{t_H D_H}(T_H - T_0) \approx T_H \quad (8)$$

where

$$\phi \equiv \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\beta_n^3}$$

It is because

$$\frac{D_H t_H}{b^2} \gg 1$$

When the heating time, t_H is moderately short, then $D_H t_H b^{-2} \approx 1$. As a result, we have

$$f_n \approx \frac{b^2}{\beta_n^2 D_H}\left[\frac{\beta_n^2 D_H}{b^2}t_H - \frac{1}{2}\left(\frac{\beta_n^2 D_H}{b^2}t_H\right)^2\right] = t_H\left(1 - \frac{1}{2}\frac{\beta_n^2 D_H}{b^2}t_H\right) \quad (9)$$

Thus, the average temperature is

$$\bar{T} \approx T_H - 2(T_H - T_0)\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\beta_n}\left(1 - \frac{1}{2}\frac{\beta_n^2 D_H}{b^2}t_H\right) \tag{10}$$

$$= T_0 + (T_H - T_0)t_H\varphi$$

where

$$\varphi \equiv \sum_{n=1}^{\infty}(-1)^{n+1}\frac{\beta_n D_H}{b^2} \tag{11}$$

This case makes the problem more complicate because material parameters such as viscosity and modulus become a function of the heat band speed through the average temperature. Furthermore, we have to use more complicate viscoelastic constitutive equation in this case of moderately short heating time. To make the problem simpler, we only consider only the case where the heating time is long.

The plastic strain rate of the static zone drawing is given by

$$\dot{e}_p = \frac{d\log l}{dt} \tag{12}$$

Because the plastic deformation is occurred only when the heat band contacts the specimen, draw ratio would be expressed as a function of heating time, t_H

$$\log\lambda = \int_0^{t_H}\dot{e}_p(t)dt \tag{13}$$

where λ is draw ratio. If heat transfer is efficient enough that the average specimen temperature $\bar{T}$ is very close to T_H, the following constitutive equation can be used

$$\dot{e}_p = \frac{\sigma}{\eta_E(T_H)} \tag{14}$$

where $\eta_E(T)$ is the elongational viscosity of polymer. The elongational viscosity is nearly independent of strain rate if the polymer does not show any strain hardening. It is notable that temperature dependence of viscosity usually follows the Arhenious equation such that

$$\eta_E(T) = \eta_E(T_{ref})\exp\left(\frac{\Delta E}{RT}\right) \tag{15}$$

Since stress is constant in this process and specimen temperature can be represented in terms of the average temperature $\bar{T}$ during the drawing process, substitution of eq. (14) into eq. (13) yields

$$\log\lambda \approx t_H\frac{\sigma}{\eta_E(T_H)} = \frac{w}{V_H}\frac{\sigma}{\eta_E(T_H)} \tag{16}$$

Although this calculation is based on crude approximations, eq.(16) shows how the process variables such as applied stress, the heat band speed and the processing temperature affect on the draw ratio in a qualitative way. Including correction to the errors due to crude approximation, we can rewrite eq. (16) as follows:

$$\lambda \approx f\left[\frac{\sigma}{a_V(V_H)a_T(T_H)}\right] \tag{17}$$

where $f(x)$ is a function which may be determined by experiment, $a_V(V_H)$ is the shift factor due to the heat band speed, and $a_T(T_H)$ is the shift factor due the processing temperature. From eq. (16), we know that the shift factors have following functional forms:

$$a_V(V_H) = c(V_H - V_H^{ref}) + 1 \tag{18}$$

$$a_T(T_H) = \frac{\eta_E(T_H)}{\eta_E(T_H^{ref})} \tag{19}$$

where c is a constant and V_H^{ref} and T_H^{ref} are, respectively, speed and temperature of the heat band at reference condition. Eq. (17) indicates that all data of draw ratio measured at different process conditions can be put on a single curve when it is plotted against the following shifted stress:

$$x \equiv \frac{\sigma}{a_V(V_H)a_T(T_H)} \tag{20}$$

It is noteworthy that the constant c is independent of materials because it depends on geometry of heat band. On the other hand, the shift factor a_T is a material function of temperature.

Continuous Process. Figure 2 illustrates the schematic diagram of continuous zone-drawing process. In the process, specimen is supplied from the left side rollers with speed of v_0 and exit through the right side rollers with speed of v_L. Before specimen enters the heating zone (zone II), the temperature of the specimen is assumed to be the ambient temperature, T_0. In the zone I, the specimen deforms like elastic solid and the deformation is recoverable. In the zone II, the temperature of the specimen rapidly elevates up to the pro-

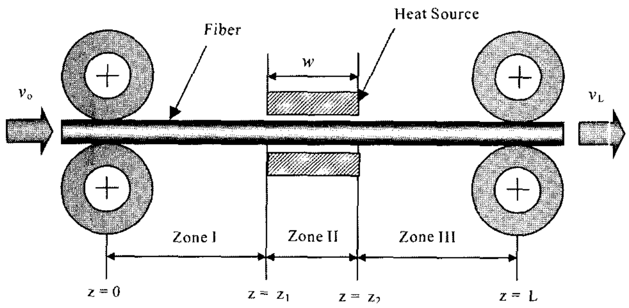

Figure 2. The geometry of continuous zone-drawing process.

cessing temperature T_H, which allows the specimen to deform like Newtonian fluid at isothermal condition. In the zone III, two scenarios are possible. The one is that efficient cooling drops the fiber temperature quickly at constant low temperature. In this case, the fiber deforms like isothermal elastic solid again. The other case is that the specimen is cooled down without any cooling device. In this case, fiber deforms like non-isothermal viscoelastic material. Temperature after the heating zone may be evolved by following equation:

$$\frac{dT}{dt} = h(T_0 - T) \tag{21}$$

where h is a heat transfer coefficient and T_0 is the ambient temperature. The time derivative in eq. (21) is the material time derivative such that

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v \cdot \nabla \tag{22}$$

This research was only focused on the first case that heating and cooling is so immediate that fiber deforms elastically in the outside of the heating zone.

For the analysis of continuous process, we assume that the fiber is cylindrical and axially symmetric. We also assume that velocity field is

$$\mathbf{v} = v_z(z)\boldsymbol{k} \tag{23}$$

and fiber is incompressible. Thus, incompressibility gives the mass balance equation as follows

$$v_z(z)R^2(z) = v_0 R_0^2 \tag{24}$$

where R_0 is the radius of fiber at $z=0$. Since we use cylindrical coordinates, the momentum balance equation for the z-component is

$$\begin{aligned} &\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta \partial v_z}{r\ \partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) \\ &= \frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{zr}) + \frac{1}{r}\frac{\sigma_{z\theta}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} \end{aligned} \tag{25}$$

The assumptions for symmetry and the velocity field lead eq. (25) to

$$\rho v_z \frac{dv_z}{dz} = \frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{zr}) + \frac{\partial \sigma_{zz}}{\partial z} \tag{26}$$

In the zone II, since the fiber becomes an isothermal Newtonian fluid, we can use the solution of isothermal spinning process with Newtonian fluid. Neglecting the inertia term, we have

$$v_z(z) = \beta \exp(\alpha z) \tag{27}$$

We use boundary conditions such that

$$\begin{aligned} v_z(z_1) &= v_1 \\ v_z(z_2) &= v_2 \end{aligned} \tag{28}$$

Then the constants α and β are given by

$$\begin{aligned} \alpha &= \frac{\log(v_2/v_1)}{z_2 - z_1} = \frac{\log(v_2/v_1)}{w} \\ \beta &= v_1 \exp\left(-\frac{z_1 \log(v_2/v_1)}{w}\right) = v_1\left(\frac{v_2}{v_1}\right)^{-\frac{z_1}{w}} \end{aligned} \tag{29}$$

We have to determine the two velocity v_1 and v_2 in terms of measurable process variables. These quantities can be determined by continuity of stress. In the zone I, since fiber is elastic, we have

$$\sigma_{zz}(z) = E(T_0)e_{zz}(z) \tag{30}$$

where $E(T_0)$ is the Young's modulus at temperature T_0 and $e_{zz}(z)$ is the strain given by

$$e_{zz} = \int_0^t \dot{e}_{zz} dt = \int_0^z \dot{e}_{zz}\frac{dz}{v_z} = \int_0^z \frac{\log v_z}{dz} dz = \log\frac{v_z(z)}{v_0} \tag{31}$$

On the other hand, stress at $z=z_1$ and $z=z_2$ can be calculated in the zone II. Since the strain rate at the two positions are given by

$$\begin{aligned} \left.\frac{\partial v_z}{\partial z}\right|_{z=z_1} &= \frac{v_1}{w}\log\frac{v_2}{v_1} \\ \left.\frac{\partial v_z}{\partial z}\right|_{z=z_2} &= \frac{v_2}{w}\log\frac{v_2}{v_1} \end{aligned} \tag{32}$$

we have

$$\begin{aligned} \sigma_{zz}(z_1) &= \eta_E(T_H)\frac{v_1}{w}\log\frac{v_2}{v_1} \\ \sigma_{zz}(z_2) &= \eta_E(T_H)\frac{v_2}{w}\log\frac{v_2}{v_1} \end{aligned} \tag{33}$$

From eqs. (30) and (32), at $z=z_1$, we have

$$E(T_0)\log\frac{v_1}{v_0} = \eta_E(T_H)\frac{v_1}{w}\log\frac{v_2}{v_1} \tag{34}$$

Now, we consider the case that cooling device is so efficient that temperature of the zone III is nearly the ambient temperature, T_0. Thus, we can consider the fiber as elastic body again. Since the fiber is melted in the zone II, the strain should be

$$e_{zz}=\int_0^z \dot{e}_{zz}\frac{dz}{v_z}+e_{zz}^R=\log\frac{v_z(z)}{v_0}+e_{zz}^R \tag{35}$$

where e_{zz}^R is the residual strain that survives after fiber passes the zone II. Thus, we have another continuity equation for stress at $z=z_2$

$$E(T_0)e_{zz}^R = \eta_E(T_H)\frac{v_2}{w}\log\frac{v_2}{v_1} \tag{36}$$

We can measure the take-up force F at $z=L$, which is given by

$$\begin{aligned} F &= \pi R_L^2 E(T_0)\left(\log\frac{v_L}{v_2}+e_{zz}^R\right) \\ &= \pi\frac{v_0}{v_L}R_0^2 E(T_0)\left(\log\frac{v_L}{v_2}+e_{zz}^R\right) \end{aligned} \tag{37}$$

From eq. (37) we can determine the residual strain:

$$e_{zz}^R = \frac{F}{\pi R_0^2 E(T_0)}\frac{v_L}{v_0}-\log\frac{v_L}{v_2} \tag{38}$$

Substitution of eq. (38) into eq. (36) gives

$$\eta_E(T_H)\frac{v_2}{w}\log\frac{v_2}{v_1}=\frac{F}{\pi R_0^2}\frac{v_L}{v_0}-E(T_0)\log\frac{v_L}{v_2} \tag{39}$$

Now we can determine the unknown velocities v_1 and v_2. By combining eqs. (34) and (39), we have

$$\log\tilde{v}_2=\left(1+\frac{\xi}{\tilde{v}_1}\right)\log\tilde{v}_1 \tag{40}$$

$$\left(\frac{\tilde{v}_2}{\xi}-1\right)\log\tilde{v}_2=\frac{\tilde{v}_2}{\xi}\log\tilde{v}_1+\varepsilon_0\lambda_a-\log\lambda_a \tag{41}$$

where

$$\tilde{v}_i=\frac{v_i}{v_0}\ \text{for } i=1 \text{ and } 2 \tag{42}$$

$$\varepsilon_0\equiv\frac{F}{\pi R_0^2 E(T_0)} \tag{43}$$

$$\lambda_a\equiv\frac{v_L}{v_0} \tag{44}$$

$$\xi\equiv\frac{E(T_0)w}{\eta_E(T_H)v_0} \tag{45}$$

It is noteworthy that ε_0 corresponds to an average strain that holds along the fiber, ξ corresponds to the Deborah number which is the ratio of processing time scale to relaxation time of materials.

Combining eqs. (40) and (41), we have

$$f(\tilde{v}_1)=\phi \tag{46}$$

where

$$\phi\equiv\varepsilon_0\lambda_a-\log\lambda_a \tag{47}$$

$$f(\tilde{v}_1)=\left(\tilde{v}_1^{\xi/\tilde{v}_1}-\frac{\xi}{\tilde{v}_1}-1\right)\log\tilde{v}_1 \tag{48}$$

If $e_{zz}^R=\log(v_2/v_0)$ then we have

$$F=\pi E(T_0)R_L^2\log\frac{v_L}{v_0} \tag{49}$$

and $\phi=0$. Since to solve eq. (46) for $\phi\neq 0$ is difficult, we assume that $\phi=0$ because we guess ϕ is very close to zero. Then, the problem becomes simple:

$$\tilde{v}_1^{\xi/\tilde{v}_1}=1+\frac{\xi}{\tilde{v}_1} \tag{50}$$

Results and Discussion

In the static process, we predicted that all data of draw ratio obtained at different conditions can be superposed on a single curve just as the linear viscoelastic data are superposed by the WLF shift factor. This kind of superposition is well known as the similarity transform or mechanical similarity in the society of mechanics. There are a lot of examples of the mechanical similarity in fluid mechanics, celestial mechanics and so on.[17,18] Eq. (16) can be obtained by dimensional analysis for the mechanical similarity, under the assumption that the draw ratio depends on the width and the speed of heat band, applied stress, viscosity of the specimen and temperature. Since the draw ratio is dimensionless and there are 4 basic dimensions (length, time, mass and temperature) and 5 governing variables mentioned above, Π-theorem leads that the draw ratio is a function of a single dimensional variable obtained by the combination of the 5 governing variables. Since we derived the eq. (16) by use of mechanics, we omit the derivation by the dimensional analysis.

We modified w/V_H into $1/a_V$ of eq. (18) because the eq. (16) was derived based on the assumption that heat transfer occurs only along the direction of thickness of the film (or radius of fiber). In fact, heat transfer along the drawing direction is also important because an immediate cooling at the boundary of heat band cannot be achieved in real system. However, eq. (16) is a good approximation because contributions from the magnitudes of the important variables are included in the equation. We expect that more sophisticate calculation should have the leading term. In eq. (16), hidden assumption is that the specimen thickness is nearly uniform in the heating zone. Although variation of the specimen thickness in the heating zone exists, it does not alter the functional form of eq. (16) too much. Thus, we believe that eq. (17) is very useful to identify the behavior of the static zone drawing process with small number of experiments.

From the help of eqs. (40) and (50), we find that the continuous model is independent of apparent draw ratio, because drawing occurs mainly in the heating zone and all deformation in the outside of the heating zone is elastic and recoverable. Since heating and cooling in this model are so immediate, inherent draw ratio defined by $\lambda \equiv v_2/v_1$ is independent of both the position of the heating zone and the distance between the two rollers as well as of the apparent draw ratio λ_a. However, we need to notice that the inherent draw ratio, λ can not be greater than the apparent draw ratio λ_a. The condition $\lambda > \lambda_a$ implies that disconnection of fiber after passing the heating zone.

Eqs. (40) and (50) imply that the inherent draw ratio is a function of only the dimensionless parameter ξ which is a ratio of time scale of heating w/v_0 to that of material properties $\eta_E(T_H)/E(T_0)$. This number is the normalized time for extension in the heating zone where fiber behaves like fluid. The normalization, of course, implies the one for materials and processing conditions.

The symbols in Figure 3 are solutions of (40) and (50) with changing the value of v_0, and the line is the result of the regression analysis. Figure 3 shows the inherent draw ratio

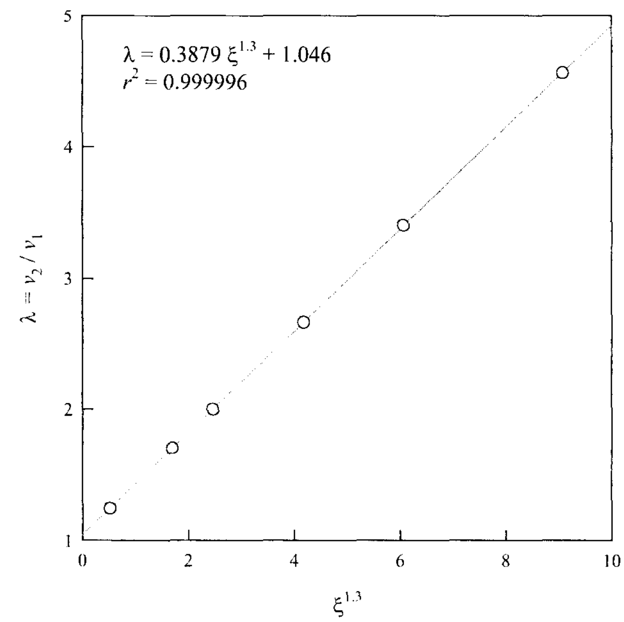

Figure 3. The inherent draw ratio as a function of the dimensionless parameter ξ. The calculation (the symbols) was done by use of following assumed data: $E(T_0) = 10^9$ Pa; $\eta_e(T_H) = 10^5$ Pa·sec; $w = 1$ cm. The line indicates the regression results from the calculated data.

as a function of the dimensionless parameter ξ. This graph implies that λ is a linear function of $\xi^{1.3}$. Including the numerical errors, we can guess that the linear function must be

$$\lambda = 0.39\xi^{1.3} + 1 \tag{51}$$

Since drawing without disconnection or other instability must satisfy $\lambda \leq \lambda_a$, we can say that acceptable processing condition should be

$$\xi \leq \xi_c \equiv \left(\frac{\lambda_a - 1}{0.39}\right)^{\frac{1}{1.3}} \tag{52}$$

where ξ is the critical Deborah number. By using eq. (45), it is straightforward that we can rewrite eq. (52) in terms of processing variables as well as material variables.

Although our model is useful to find acceptable process window with small number of experiments, it is impossible to analyze the stability conditions when the roller speed is increased too high for high production rate. As the roller speed increases, it is clear that there must be an instability such as draw resonance because of vibration of roller, nonlinearity of material properties and so on. In order to analyze the high speed process, we need more sophisticate constitutive equations for each zone as well as more detailed analysis for heat transfer.

Conclusions

Depending on the mechanism of the process, two mathematical models for zone drawing process have been developed in terms of processing condition and material properties. The mathematical models can predict the behavior of the processes and give a guideline for the limit of acceptable conditions and a tool for analysis of experimental data.

Most papers on the relationship between physical properties of zone-drawn polymer and process conditions, lead to the result that the physical properties mainly depends on the orientation of polymer chains or crystals and the orientation is a function of draw ratio. However, they did not find a simple relationship between the draw ratio and process conditions. Our model for the static zone drawing provides the universal behavior of draw ratio, which puts all data of draw ratio measured at different conditions on a single curve. It can be called stress-temperature-heat band speed superposition.

In the continuous zone drawing, it is important to find an acceptable processing condition that must be changed as different materials are used. Our model for the continuous process provides the limit of the acceptable processing condition. Furthermore, the model also provides an insight that the inherent draw ratio can be expressed in a simple form, other words, as a function of a dimensionless parameter called Deborah number defined in this study.

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